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# Numerical results using the conforming VEM for the convection-diffusion-reaction equation with variable coefficients.

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## Abstract

This document presents the results of a set of preliminary numerical experiments using several possible *conforming virtual element approximations* of the convection-reaction-diffusion equation with variable coefficients.

*Key words:* High-order method, unstructured polygonal mesh, virtual element method, diffusion, convection-dominated, reaction problem

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## 1. Virtual formulations for variable coefficients

The formulations considered herein are described in a companion paper entitled:

The Conforming Virtual Element Method for the convection-diffusion-reaction equation with variable coefficients

by the same Authors. All these formulations may achieve any order of accuracy in the diffusive regime. In particular, we document the behavior of these methods for the case of constant and variable coefficients. At present, we may distinguish between two different kind of conforming implementations: the *internal* and the *external* formulation. The difference between the two is in the way the projection operators act on the derivatives (laplacian, gradient) of the bilinear forms of the weak formulation of the partial differential equation. For the convection-dominated case, the streamline diffusion stabilization (aka SUPG) is also considered. Nonetheless, the stabilization term  $s_h^E(\underline{\phi}, \underline{\phi}^T)$  is present only when the scheme is working in the convection-dominated regime. We consider the following three possible formulations. Variants can be designed by combining differently the projection operators.

(i) The **external formulation**; the local bilinear forms and the right-hand side functional are given by:

$$A_h^E(\underline{\phi}, \underline{\phi}^T) := a_h^E(\underline{\phi}, \underline{\phi}^T) + b_h^E(\underline{\phi}, \underline{\phi}^T) + c_h^E(\underline{\phi}, \underline{\phi}^T) + s_h^E(\underline{\phi}, \underline{\phi}^T),$$

with

$$\begin{aligned}
a_h^E(\underline{\phi}, \underline{\phi}^T) &:= \int_E K \Pi_{k-1}^0(\nabla \underline{\phi}) \cdot \Pi_{k-1}^0(\nabla \underline{\phi}^T) dV + \text{stab}[a_h^E], \\
\text{stab}[a_h^E] &:= \text{Trace}(K) h_E^{d-2} (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi}), \\
b_h^E(\underline{\phi}, \underline{\phi}^T) &:= \int_E \boldsymbol{\beta} \cdot \Pi_{k-1}^0(\nabla \underline{\phi}) \Pi_k^0(\underline{\phi}^T) dV, \\
c_h^E(\underline{\phi}, \underline{\phi}^T) &:= \int_E \Pi_k^0(\underline{\phi}) \Pi_k^0(\underline{\phi}^T) dV, \\
s_h^E(\underline{\phi}, \underline{\phi}^T) &:= \tau_E \int_E [\boldsymbol{\beta} \cdot \Pi_{k-1}^0(\nabla \underline{\phi})] [\boldsymbol{\beta} \cdot \Pi_{k-1}^0(\nabla \underline{\phi}^T)] dV + \tau_E \text{stab}[s_h^E], \\
\text{stab}[s_h^E] &:= |\boldsymbol{\beta}|^2 h_E^{d-2} (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi}),
\end{aligned}$$

and

$$(f_h, \underline{\phi}^T)_E := \int_E f_h (\underline{\phi}^T + \boldsymbol{\beta} \cdot \Pi_k^0(\underline{\phi}^T)) dV$$

(ii) The **internal formulation using  $\Pi_k^\nabla$** ; local bilinear forms and right-hand side functional are given by:

$$A_h^E(\underline{\phi}, \underline{\phi}^T) := a_h^E(\underline{\phi}, \underline{\phi}^T) + b_h^E(\underline{\phi}, \underline{\phi}^T) + c_h^E(\underline{\phi}, \underline{\phi}^T) + s_h^E(\underline{\phi}, \underline{\phi}^T),$$

with

$$\begin{aligned}
a_h^E(\underline{\phi}, \underline{\phi}^T) &:= a_h^{E,0}(\underline{\phi}, \underline{\phi}^T) + \text{stab}[a_h^E] \\
a_h^{E,0}(\underline{\phi}, \underline{\phi}^T) &:= \begin{cases} \int_E K \nabla \Pi_k^\nabla(\underline{\phi}) \cdot \nabla \Pi_k^\nabla(\underline{\phi}^T) dV & [\text{constant } K] \\ \int_E \nabla \tilde{\Pi}^{\nabla, \phi}(\underline{\phi}) \cdot \nabla \Pi_k^\nabla(\underline{\phi}^T) dV & [\text{variable } K] \end{cases}
\end{aligned}$$

$$\text{stab}[a_h^E] := \text{Trace}(K) h_E^{d-2} (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi}),$$

$$b_h^E(\underline{\phi}, \underline{\phi}^T) := \int_E [\boldsymbol{\beta} \cdot \nabla \Pi_k^\nabla(\underline{\phi})] \Pi_k^0(\underline{\phi}^T) dV,$$

$$c_h^E(\underline{\phi}, \underline{\phi}^T) := \int_E \Pi_k^0(\underline{\phi}) \Pi_k^0(\underline{\phi}^T) dV,$$

$$s_h^E(\underline{\phi}, \underline{\phi}^T) := \tau_E \int_E [\boldsymbol{\beta} \cdot \nabla \Pi_k^\nabla(\underline{\phi})] [\boldsymbol{\beta} \cdot \nabla \Pi_k^\nabla(\underline{\phi}^T)] dV + \tau_E \text{stab}[s_h^E],$$

$$\text{stab}[s_h^E] := |\boldsymbol{\beta}|^2 h_E^{d-2} (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \boldsymbol{\Pi}_k^{\nabla, \phi}),$$

and

$$(f_h, \underline{\phi}^T)_E := \int_E f_h (\underline{\phi}^T + \boldsymbol{\beta} \cdot \Pi_k^0(\underline{\phi}^T)) dV.$$

(iii) The **internal-external formulation**; local bilinear forms and right-hand side functional are given by:

$$A_h^E(\underline{\phi}, \underline{\phi}^T) := a_h^E(\underline{\phi}, \underline{\phi}^T) + b_h^E(\underline{\phi}, \underline{\phi}^T) + c_h^E(\underline{\phi}, \underline{\phi}^T) + s_h^E(\underline{\phi}, \underline{\phi}^T)$$

with

$$\begin{aligned}
a_h^E(\underline{\phi}, \underline{\phi}^T) &:= a_h^{E,0}(\underline{\phi}, \underline{\phi}^T) + \text{stab}[a_h^E] \\
a_h^{E,0}(\underline{\phi}, \underline{\phi}^T) &:= \begin{cases} \int_E \mathbf{K} \nabla \Pi_k^\nabla(\underline{\phi}) \cdot \nabla \Pi_k^\nabla(\underline{\phi}^T) dV & [\text{constant } \mathbf{K}] \\ \int_E \nabla \tilde{\Pi}^{\mathbf{K}\nabla}(\underline{\phi}) \cdot \nabla \Pi_k^\nabla(\underline{\phi}^T) dV & [\text{variable } \mathbf{K}] \end{cases} \\
\text{stab}[a_h^E] &:= \text{Trace}(\mathbf{K}) h_E^{d-2} (\mathbf{I} - \mathbf{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \mathbf{\Pi}_k^{\nabla, \phi}), \\
b_h^E(\underline{\phi}, \underline{\phi}^T) &:= \int_E [\boldsymbol{\beta} \cdot \Pi_{k-1}^0(\nabla \underline{\phi})] \Pi_k^0(\underline{\phi}^T) dV, \\
c_h^E(\underline{\phi}, \underline{\phi}^T) &:= \int_E \Pi_k^0(\underline{\phi}) \Pi_k^0(\underline{\phi}^T) dV, \\
s_h^E(\underline{\phi}, \underline{\phi}^T) &:= \tau_E \int_E [\boldsymbol{\beta} \cdot \nabla \Pi_k^{\boldsymbol{\beta}}(\underline{\phi})] [\boldsymbol{\beta} \cdot \nabla \Pi_k^{\boldsymbol{\beta}}(\underline{\phi}^T)] dV + \tau_E \text{stab}[s_h^E], \\
\text{stab}[s_h^E] &:= |\boldsymbol{\beta}|^2 h_E^{d-2} (\mathbf{I} - \mathbf{\Pi}_k^{\nabla, \phi})^T (\mathbf{I} - \mathbf{\Pi}_k^{\nabla, \phi}),
\end{aligned}$$

and

$$(f_h, \underline{\phi}^T)_E := \int_E f_h (\underline{\phi}^T + \boldsymbol{\beta} \cdot \Pi_k^0(\underline{\phi}^T)) dV$$

This formulation differs from the previous ones because we use the internal (elliptic) projection in  $a_h^E$  and the external projection for the convection term. Again, the bilinear form  $a_h^E$  has a different definition for constant and variable diffusion tensors  $\mathbf{K}$ .

## 2. Convergence behavior in diffusive regime

### 2.1. Test Cases

To assess the convergence behavior of the possible formulations presented in the previous section, we consider the convergence (in the diffusive regime) for the following combinations of the equation's coefficients.

#### Constant Coefficients:

- *Test Case A*:  $\mathbf{K} = \mathbf{I}$ ,  $\boldsymbol{\beta} = (1, 1)^T$ ,  $c = 1$ ;
- *Test Case B*:  $\mathbf{K} = \mathbf{I}$ ,  $\boldsymbol{\beta} = 0$ ,  $c = 0$ ;
- *Test Case C*:  $\mathbf{K} = \mathbf{I}$ ,  $\boldsymbol{\beta} = (1, 1)^T$ ,  $c = 0$ ;
- *Test Case D*:  $\mathbf{K} = \mathbf{I}$ ,  $\boldsymbol{\beta} = 0$ ,  $c = 1$ ;

#### Variable Coefficients:

Now, let

$$\mathbf{K}' = (x^2 + y^3 + 1) \exp(x + y) \mathbf{I}, \quad \boldsymbol{\beta}' = \begin{bmatrix} 2(-2y^2 - x + 1) \\ 3(3x^2 - 2y + 3) \end{bmatrix}, \quad c' = x^2 + y^3 + 1.$$

We consider the following combinations:

- *Test Case A*:  $\mathbf{K} = \mathbf{K}'$ ,  $\boldsymbol{\beta} = \boldsymbol{\beta}'$ ,  $c = c'$ ;
- *Test Case B*:  $\mathbf{K} = \mathbf{K}'$ ,  $\boldsymbol{\beta} = 0$ ,  $c = 0$ ;

- *Test Case C*:  $\mathbf{K} = \mathbf{K}'$ ,  $\boldsymbol{\beta} = \boldsymbol{\beta}'$ ,  $c = 0$ ;
- *Test Case D*:  $\mathbf{K} = \mathbf{K}'$ ,  $\boldsymbol{\beta} = 0$ ,  $c = c'$ ;

## 2.2. Meshes

We consider the following *six* meshes:

- *squares*:  $n \times n$  partitions of  $\Omega = [0, 1] \times [0, 1]$  with  $n = 5, 10, 20, 40, 80, 160$ ; Fig. 2 (bottom) shows the first two meshes of the sequence;
- *remapped quadrilaterals*: the mesh is partitioned as for the case of squares; then, the internal vertices are remapped from  $(\xi, \eta)$  to  $(x, y)$  by using the smooth coordinate transformation:

$$\begin{aligned} x &= \xi + w_x \sin(2\pi\xi) \sin(2\pi\eta), \\ y &= \eta + w_y \sin(2\pi\xi) \sin(2\pi\eta), \end{aligned}$$

where  $w_x = 0.075$  and  $w_y = 0.075$ ; Fig. 3 (top) shows the first two meshes of the sequence;

- *randomized quadrilaterals* mesh is partitioned as for regular mesh; then, the internal vertices are displaced randomly by:

$$\begin{aligned} x &= \xi - w_x h_x / 2 + h_x \text{double}(\text{random()}) / \text{double}(\text{RAND\_MAX}), \\ y &= \eta - w_y h_y / 2 + h_y \text{double}(\text{random()}) / \text{double}(\text{RAND\_MAX}), \end{aligned}$$

where  $w_x = 0.4$  and  $w_y = 0.4$ ,  $h_x = h_y = 1/n$ , and `random()` and `RAND_MAX` are C/C++ items (from standard library); Fig. 2 (top) shows the first two meshes of the sequence;

- *regular hexagons*: these meshes are built on regular partitions of  $n \times n$  cell with  $n = 5, 10, 20, 40, 80, 160$ ; Fig. 1 (top) shows the first two meshes of the sequence;
- *mainly hexagonal meshes*: each mesh built by taking the mesh of remapped quadrilaterals, dividing quadrilaterals into two triangles and dualizing the resulting triangular mesh by connecting the barycenters of the triangles; Fig. 1 (bottom) shows the first two meshes of the sequence;
- *the triangle-based criss-cross meshes*: each mesh is obtained by cutting each square cells into four subcell along the two diagonals simultaneously; Fig. 3 (bottom) shows the first two meshes of the sequence.

## 2.3. Exact solutions

We consider the following exact solutions:

- solution with zero trace on the boundary (no effects from boundary conditions):

$$u_1(x, y) = \sin(2\pi x) \sin(2\pi y);$$

- solution with zero forcing term:

$$u_2(x, y) = \sin(\pi x) \exp(\pi y);$$

- full solution with non-zero Dirichlet boundary and non-zero forcing term:

$$u_3(x, y) = \left( x - e^{\frac{2(x-1)}{\mathbf{K}_{xx}}} \right) \left( y^2 - e^{\frac{3(y-1)}{\mathbf{K}_{yy}}} \right)$$

where  $\mathbf{K}_{xx}$  and  $\mathbf{K}_{yy}$  are the diagonal entries of  $\mathbf{K}$ .

## 2.4. Approximation errors

In all the possible combinations, we define the approximation error as  $u - \Pi_k^0(u_h)$  where  $u$  is the exact solution (from the previous subsection),  $u_h$  is the numerical solution and  $\Pi_k^0(u_h)$  is the  $L^2$ -orthogonal projection onto the polynomials of degree up to  $k$ . The errors are measured by using the  $L^2$ -norm and the  $H^1$ -seminorm.

## 3. Streamline-diffusion stabilization

We consider the following data:

$$\mathbf{K} = \mathbb{I}, \quad \boldsymbol{\beta} = (1, 3)^T, \quad c = 0,$$

and the following boundary conditions:

$$u(x, y) = \begin{cases} 1 & x \leq 1/3, y = 0 \quad \text{or} \quad x = 0 \\ 0 & \text{otherwise.} \end{cases}$$

The solution of this test case shows a boundary layer on the top edge of  $\Omega$  for  $x = 1$  and  $y \leq 2/3$ , and an internal layer along the line connecting the points  $(1/3, 0)$  and  $(2/3, 0)$ .

### 3.1. Numerical results for the External VEM

- Fig. 364 shows the solution obtained by applying the External VEM to a mesh of quadrilateral without including any stabilization term. The Internal VEM produces a similar results (not shown here).
- Figs. 365-366 show the solution (back and front view) obtained by applying the External VEM to a mesh of quadrilateral including the streamline diffusion stabilization term for  $\tau = 0.15, 0.3, 0.45, 0.6 h_E/|\boldsymbol{\beta}|$ .
- Figs. 367-368 show the solution (back and front view) obtained by applying the External VEM to a mesh of quadrilateral including the streamline diffusion stabilization term for  $\tau = 0.15, 0.3, 0.45, 0.6 h_E/|\boldsymbol{\beta}|$ .
- Fig. 370 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  to a mesh of regular hexagons (left panels) and smoothly remapped hexagon (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 369 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  to a mesh of squares (left panels) and regular triangles (criss-cross) (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 371 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  to a mesh of smoothly remapped squares (left panels) and randomized squares (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 372 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  to a mesh of regular triangles (left and right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 373 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  and the stabilization of the stabilization with  $\tau' = 0.1 h_E/|\boldsymbol{\beta}|$  (left panel) and  $\tau' = 1.2 h_E/|\boldsymbol{\beta}|$  (right panel) to a square mesh. Back view (top panels) and front view (bottom panels) are shown.
- Fig. 374 compares the solutions obtained by applying the External VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E/|\boldsymbol{\beta}|$  and the stabilization of the stabilization with  $\tau' = 0.1 h_E/|\boldsymbol{\beta}|$  (left

panel) and  $\tau' = 1.2 h_E / |\beta|$  (right panel) to a mesh of regular hexagons. Back view (top panels) and front view (bottom panels) are shown.

### 3.2. Numerical results for the Internal VEM

- Fig. 376 compares the solutions obtained by applying the Internal VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E / |\beta|$  to a mesh of regular hexagons (left panels) and smoothly remapped hexagon (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 375 compares the solutions obtained by applying the Internal VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E / |\beta|$  to a mesh of squares (left panels) and regular triangles (criss-cross) (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 377 compares the solutions obtained by applying the Internal VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E / |\beta|$  to a mesh of smoothly remapped squares (left panels) and randomized squares (right panels). Back view (top panels) and front view (bottom panels) are shown.
- Fig. 378 compares the solutions obtained by applying the Internal VEM including the streamline diffusion stabilization term for  $\tau = 0.3 h_E / |\beta|$  to a mesh of regular triangles (left and right panels). Back view (top panels) and front view (bottom panels) are shown.

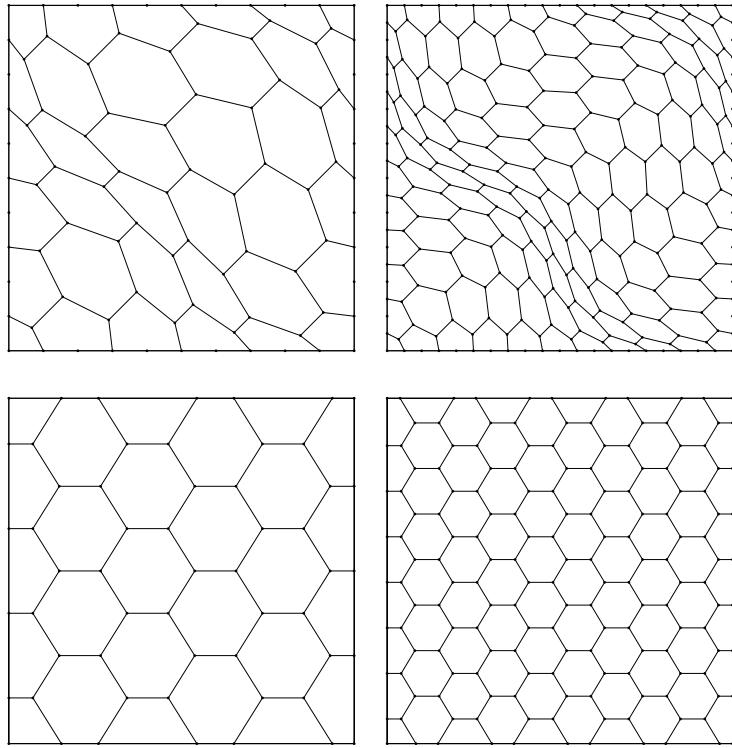


Fig. 1. First two meshes of the sequences of remapped hexagons (top) and regular hexagons (bottom).

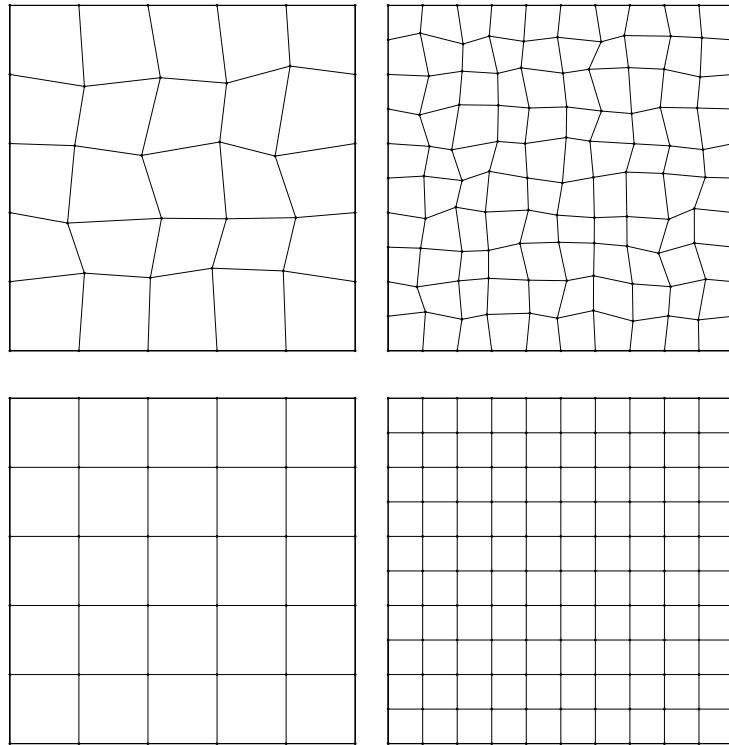


Fig. 2. First two meshes of the sequences of randomized quadrilaterals (top) and regular quadrilaterals (bottom).

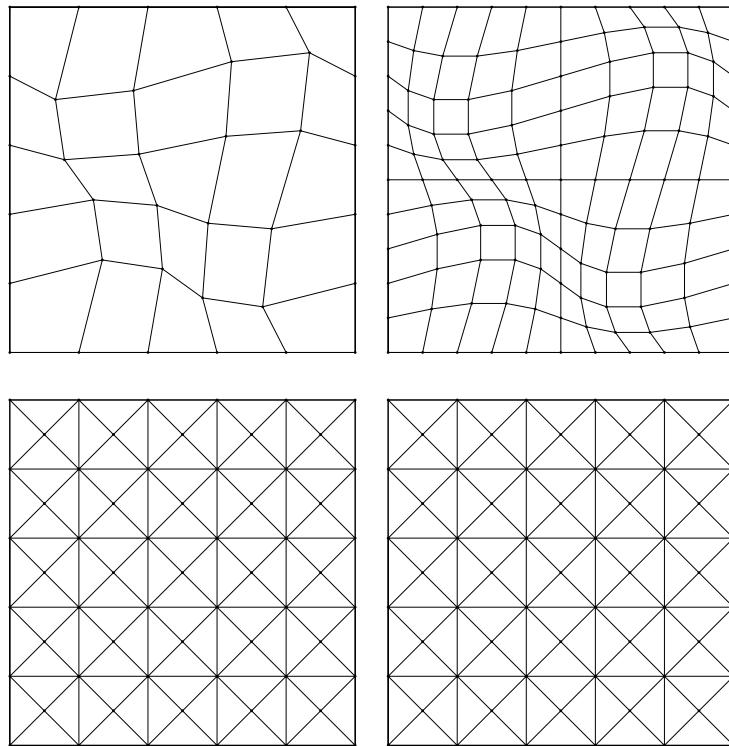


Fig. 3. First two meshes of the sequences of remapped quadrilaterals (top) and regular triangle-based (criss-cross) meshes (bottom).

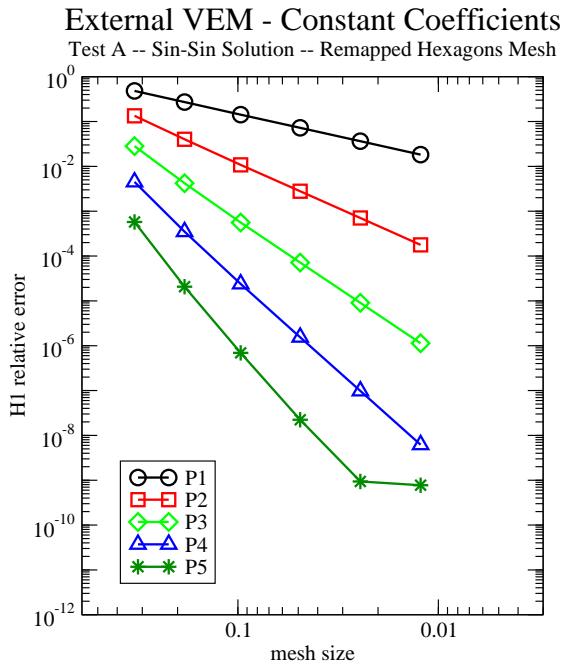
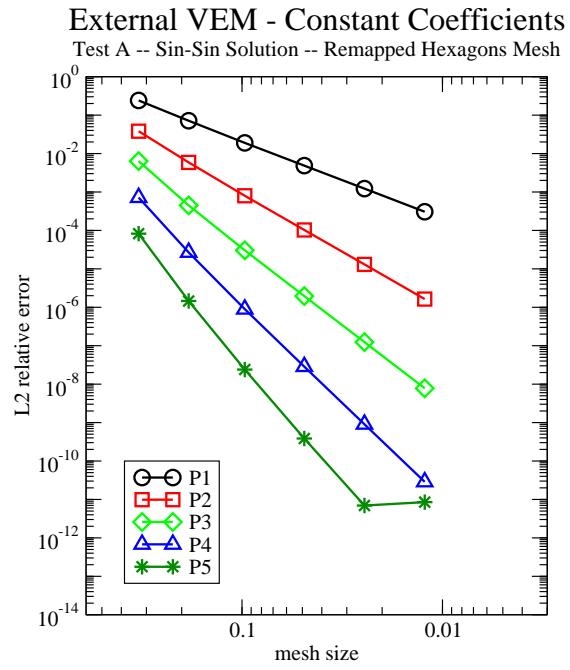


Fig. 4. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

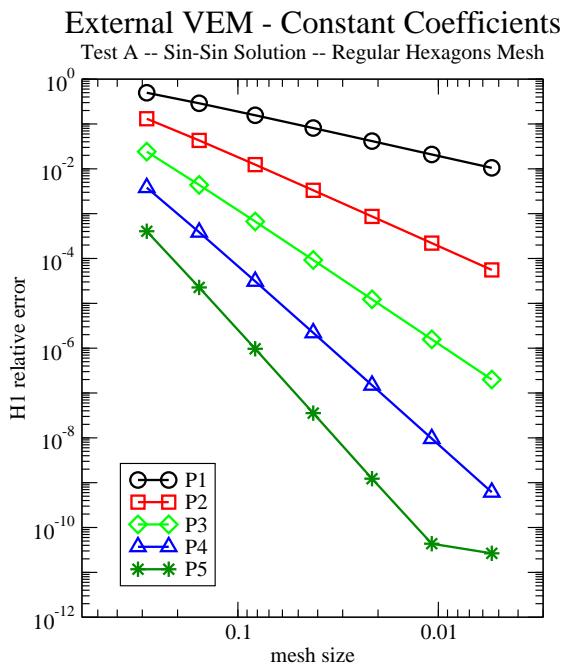
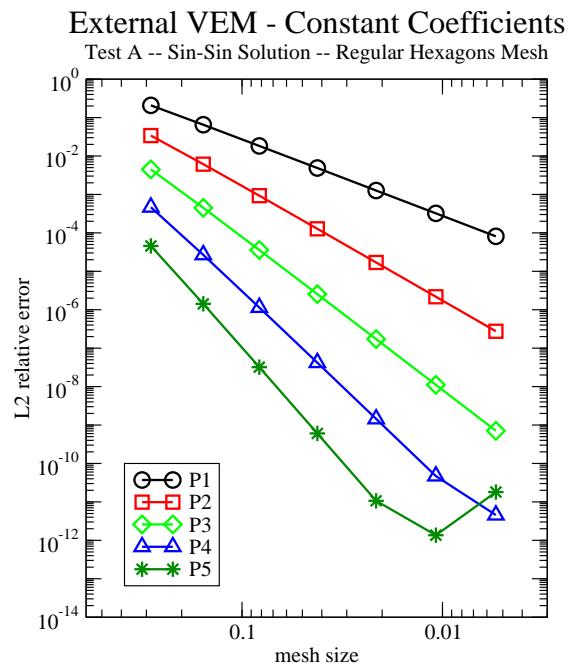


Fig. 5. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

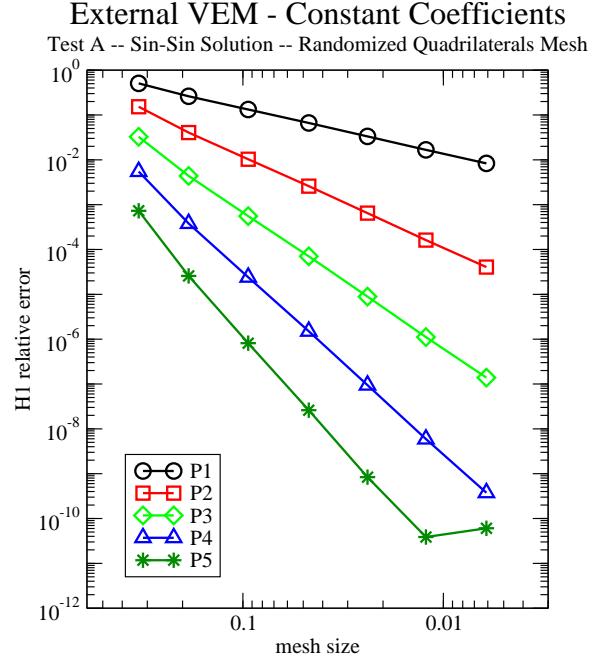
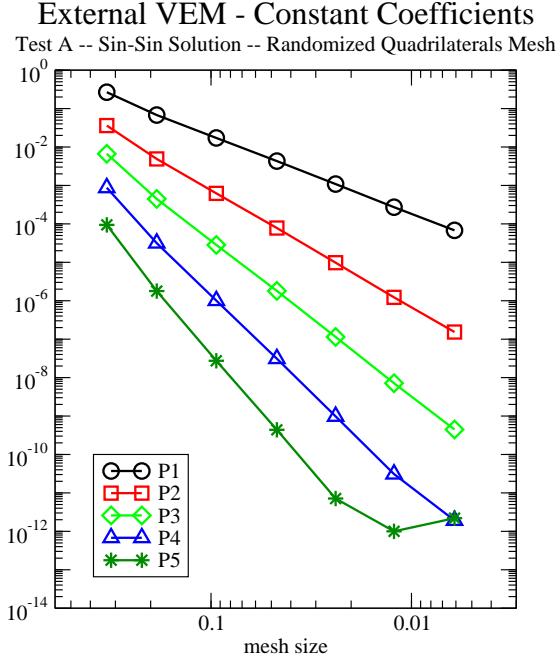


Fig. 6. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

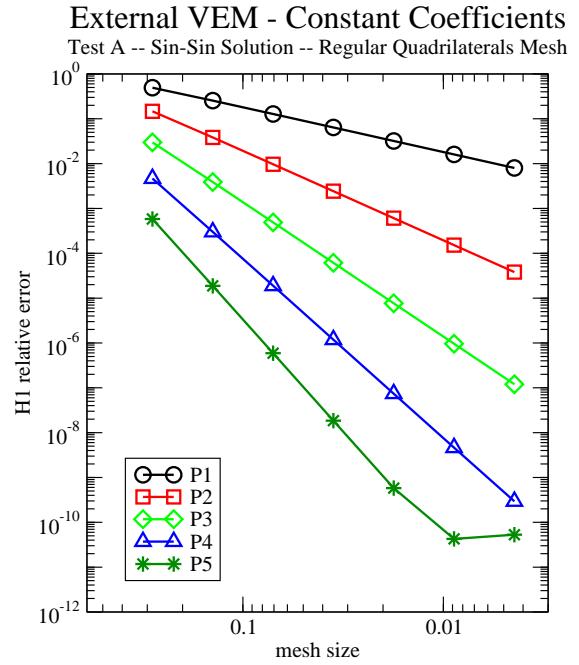
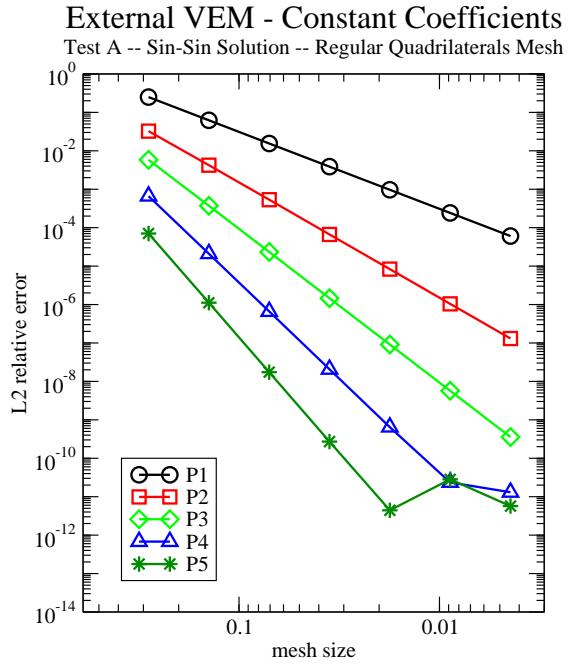


Fig. 7. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

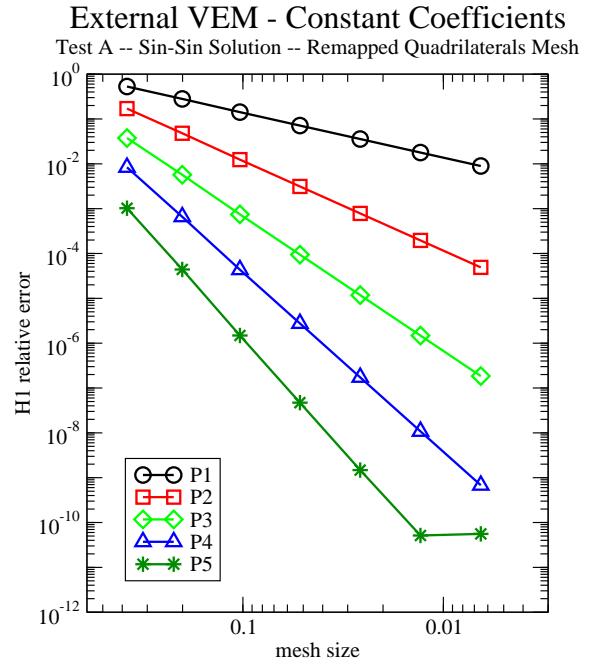
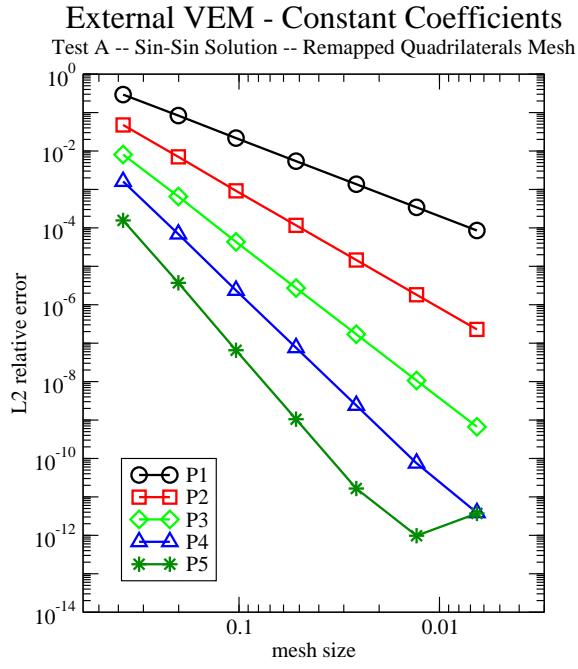


Fig. 8. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

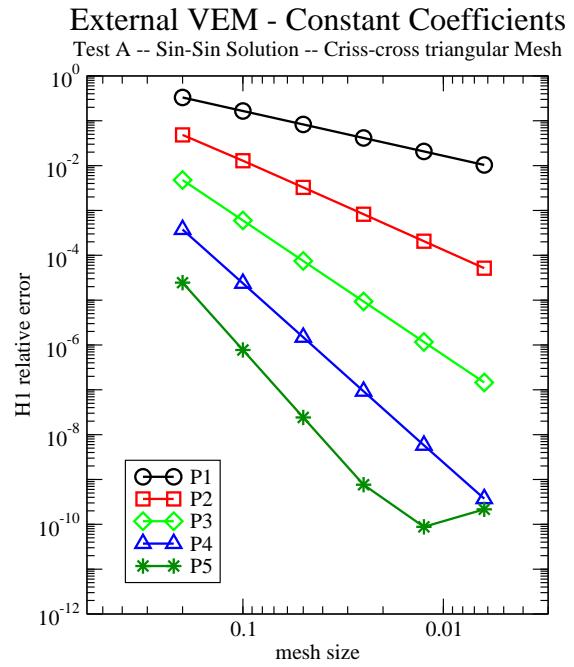
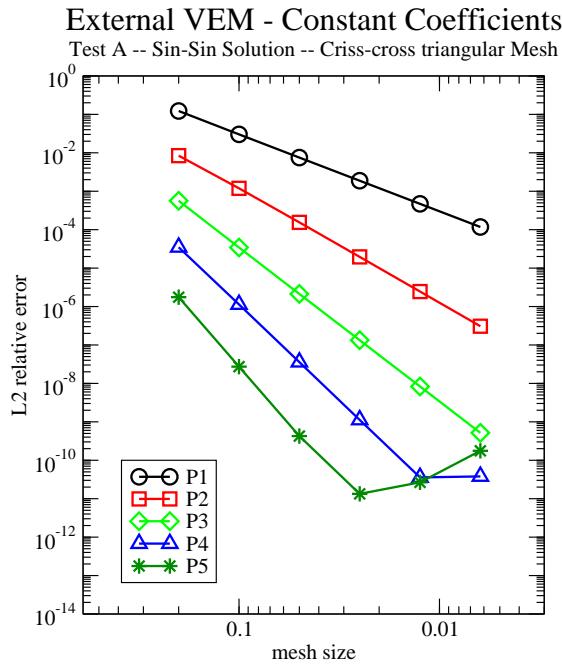


Fig. 9. External VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

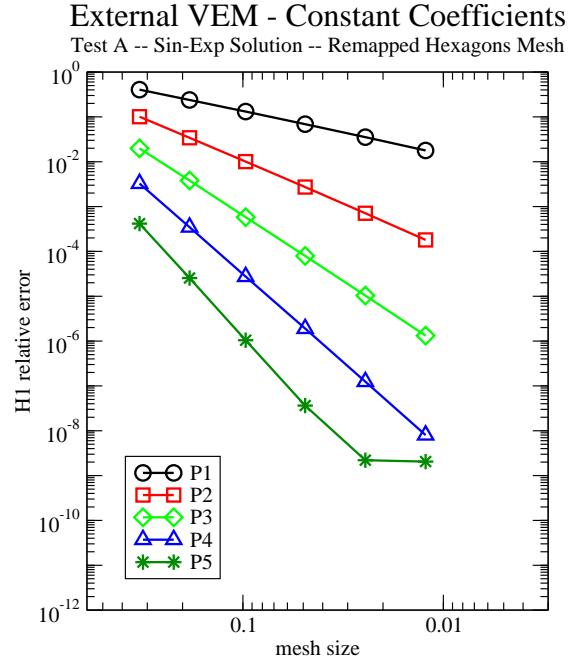
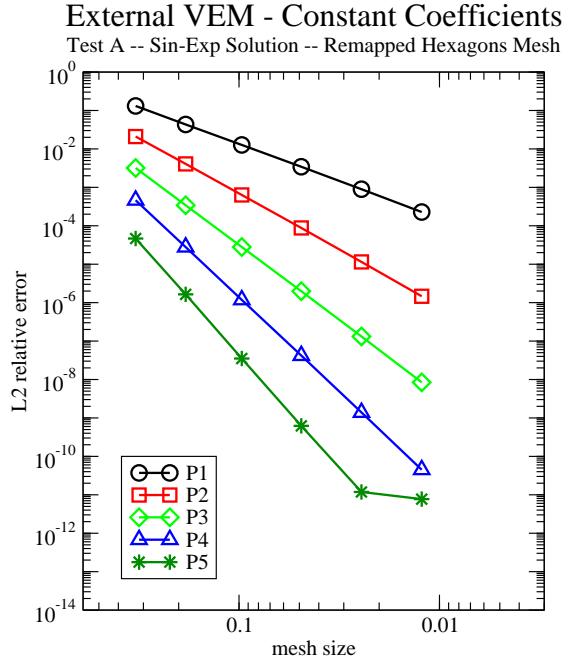


Fig. 10. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

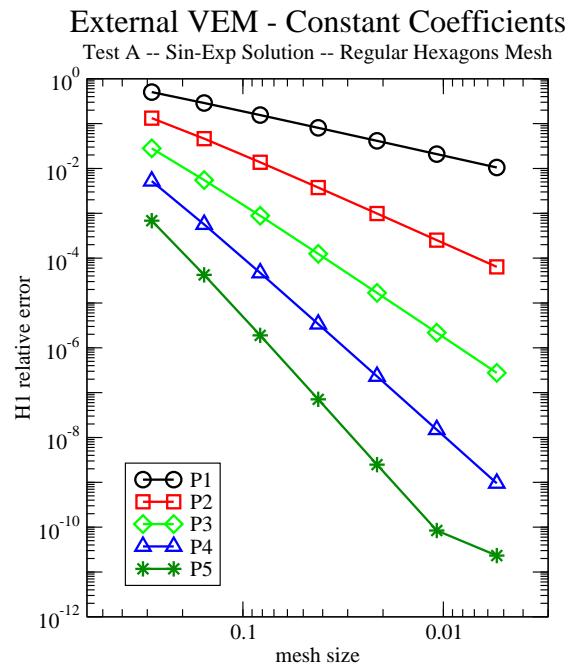
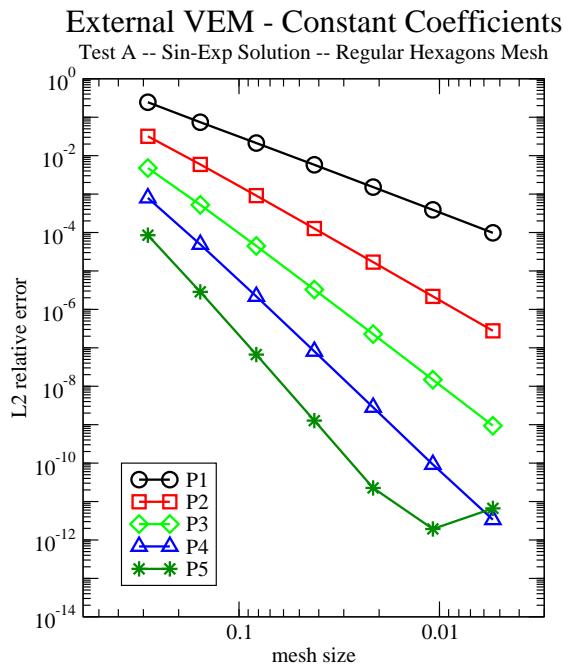


Fig. 11. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

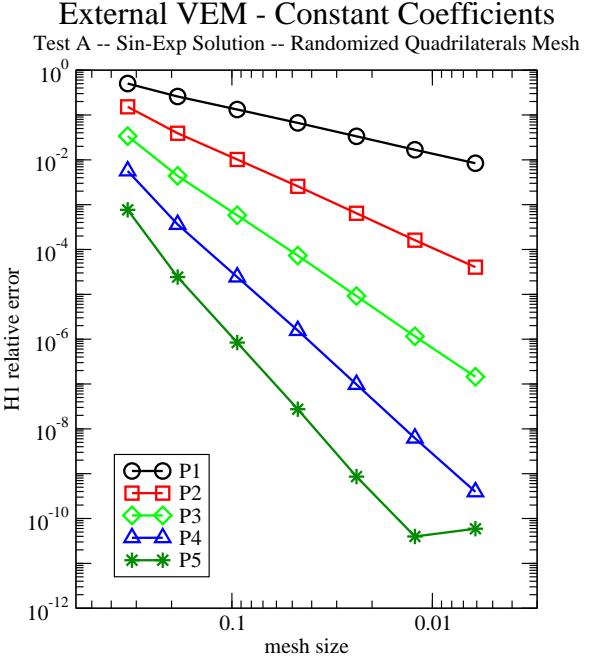
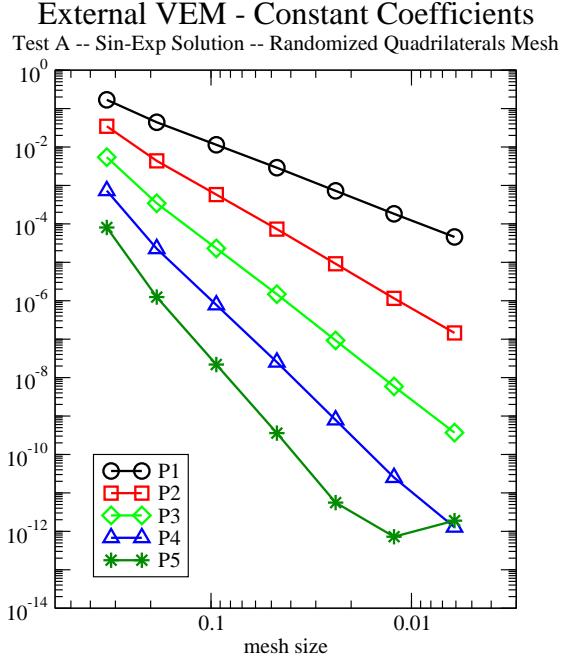


Fig. 12. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

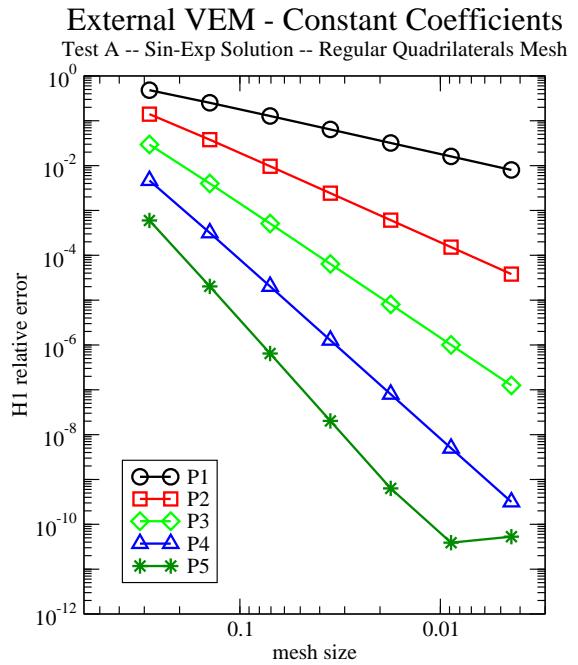
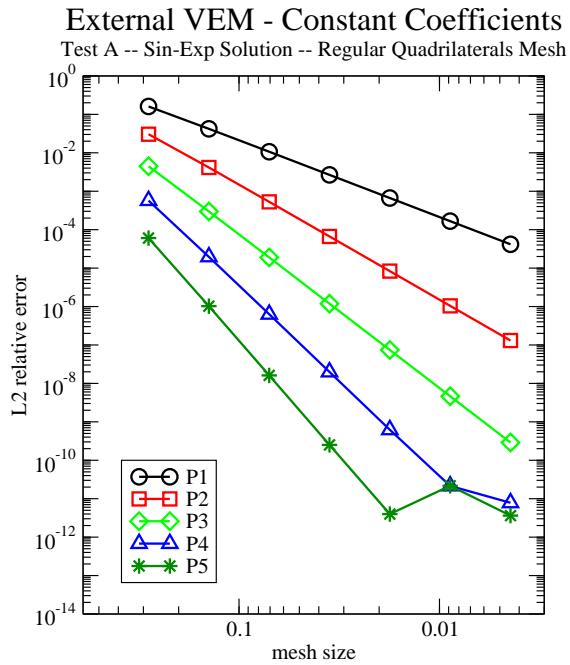


Fig. 13. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

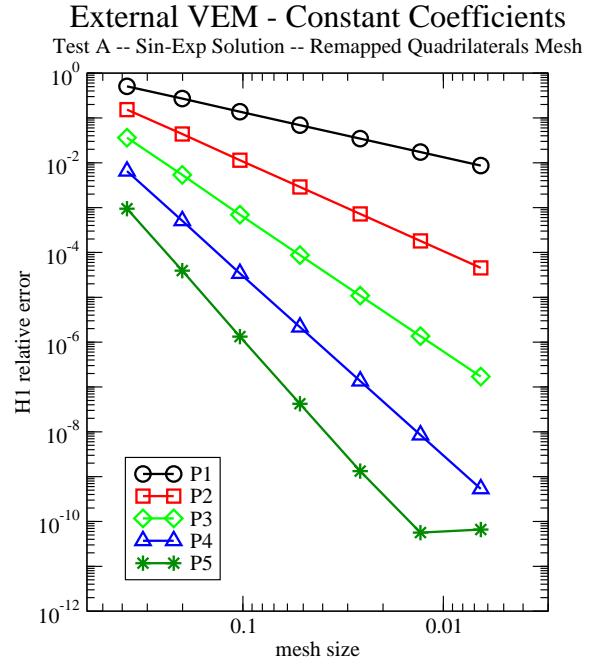
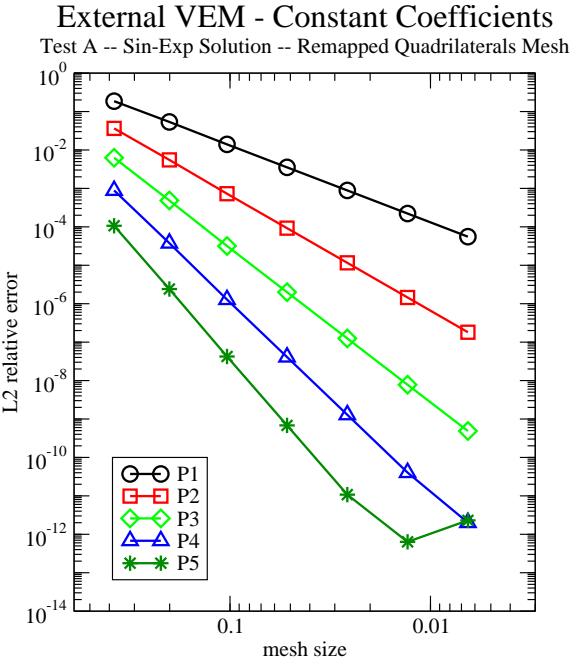


Fig. 14. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

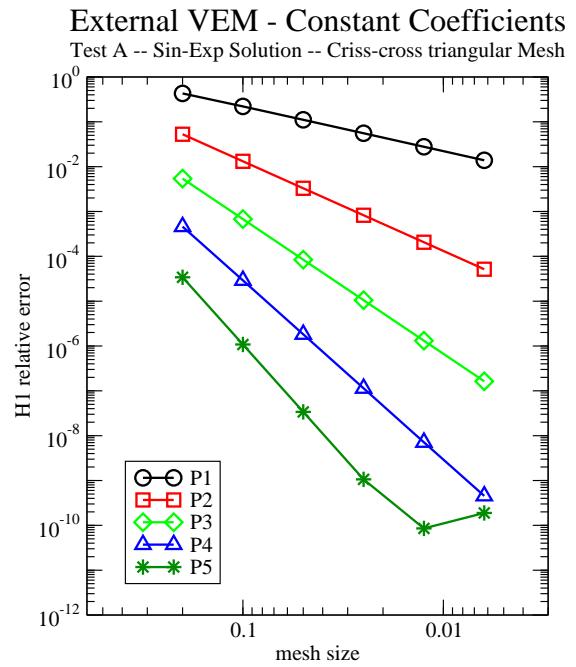
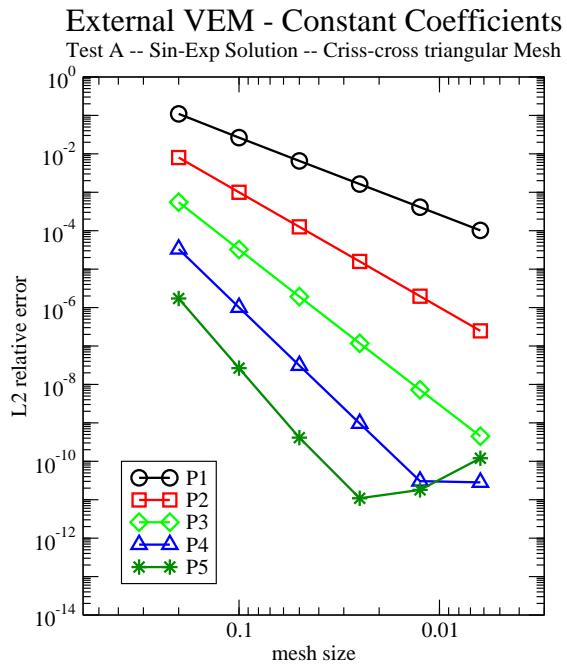


Fig. 15. External VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

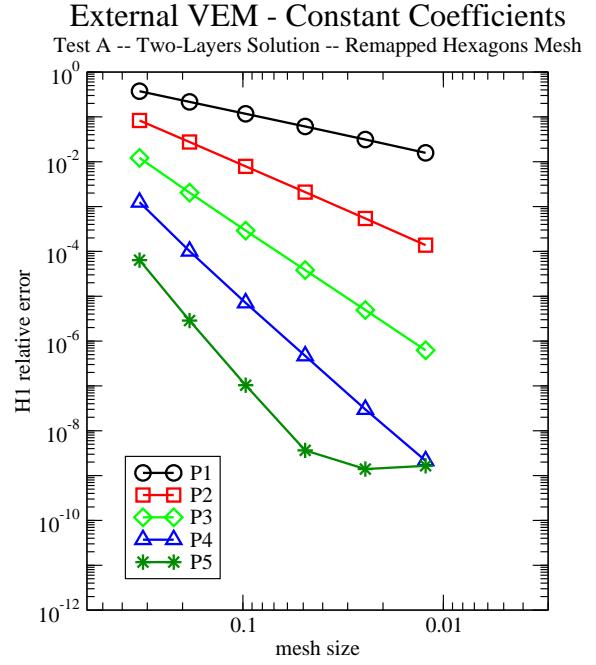
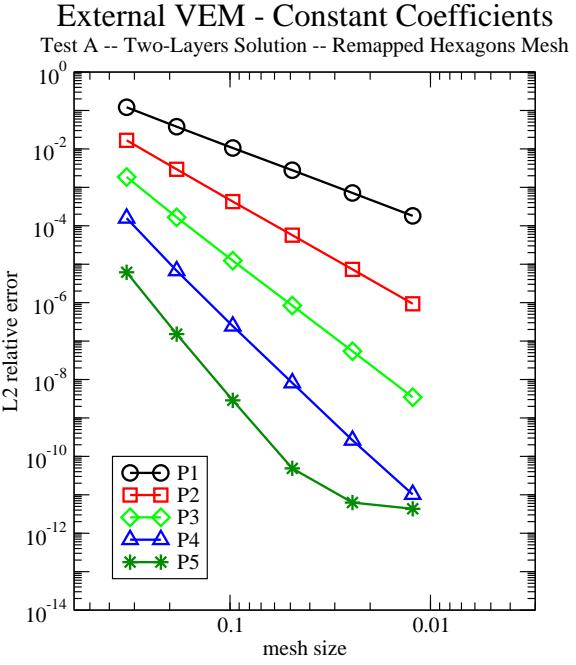


Fig. 16. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

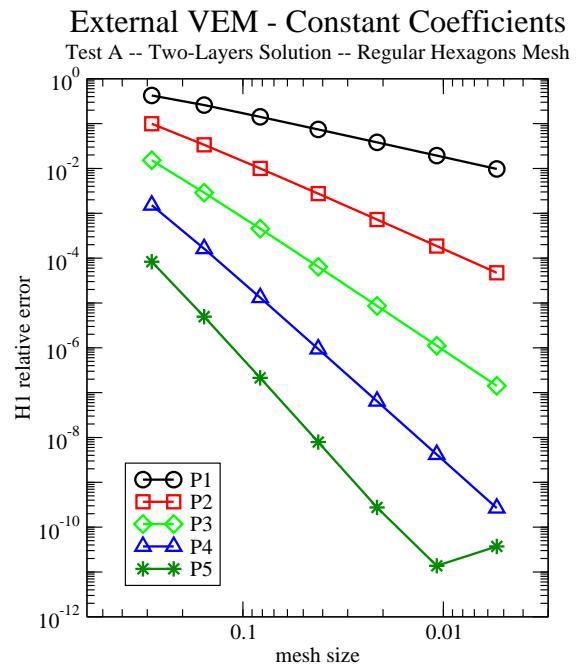
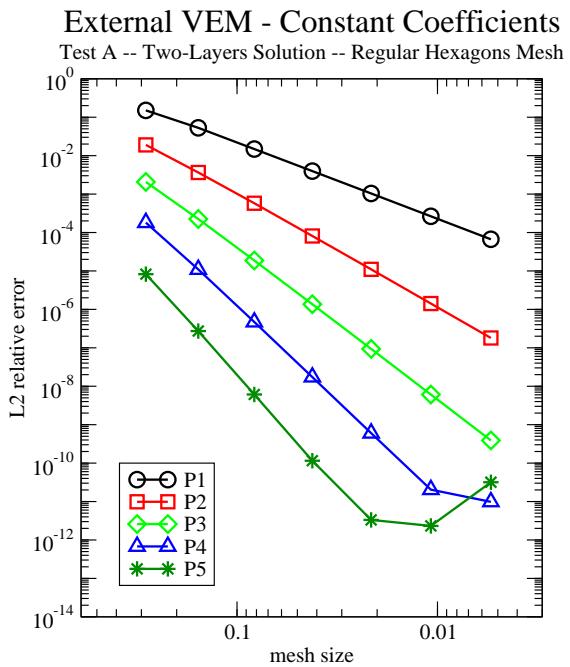


Fig. 17. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular hexagons.

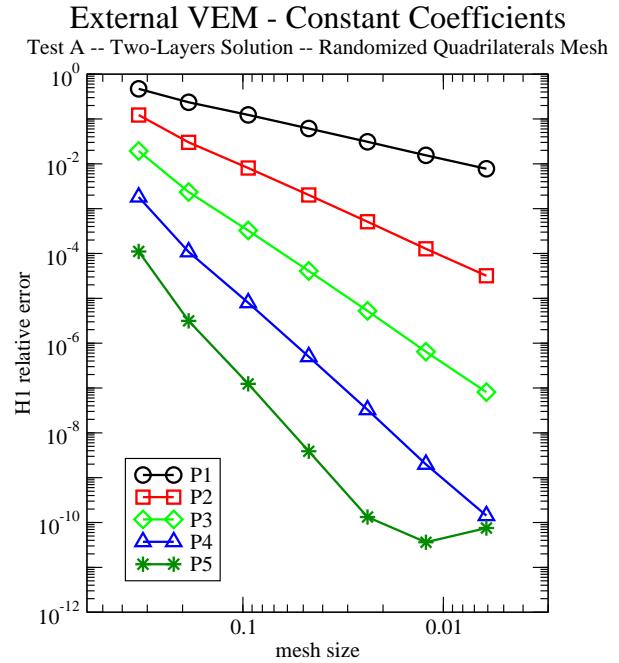
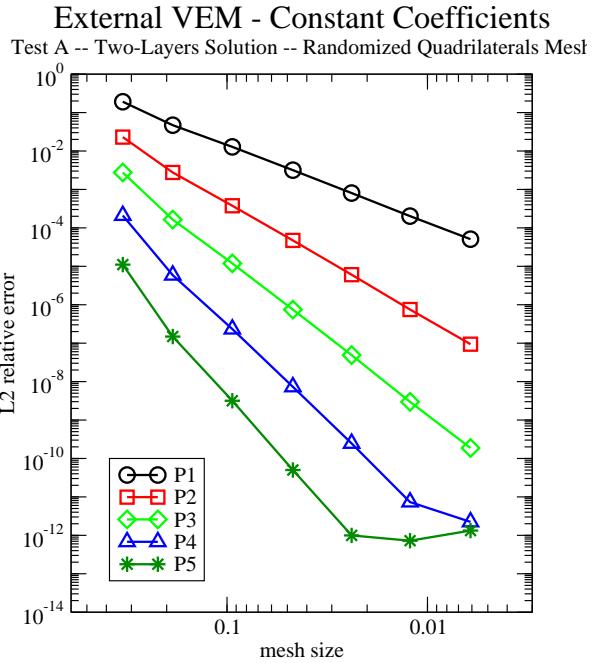


Fig. 18. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

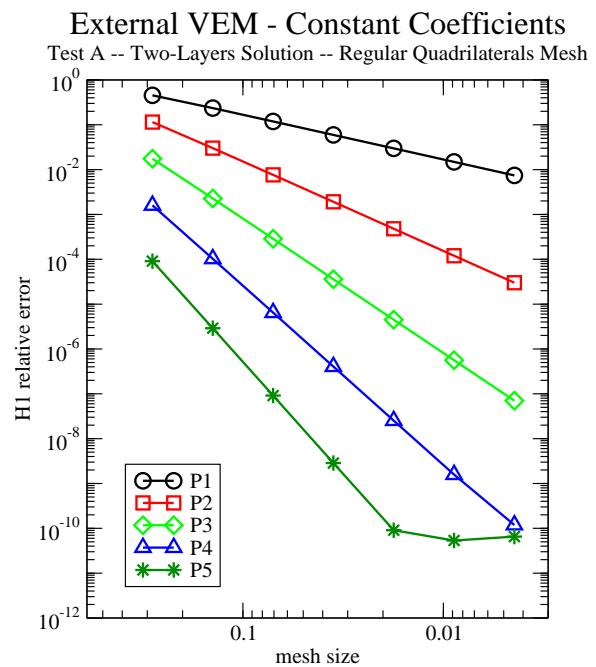
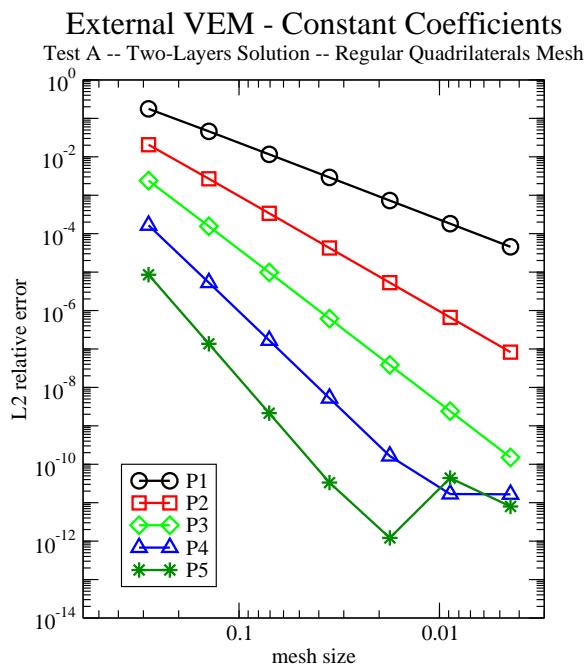


Fig. 19. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

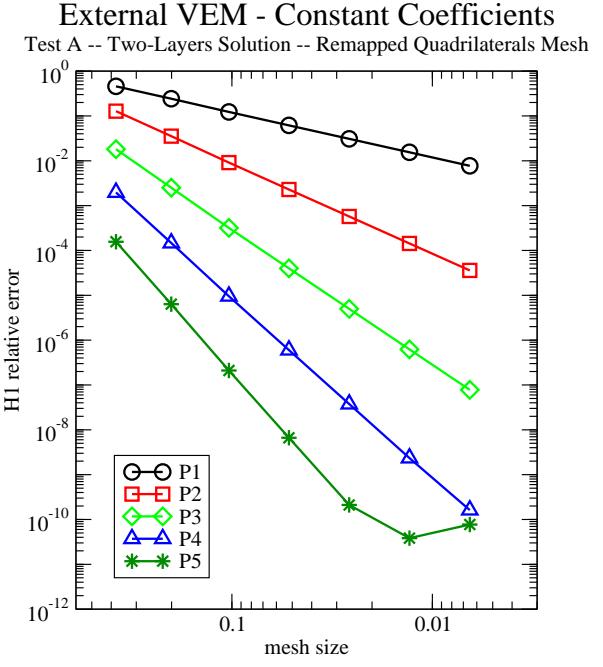
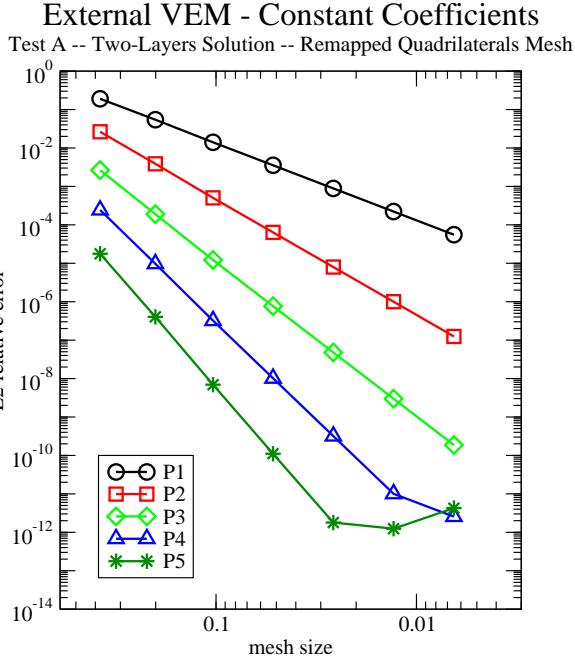


Fig. 20. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

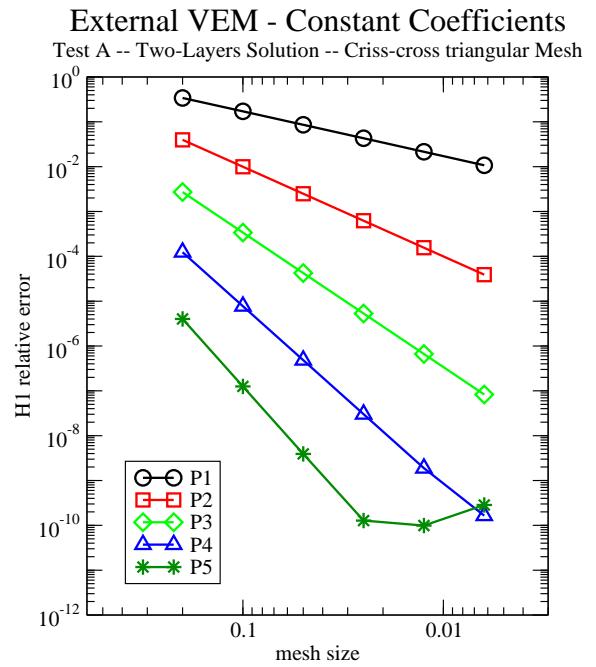
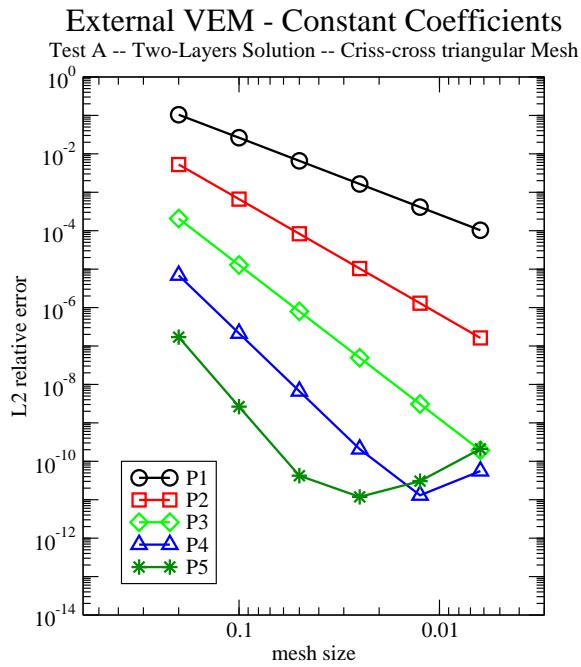


Fig. 21. External VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

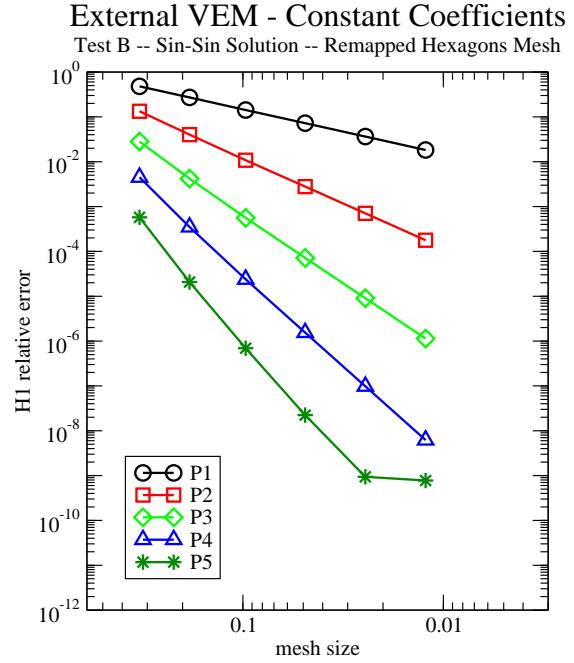
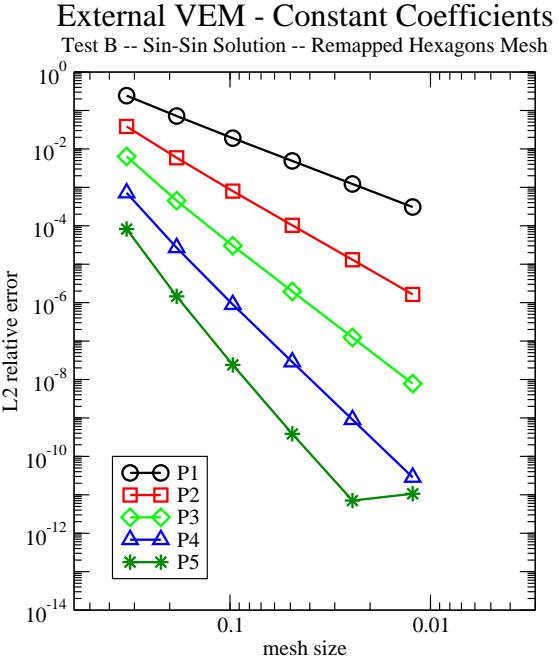


Fig. 22. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of smoothly remapped hexagons.

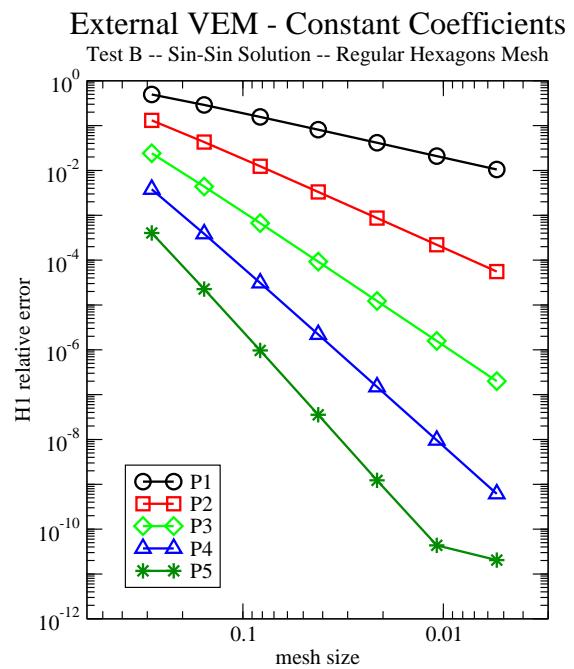
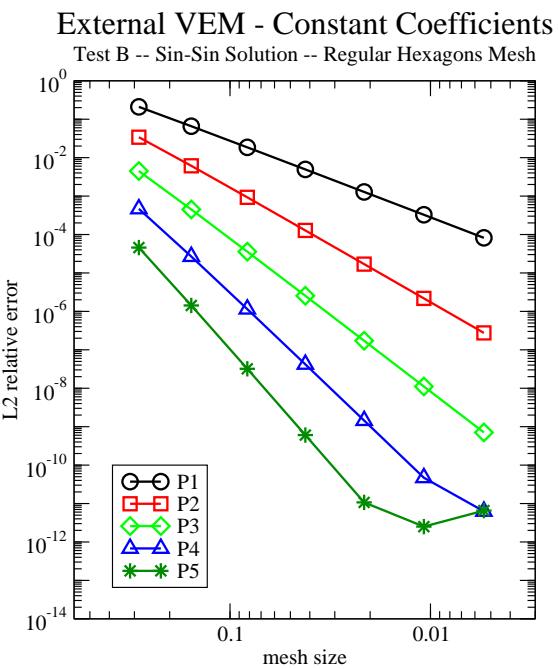


Fig. 23. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular hexagons.

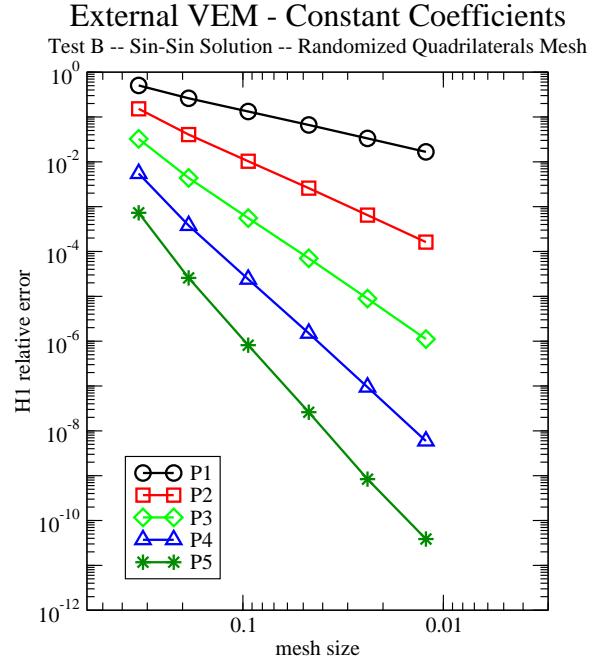
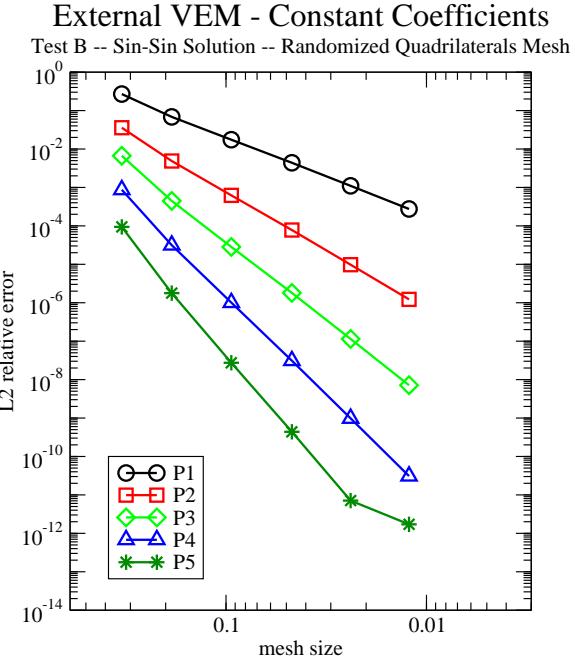


Fig. 24. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of randomized quadrilateral cells.

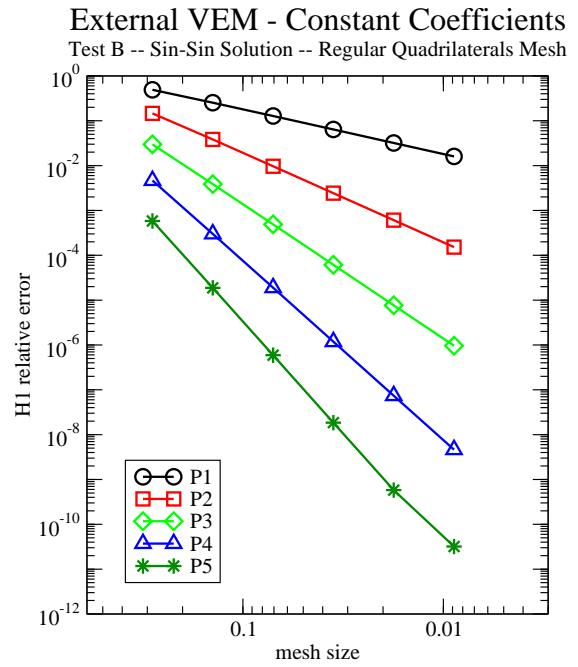
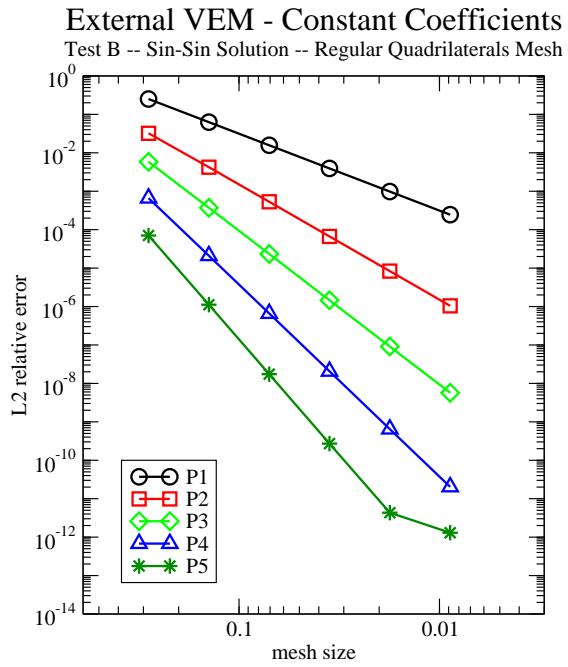


Fig. 25. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular quadrilateral cells (squares).

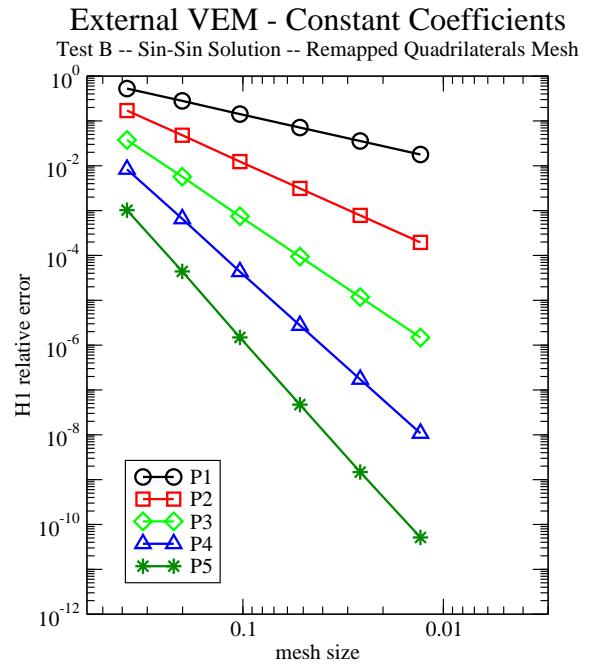
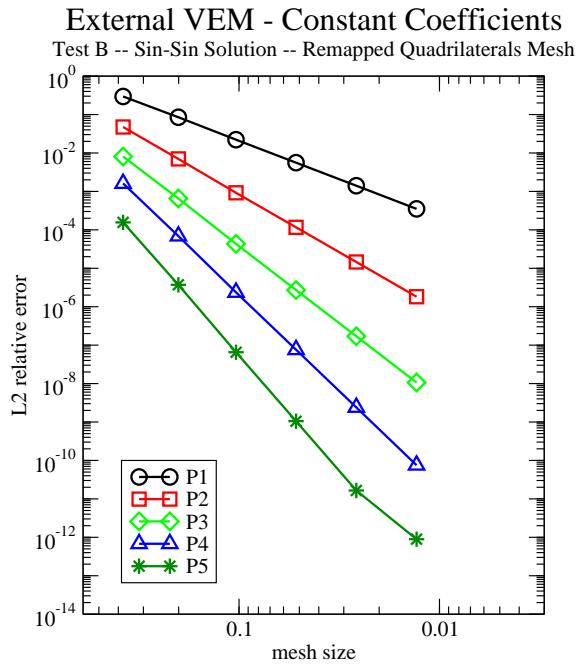


Fig. 26. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

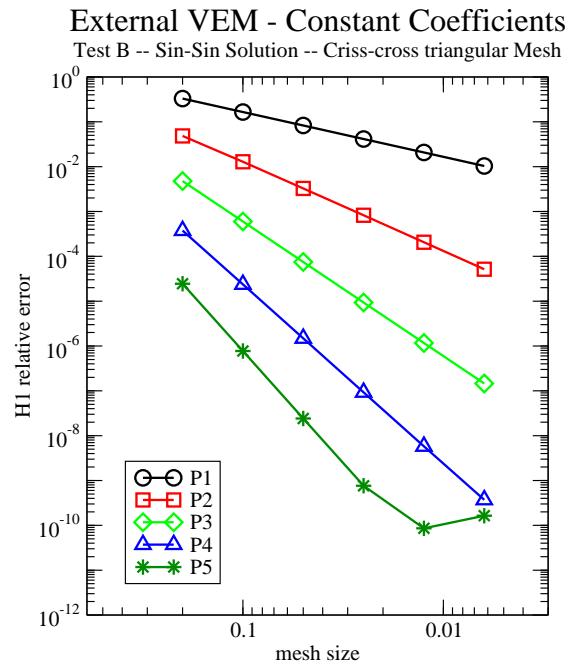
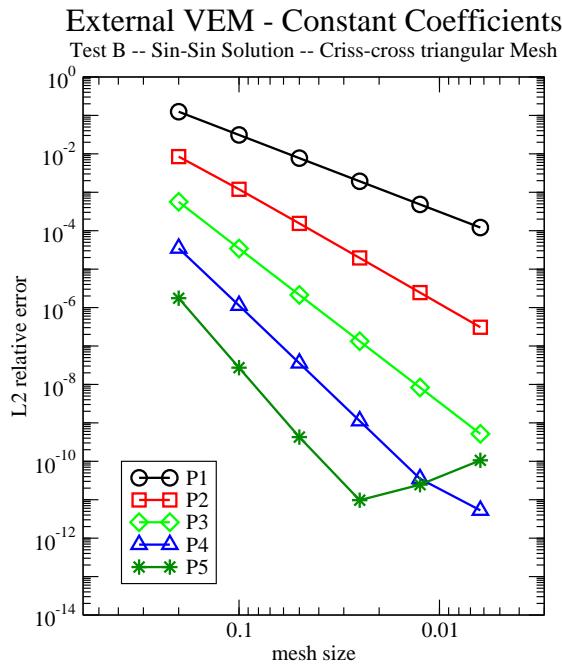


Fig. 27. External VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

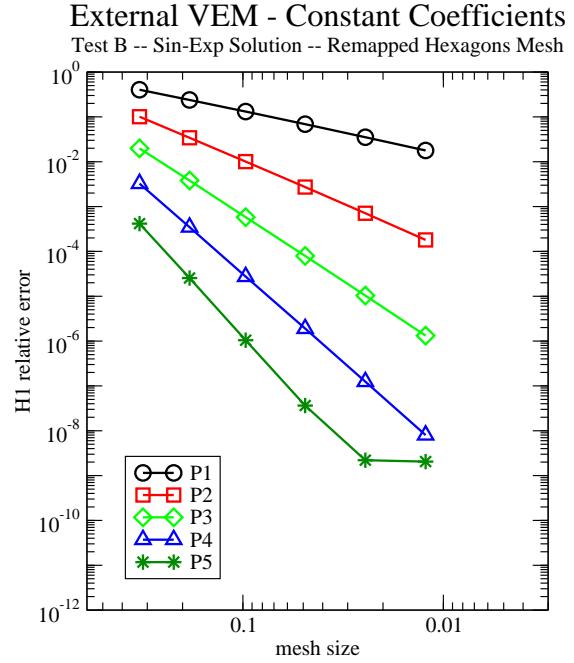
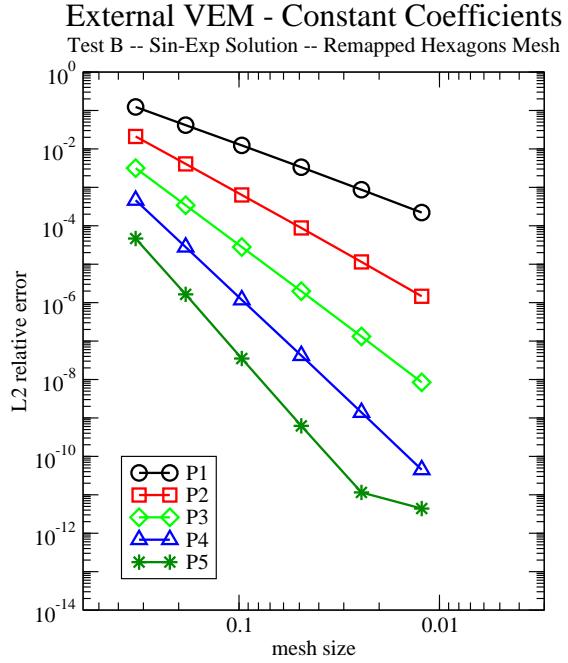


Fig. 28. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of smoothly remapped hexagons.

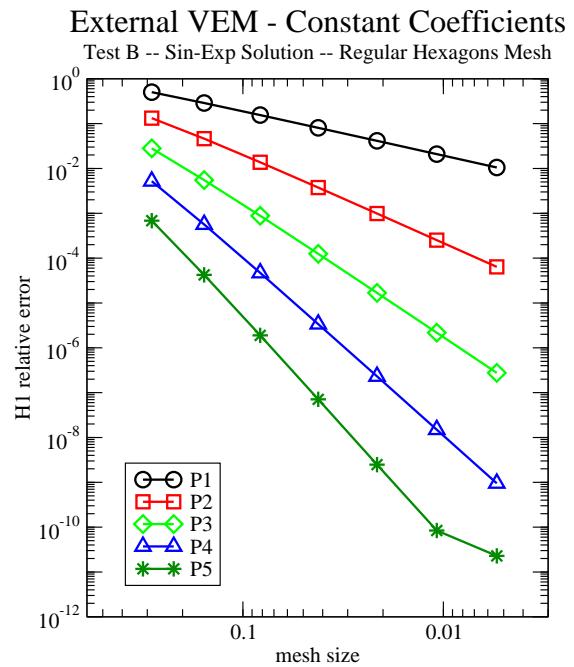
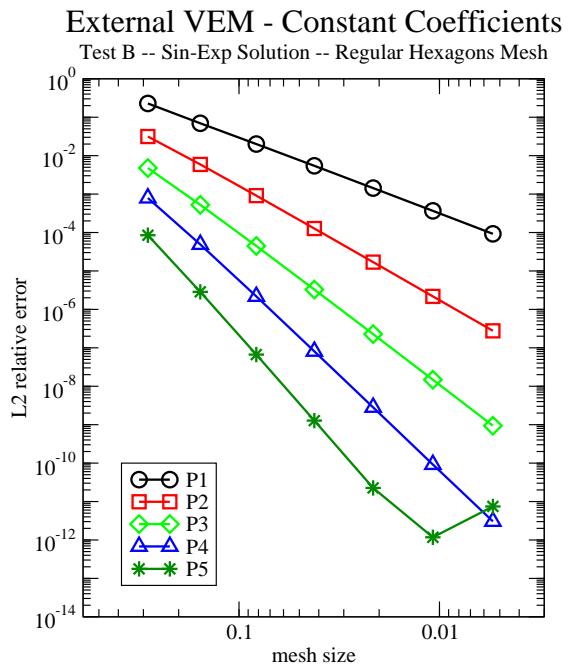


Fig. 29. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular hexagons.

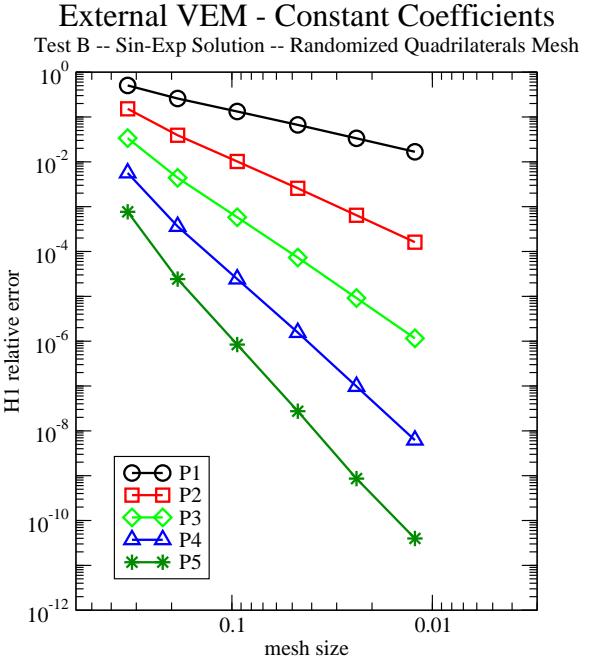
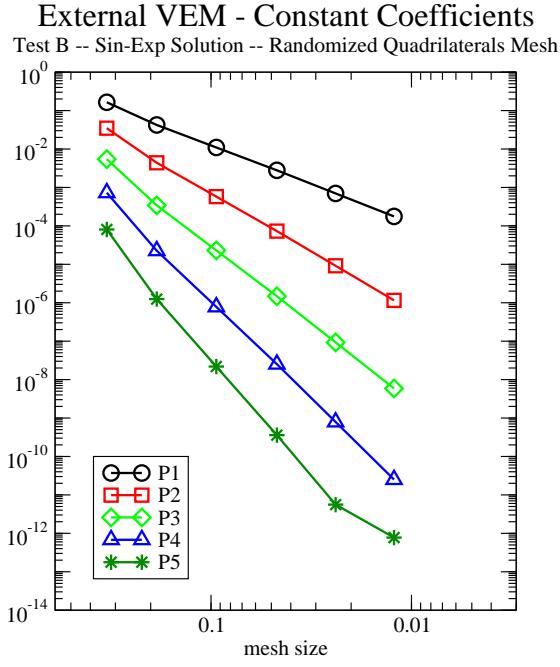


Fig. 30. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of randomized quadrilateral cells.

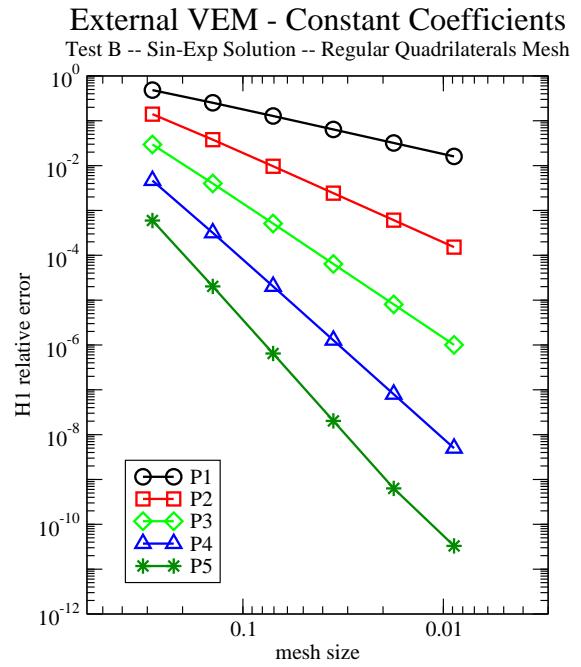
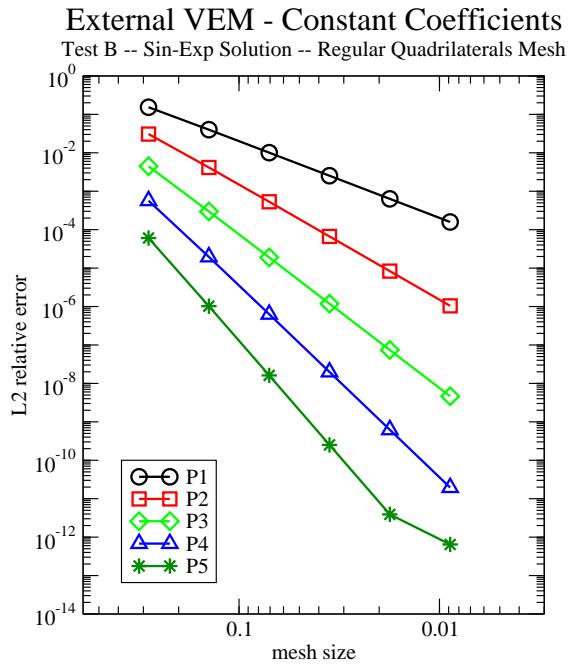


Fig. 31. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular quadrilateral cells (squares).

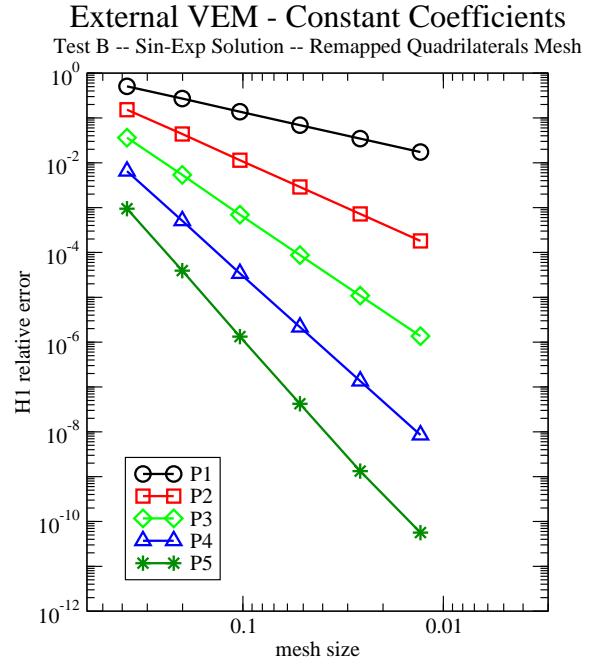
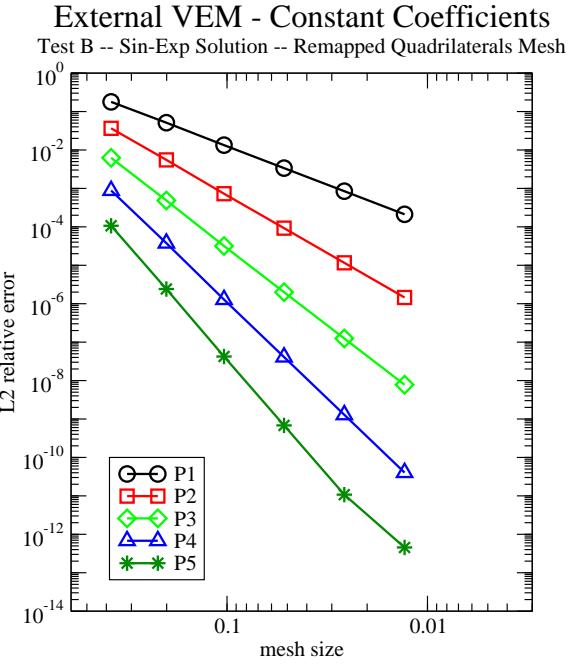


Fig. 32. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

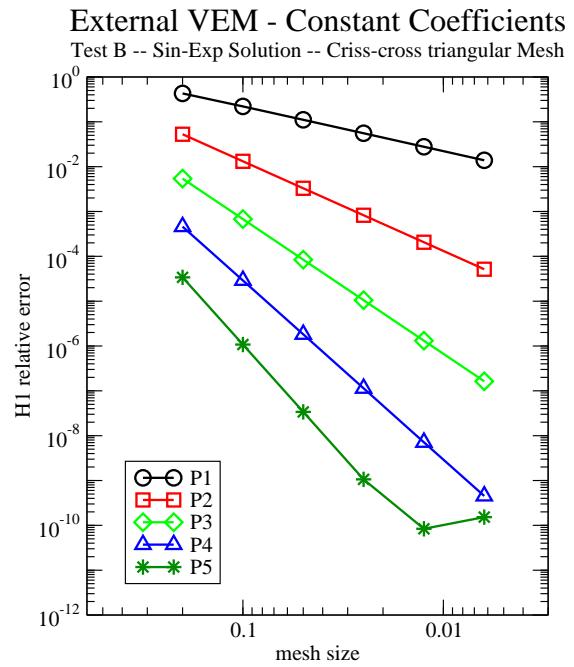
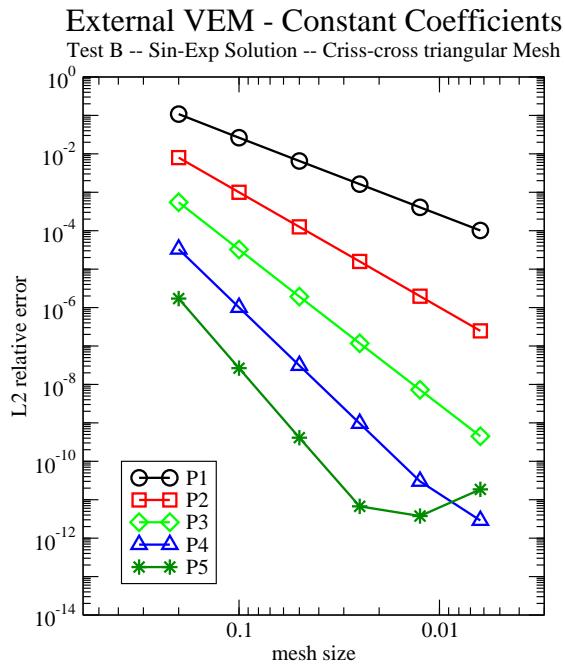


Fig. 33. External VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

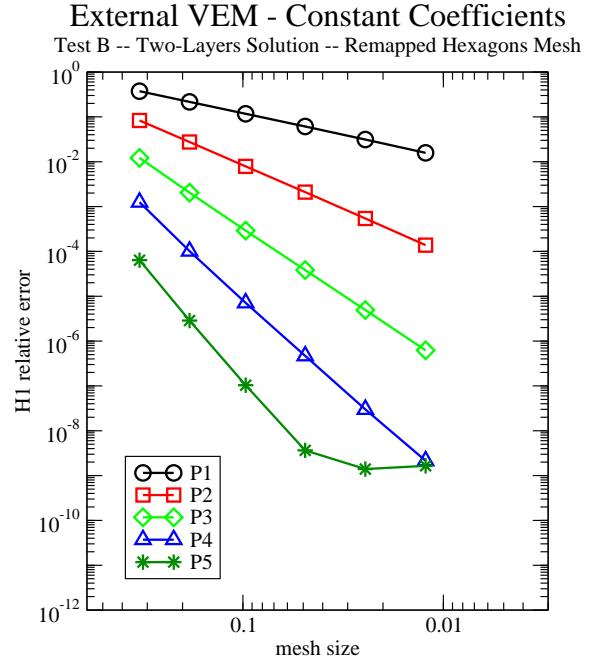
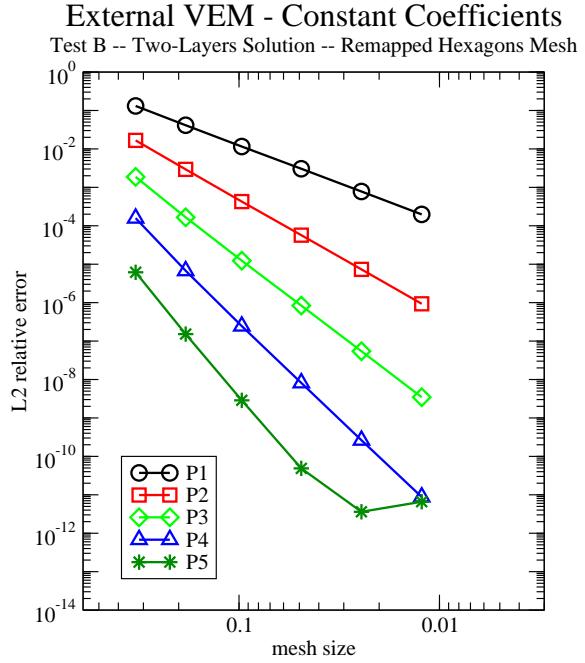


Fig. 34. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of smoothly remapped hexagons.

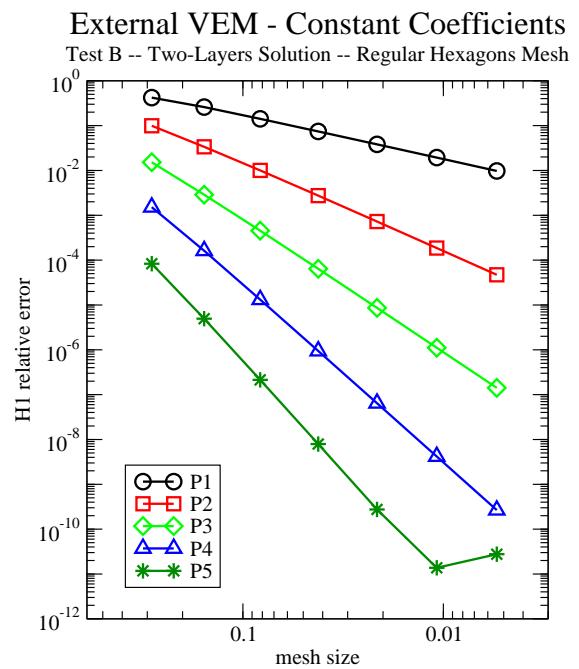
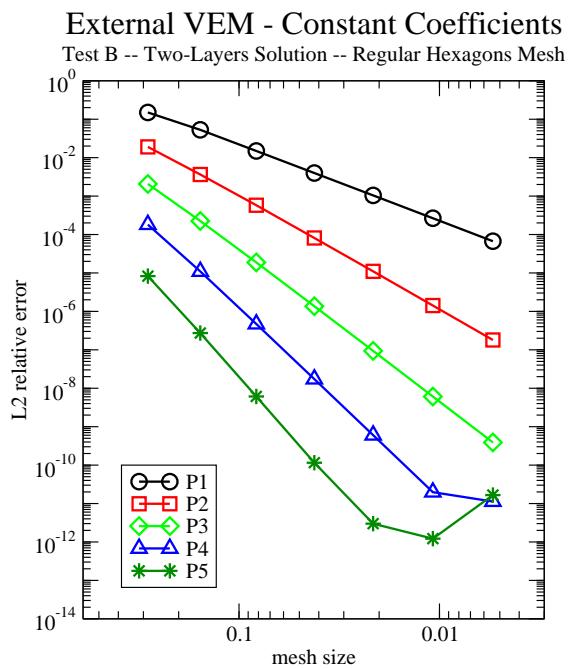


Fig. 35. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular hexagons.

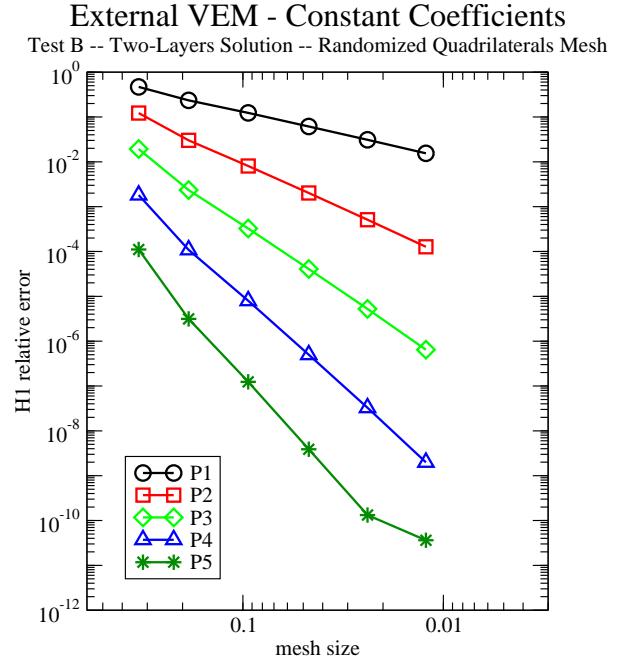
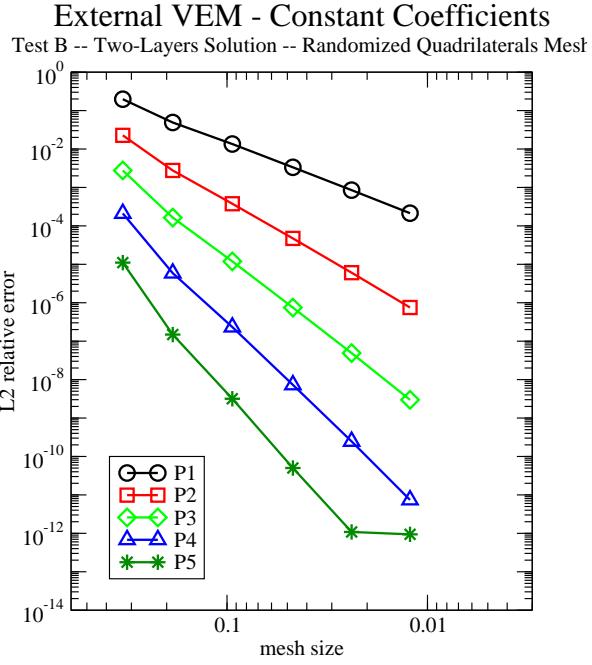


Fig. 36. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of randomized quadrilateral cells.

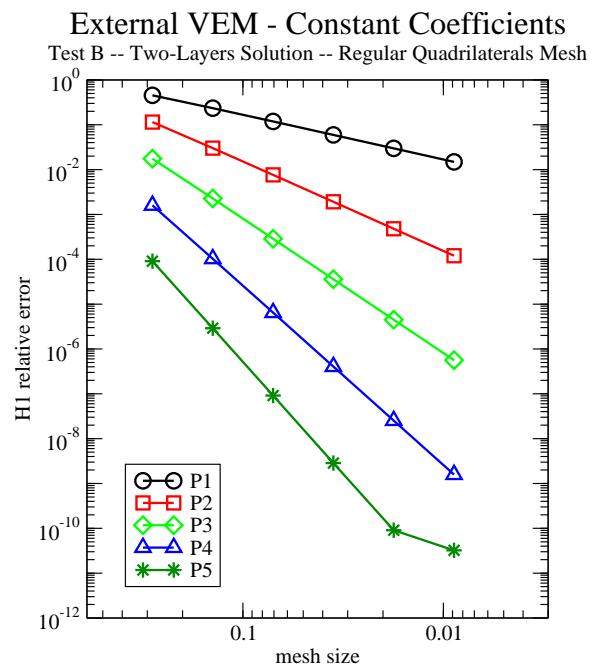
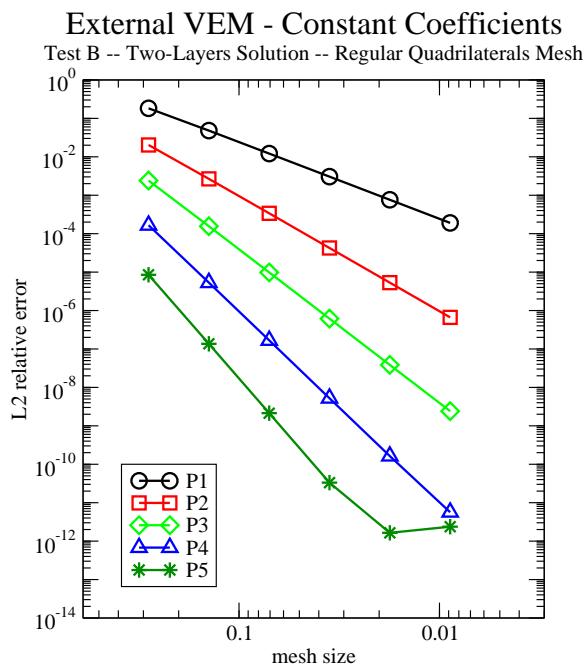


Fig. 37. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular quadrilateral cells (squares).

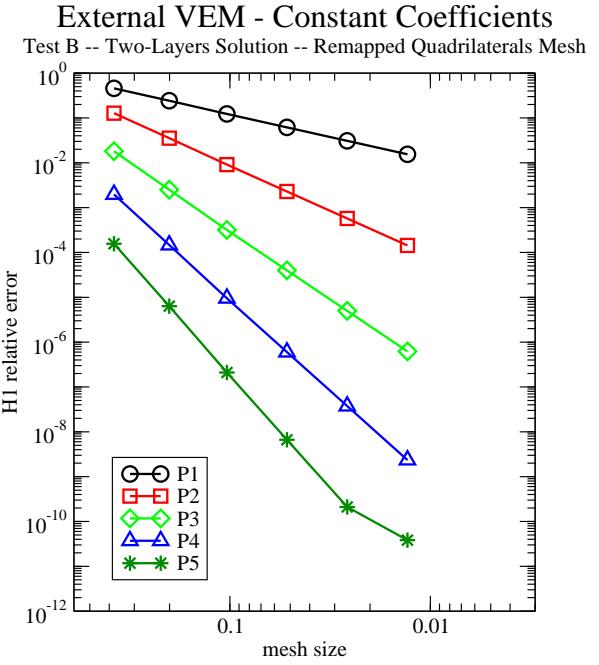
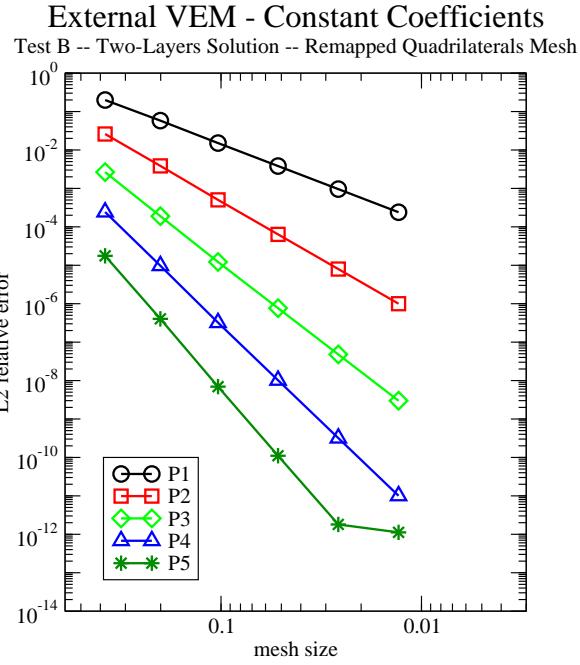


Fig. 38. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

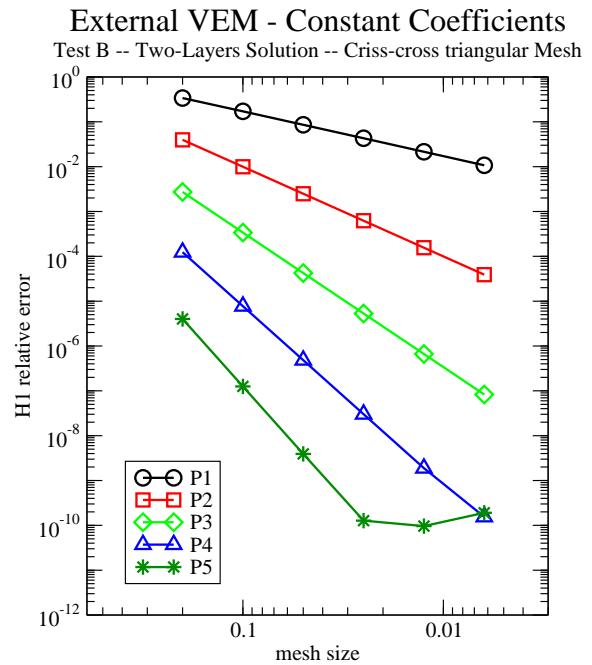
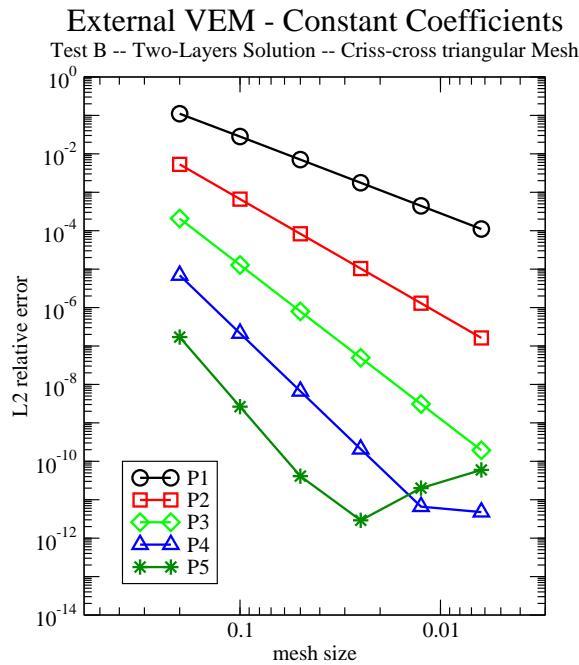


Fig. 39. External VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular triangular cells, (criss-cross).

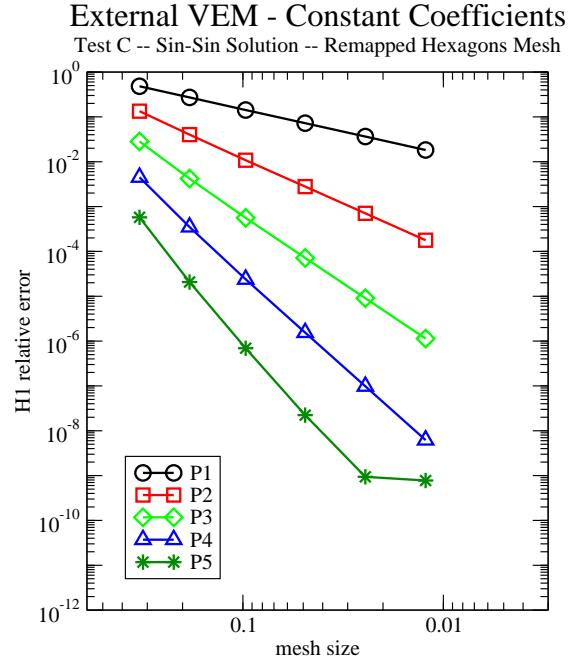
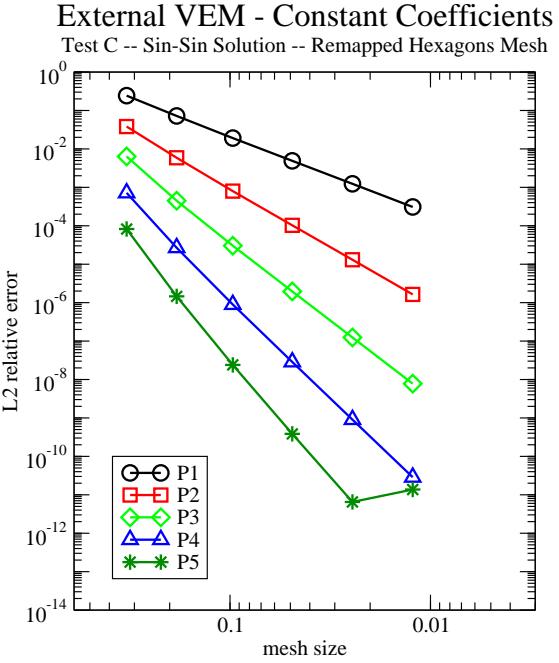


Fig. 40. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

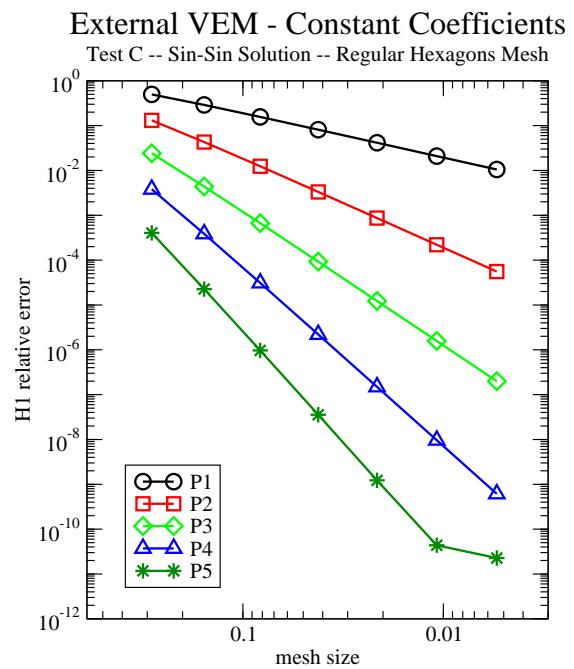
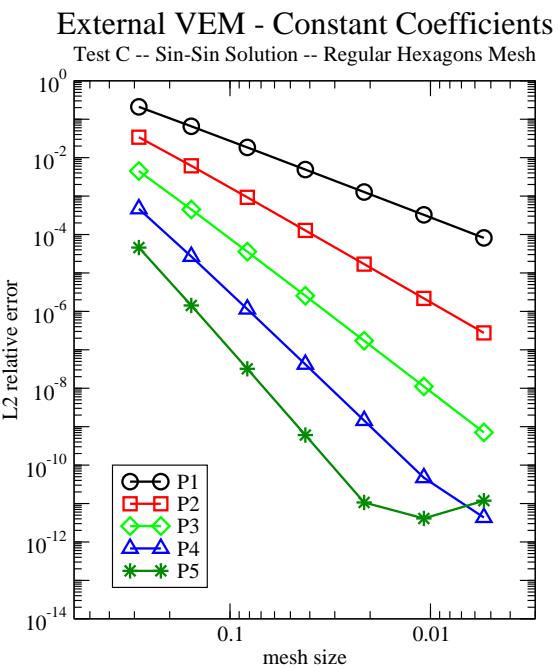


Fig. 41. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

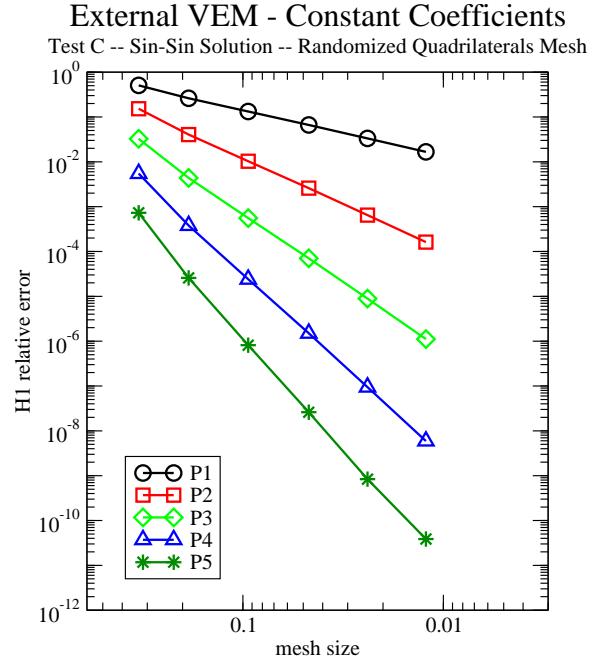
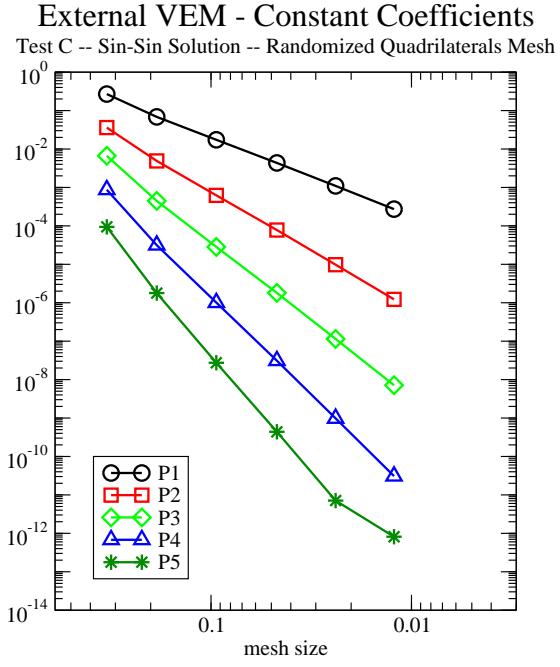


Fig. 42. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

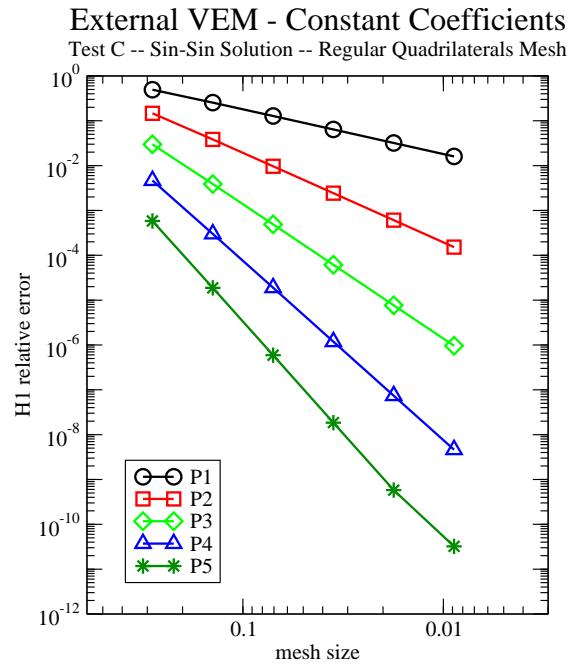
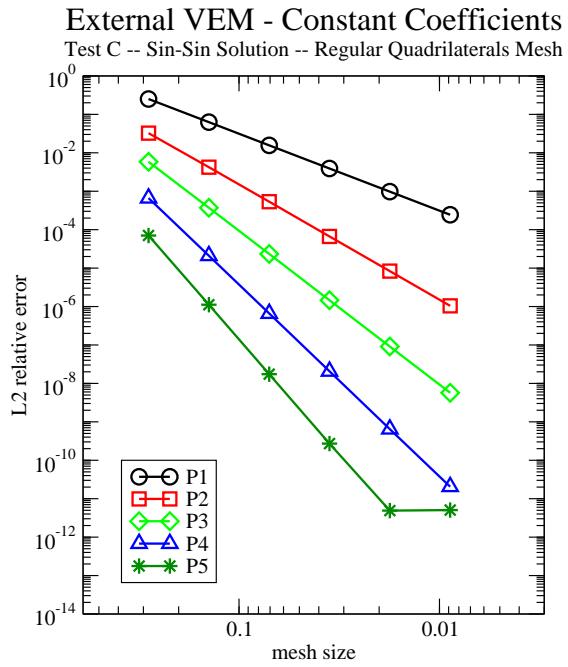


Fig. 43. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

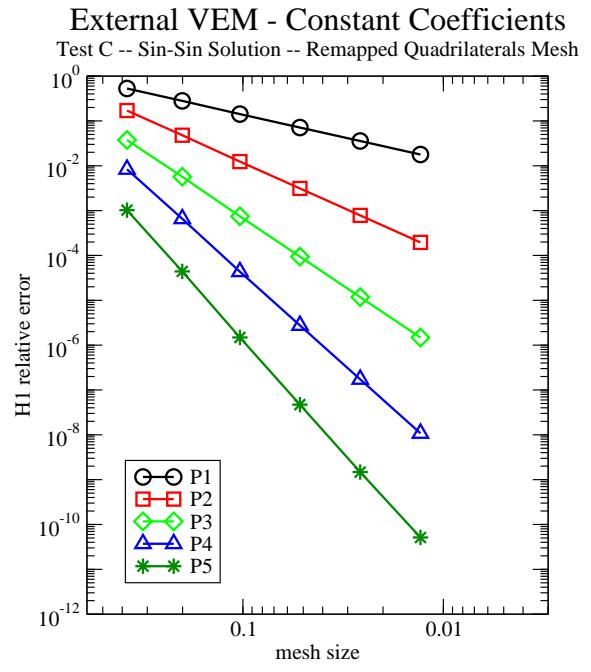
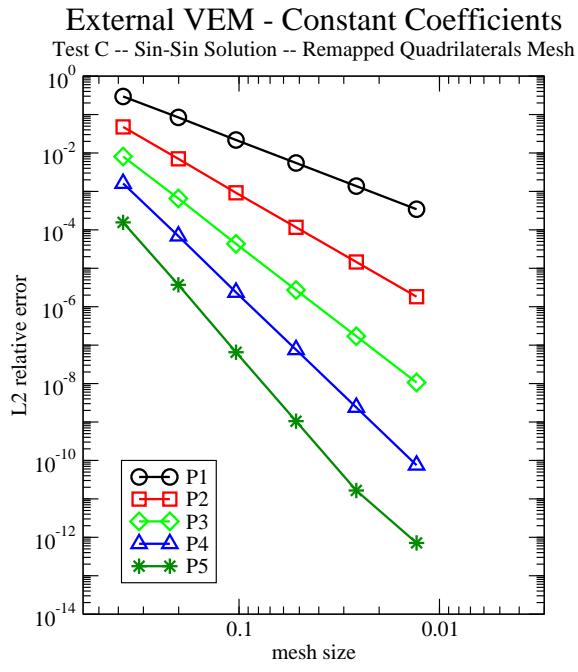


Fig. 44. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

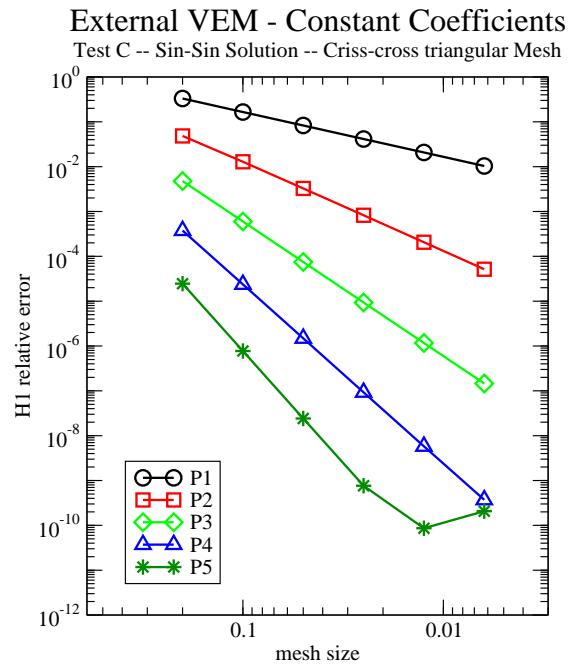
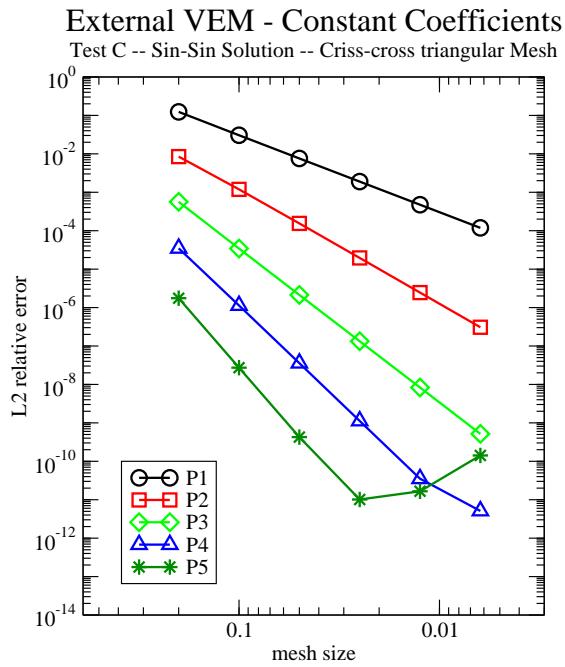


Fig. 45. External VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

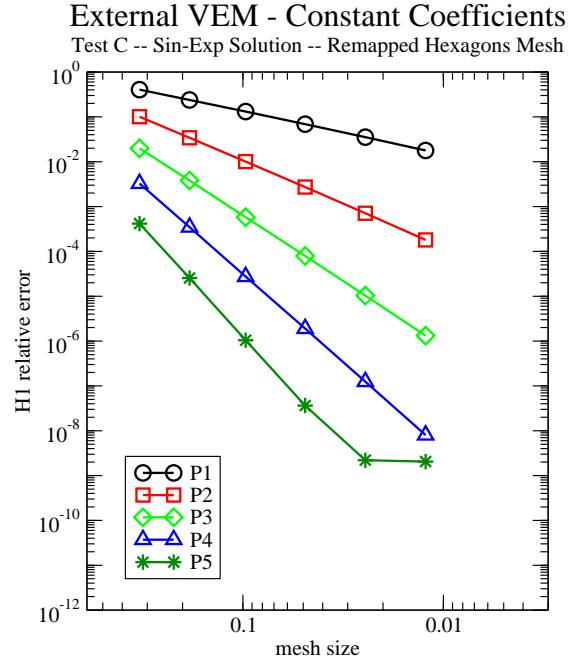
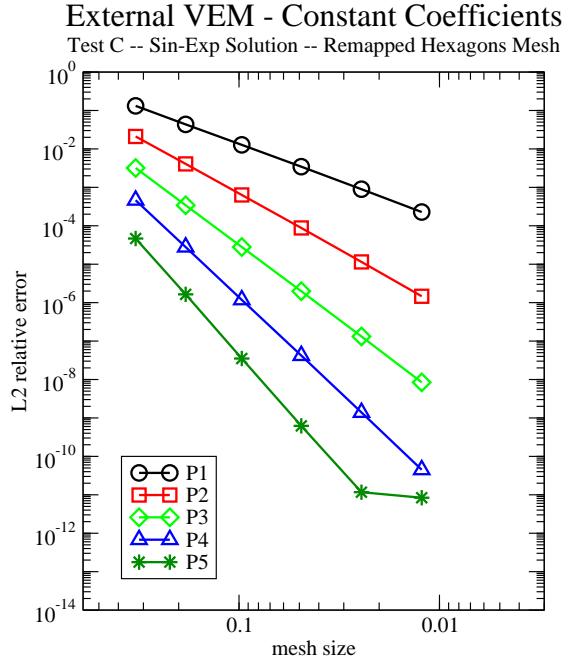


Fig. 46. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

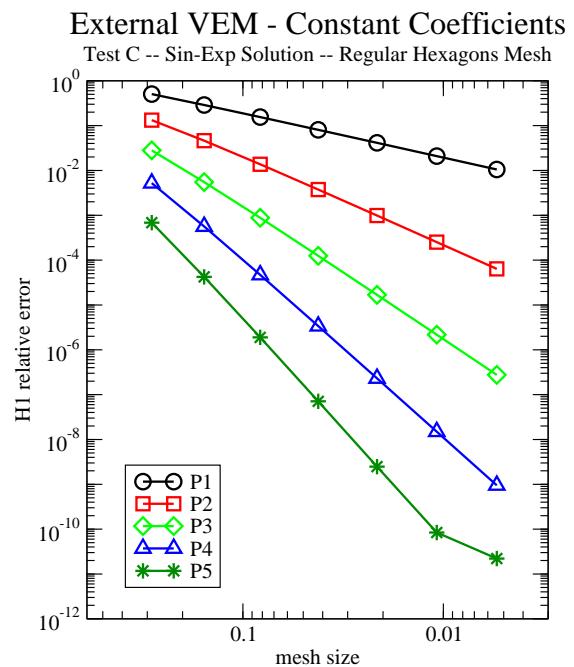
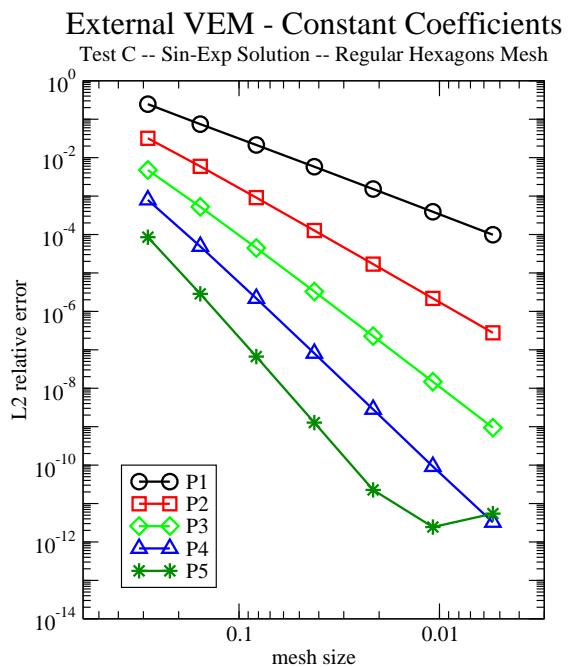


Fig. 47. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

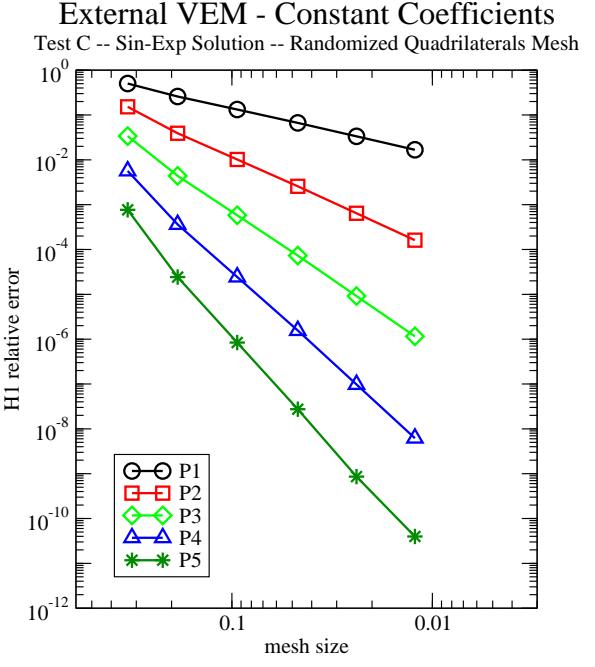
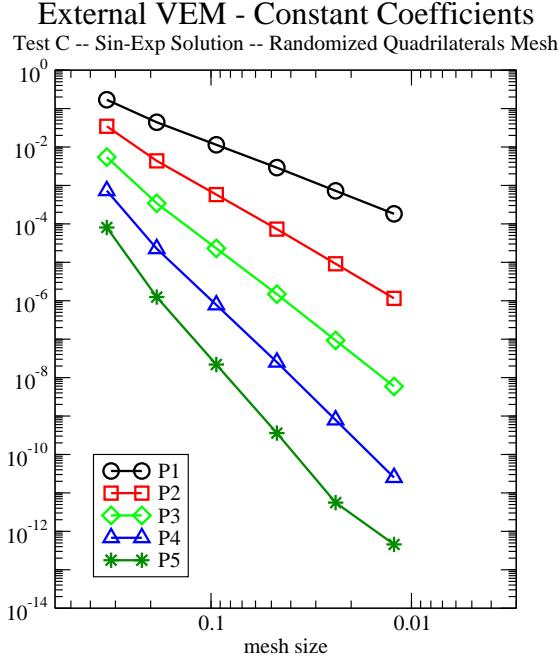


Fig. 48. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

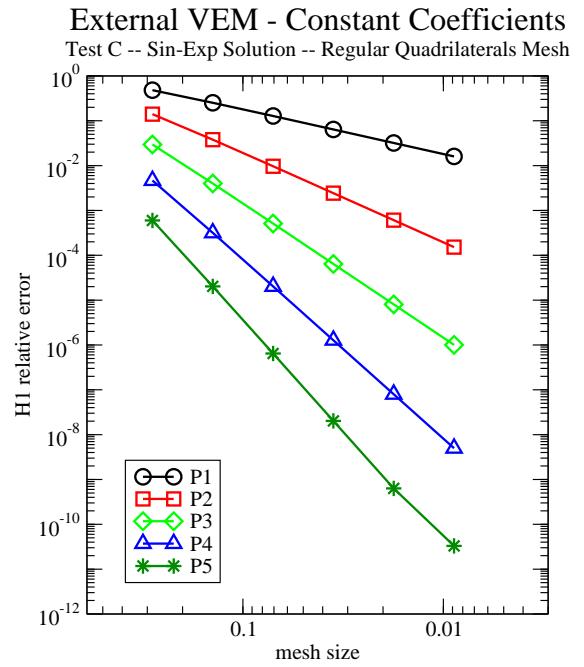
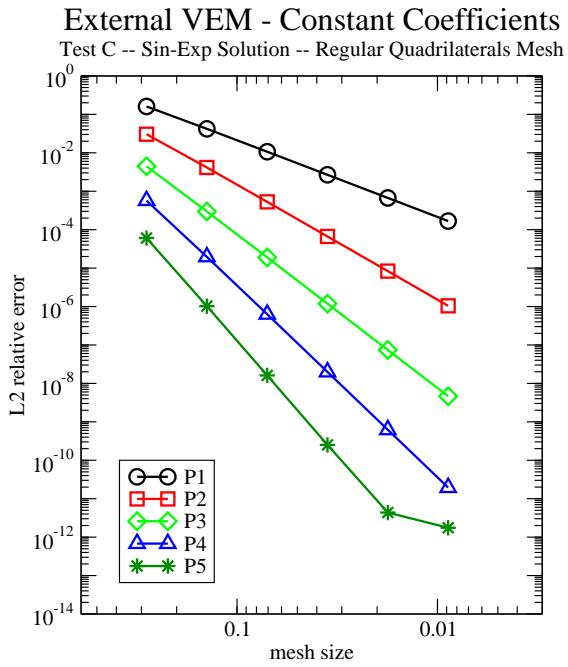


Fig. 49. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

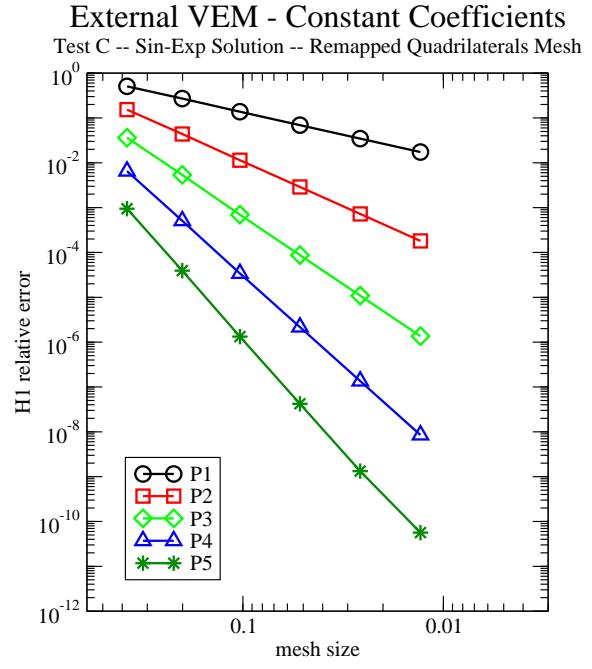
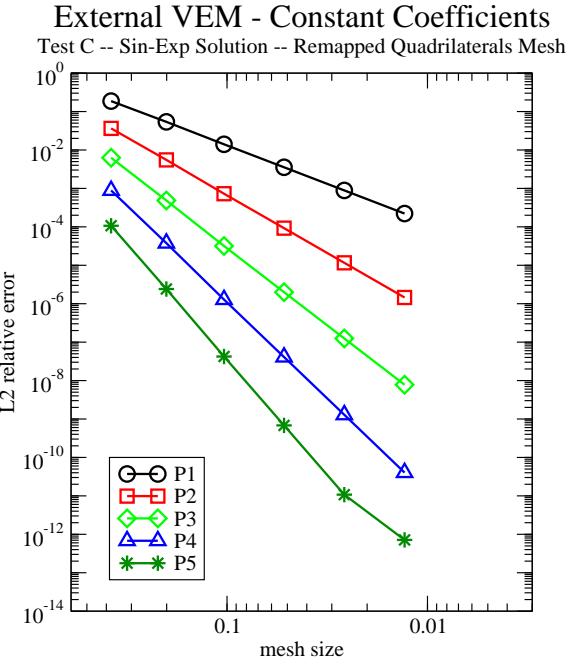


Fig. 50. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

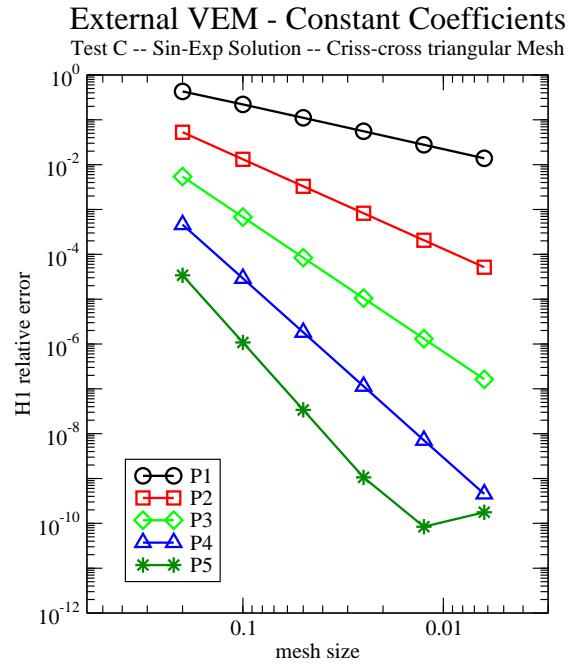
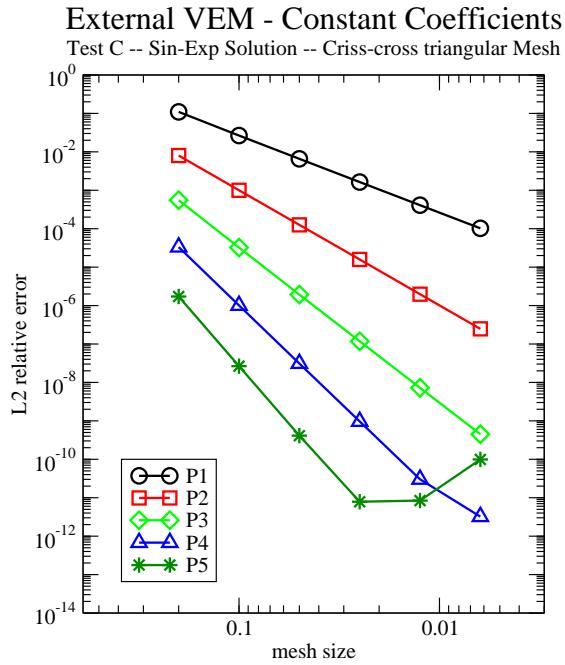


Fig. 51. External VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

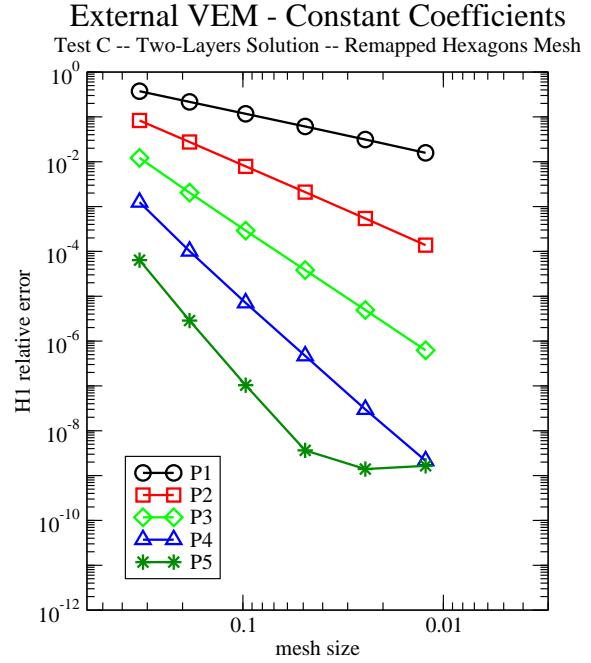
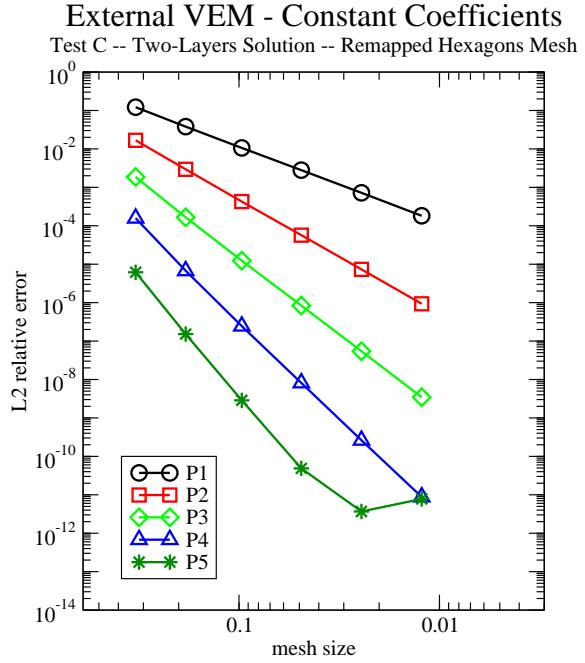


Fig. 52. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

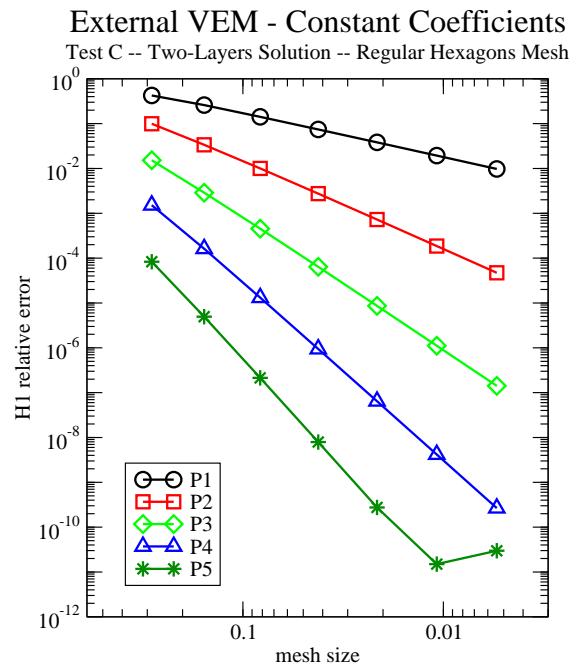
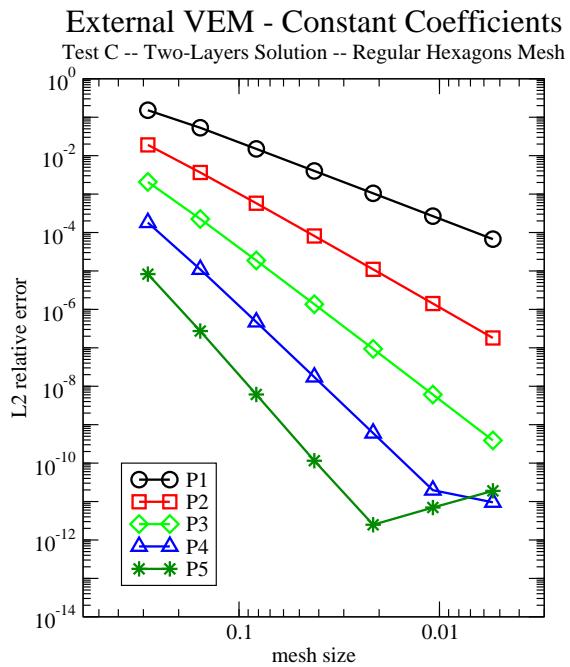


Fig. 53. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular hexagons.

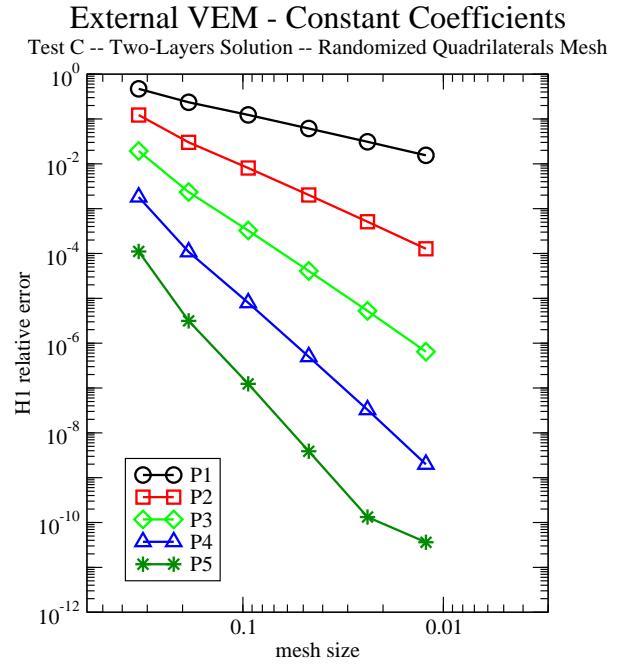
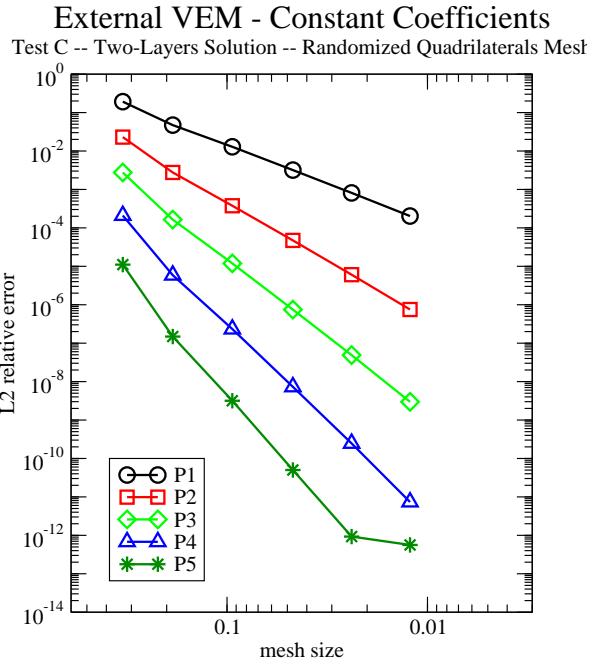


Fig. 54. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

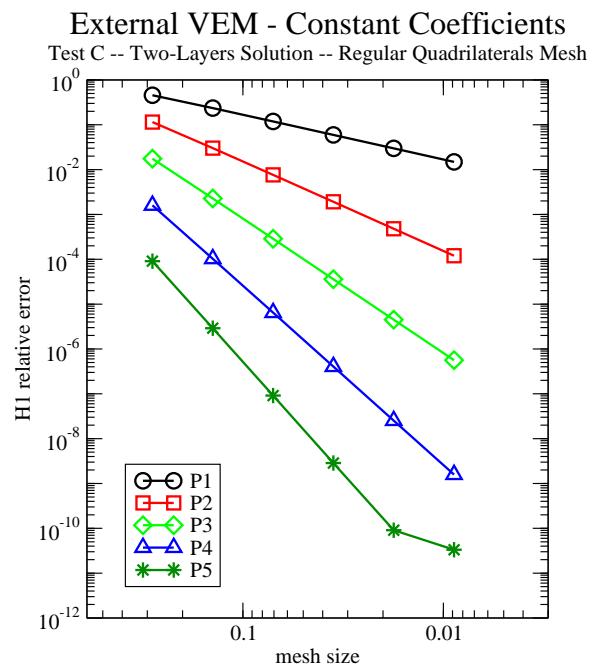
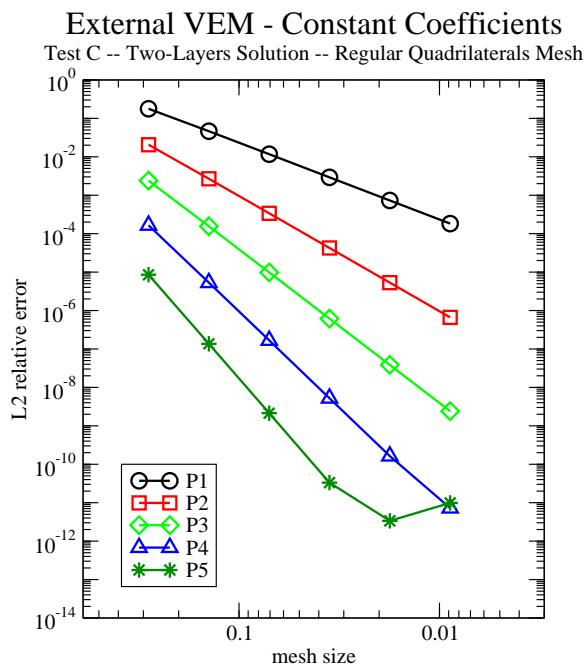


Fig. 55. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

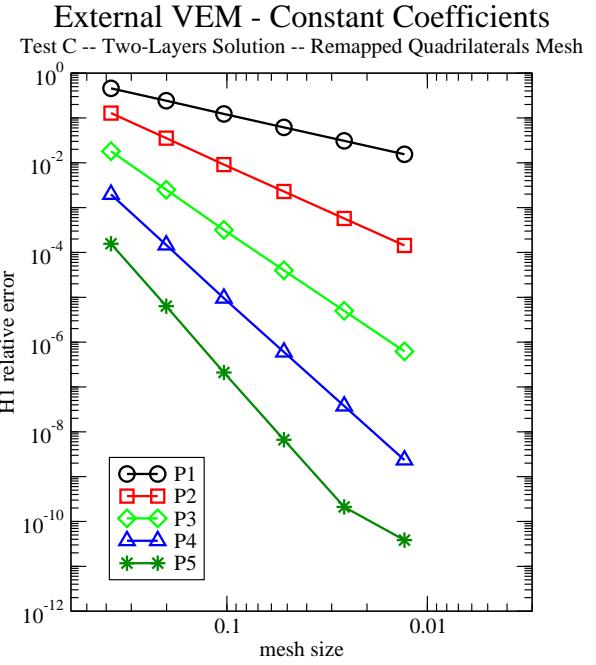
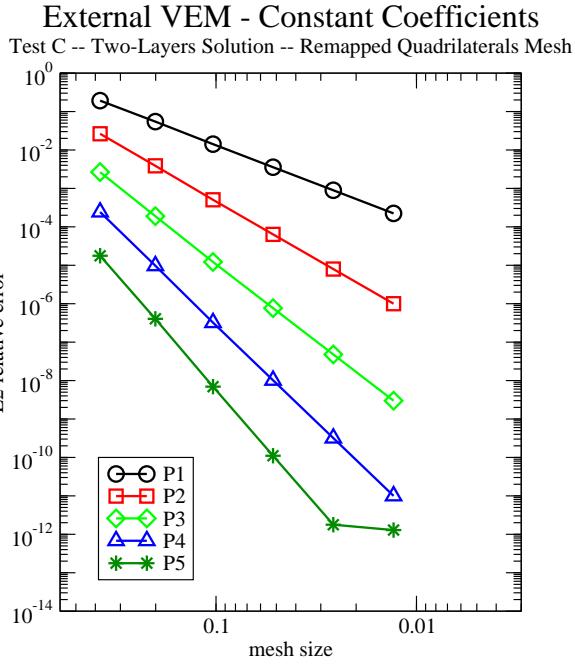


Fig. 56. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

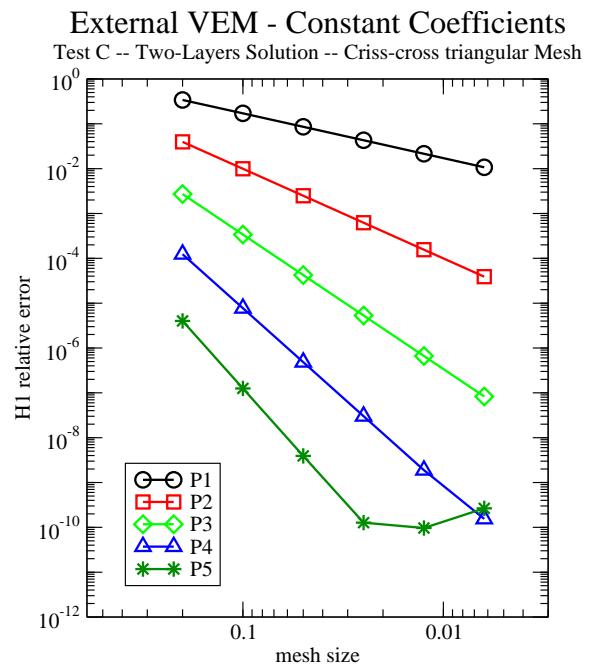
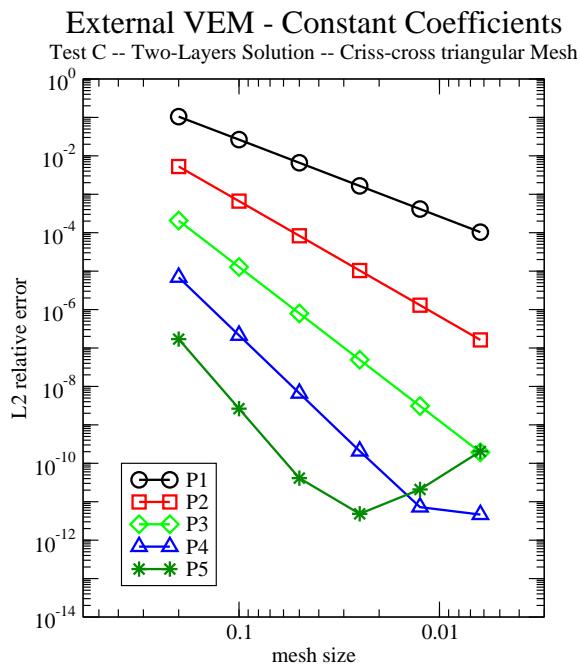


Fig. 57. External VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

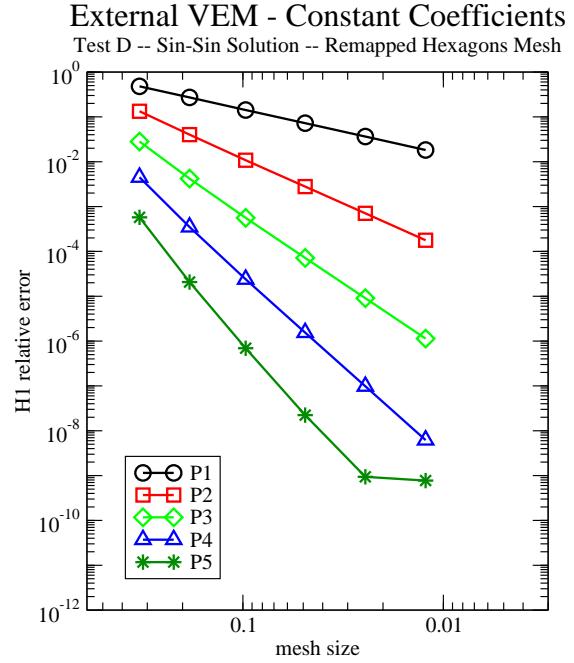
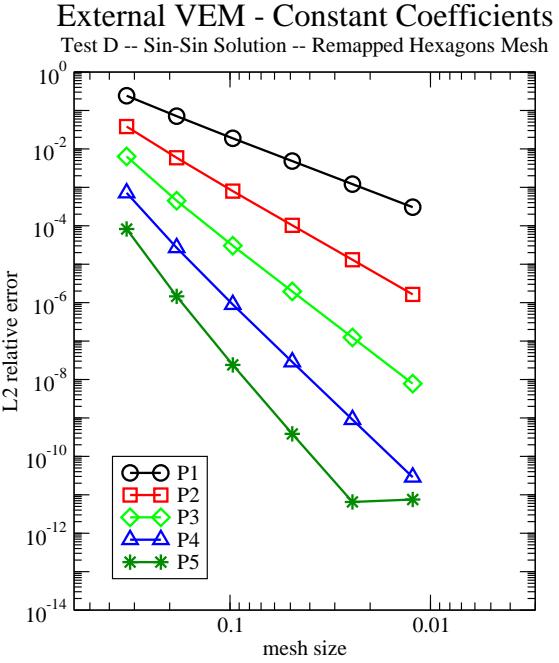


Fig. 58. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of smoothly remapped hexagons.

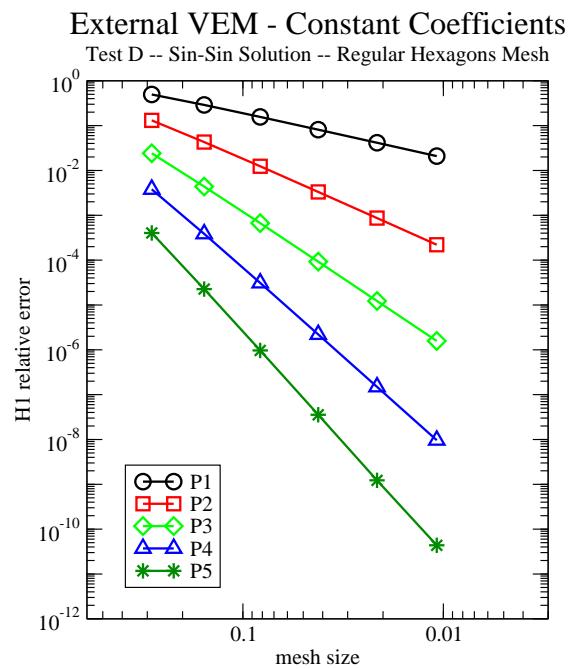
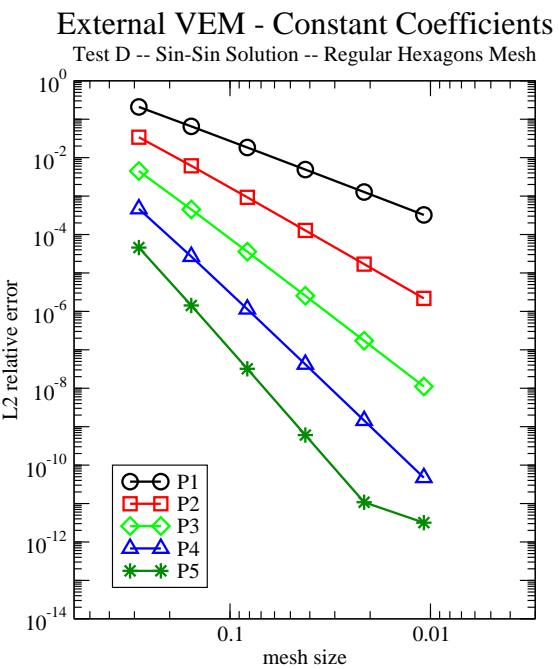


Fig. 59. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular hexagons.

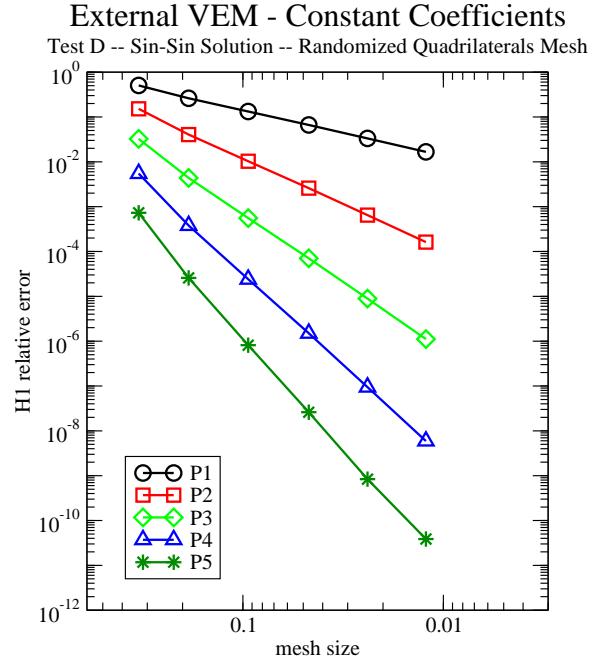
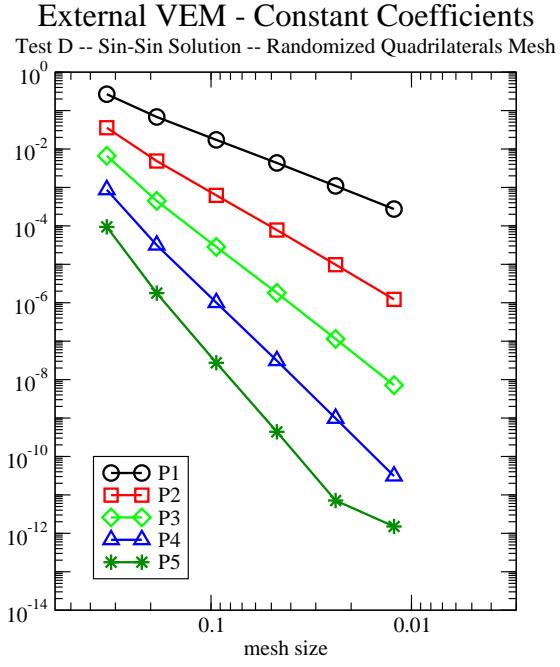


Fig. 60. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of randomized quadrilateral cells.

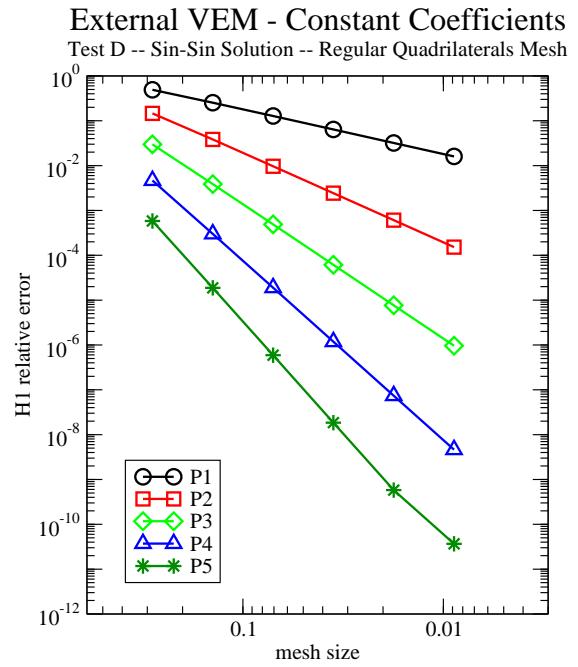
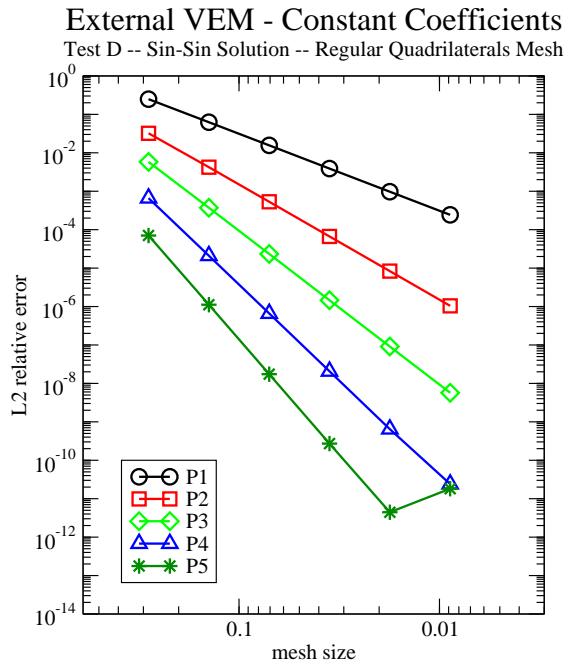


Fig. 61. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular quadrilateral cells (squares).

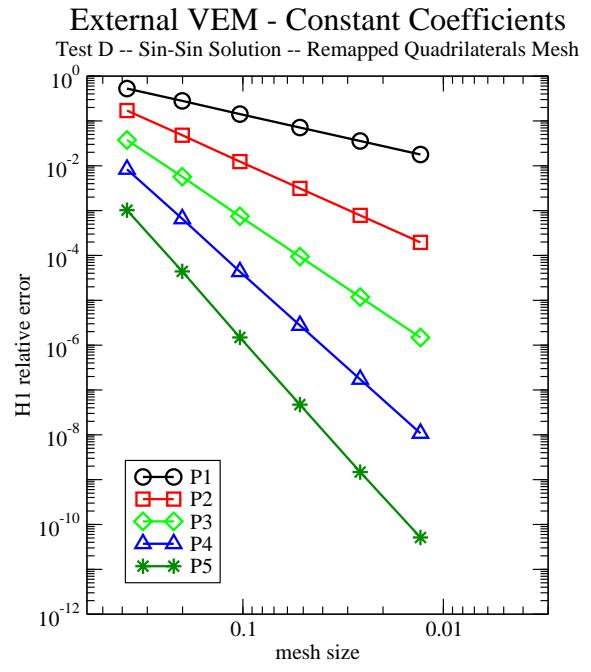
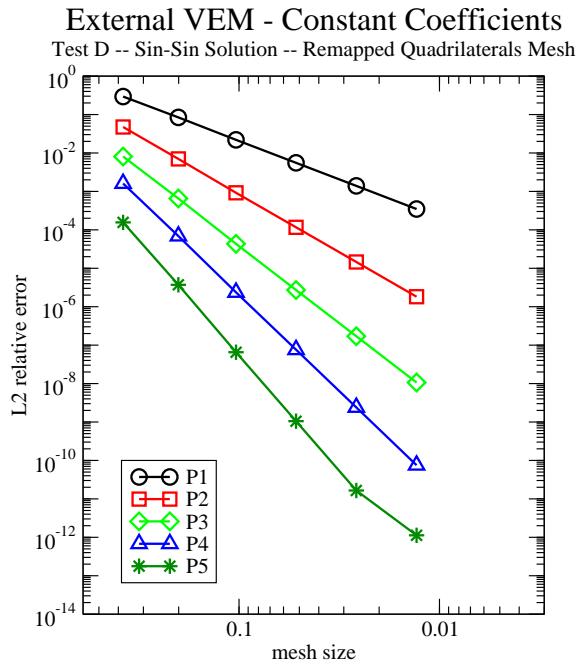


Fig. 62. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

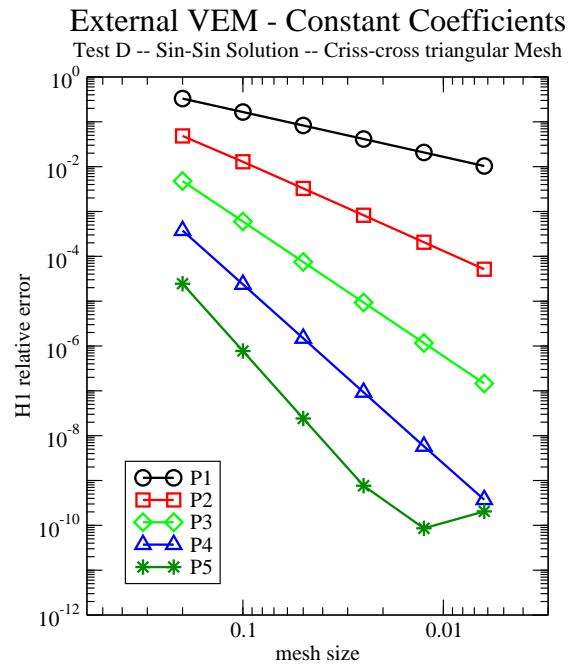
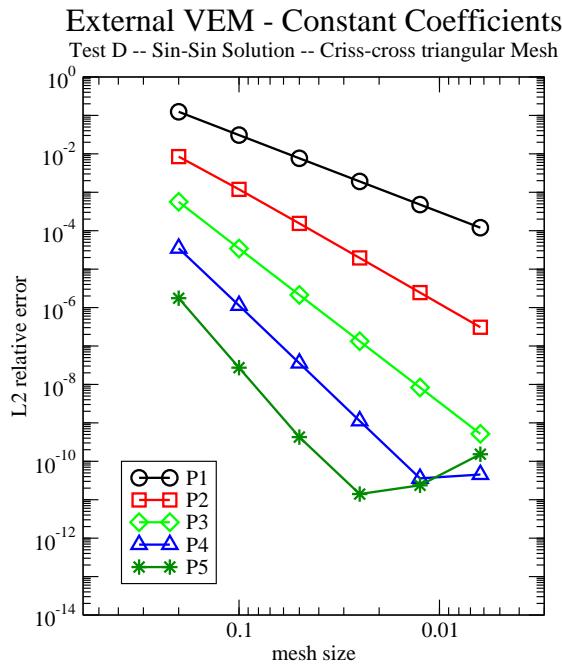


Fig. 63. External VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

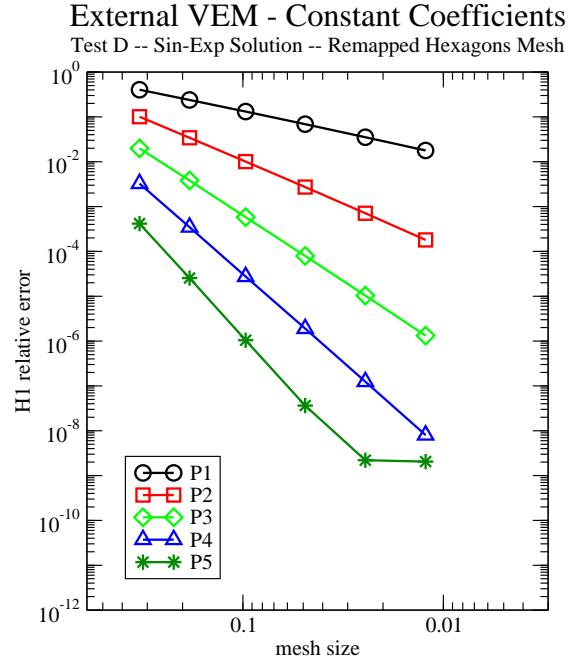
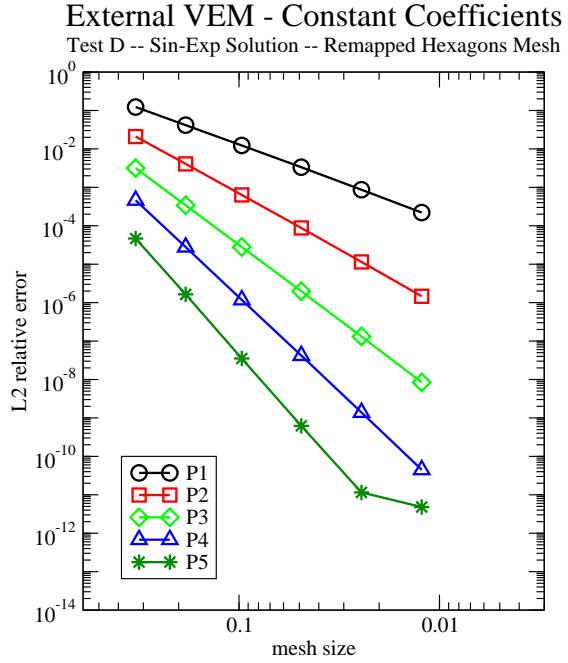


Fig. 64. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of smoothly remapped hexagons.

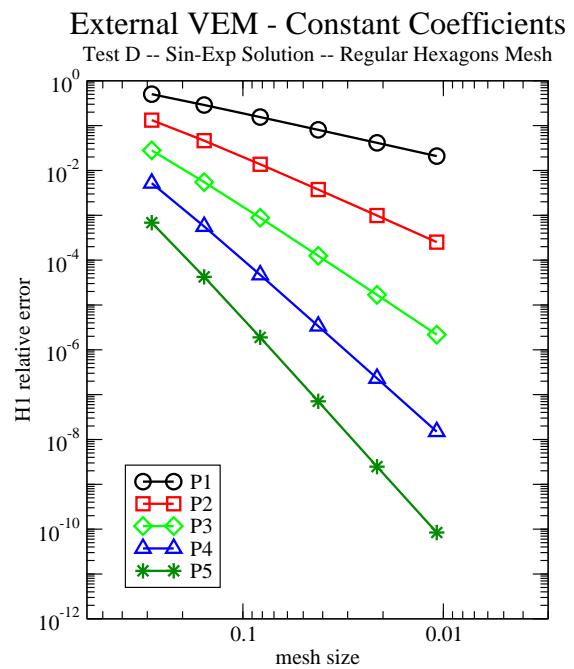
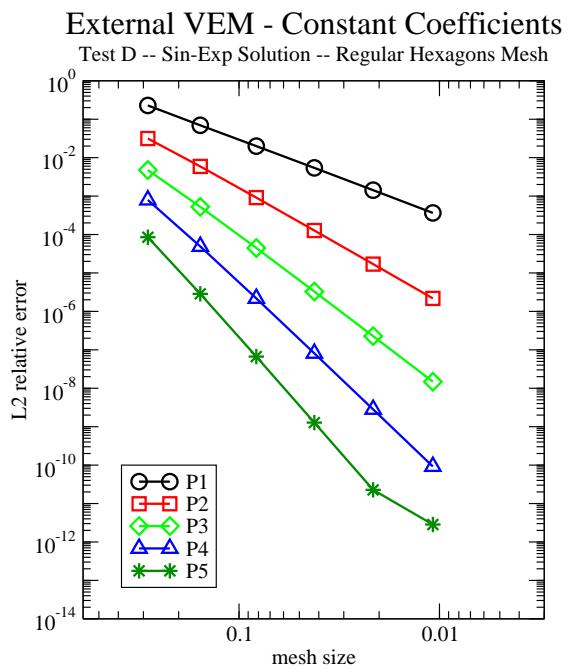


Fig. 65. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular hexagons.

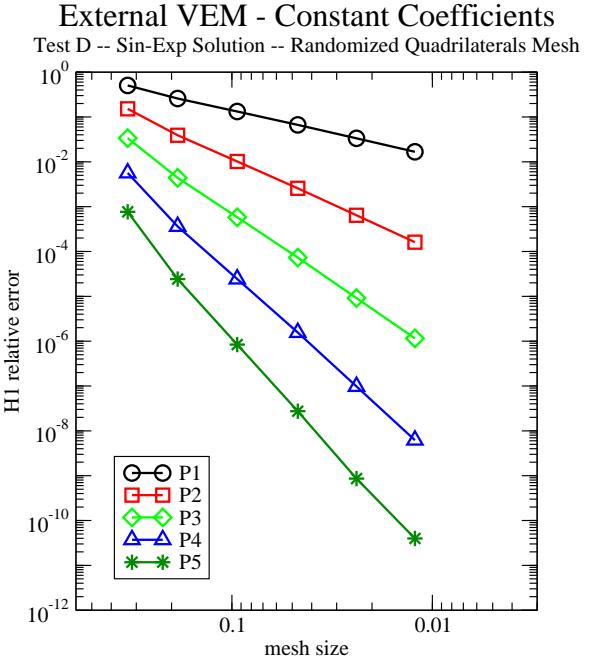
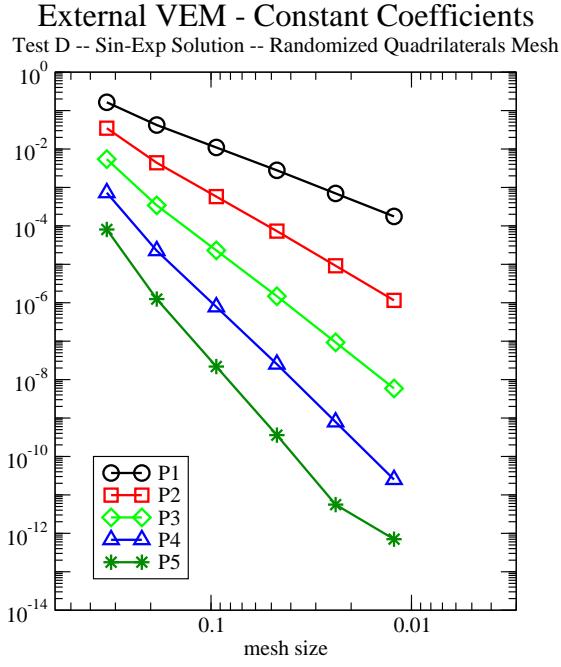


Fig. 66. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of randomized quadrilateral cells.

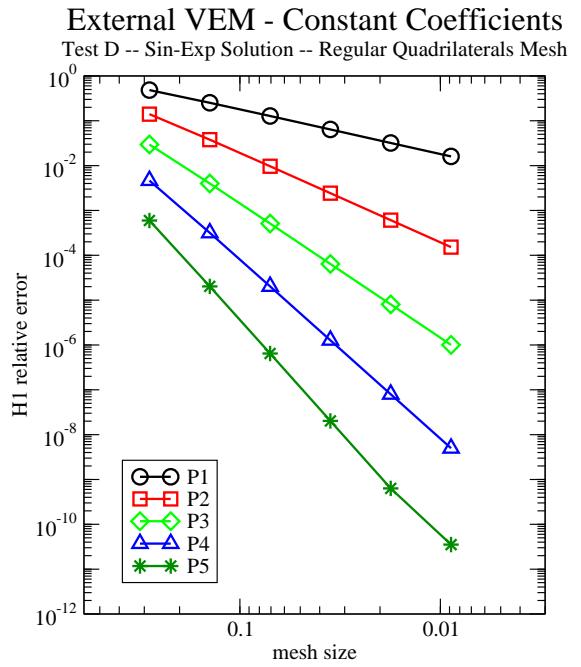
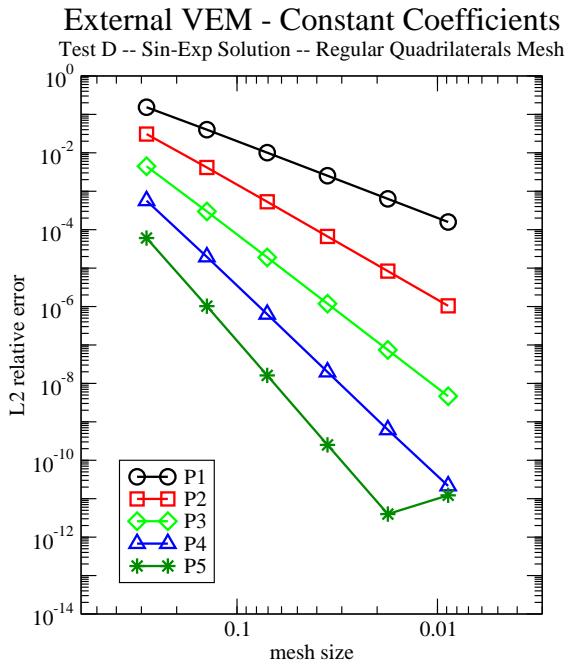


Fig. 67. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular quadrilateral cells (squares).

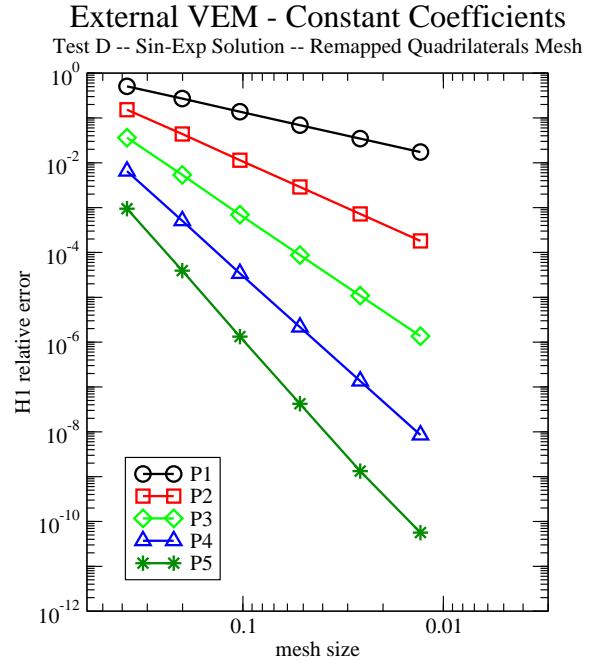
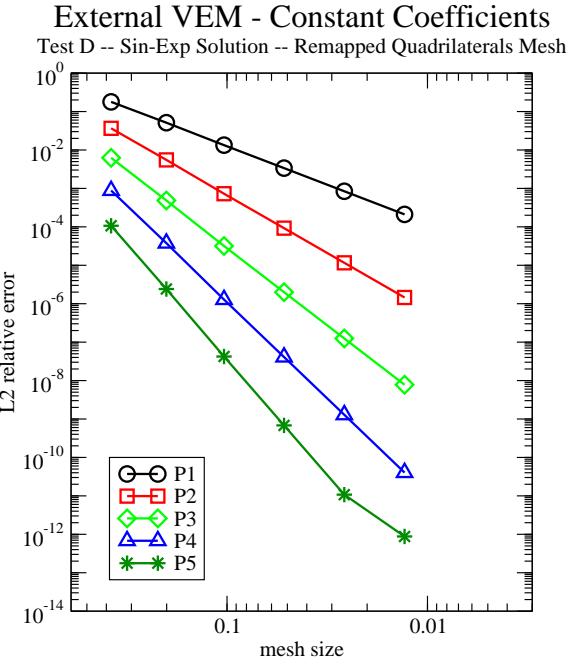


Fig. 68. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

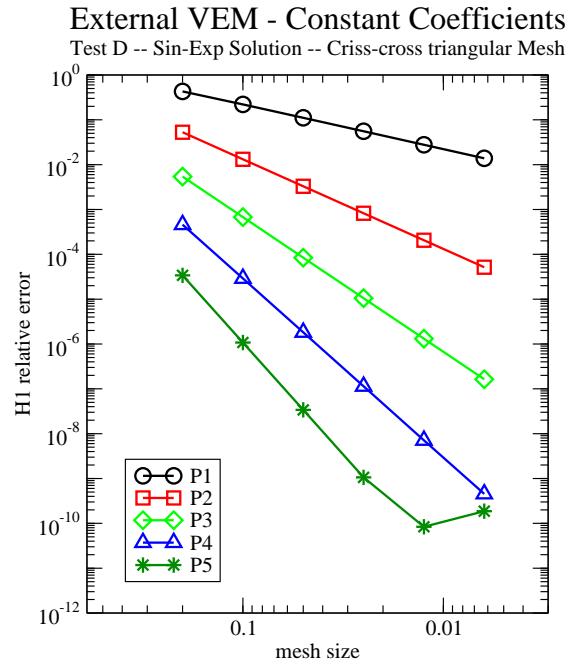
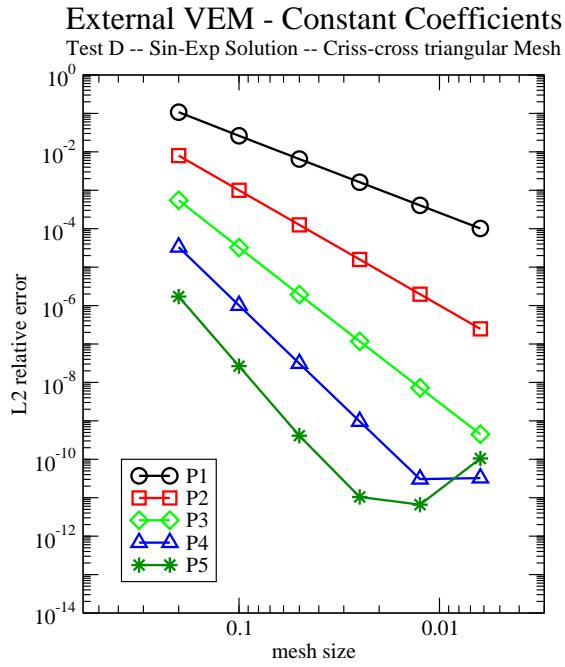


Fig. 69. External VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

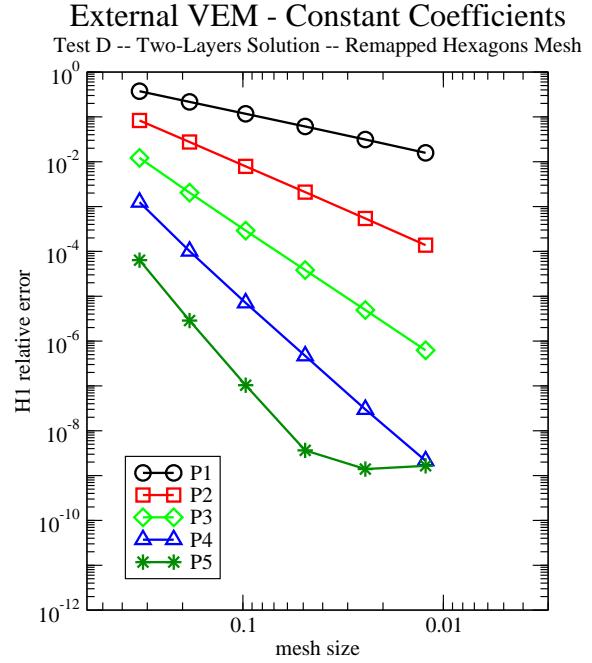
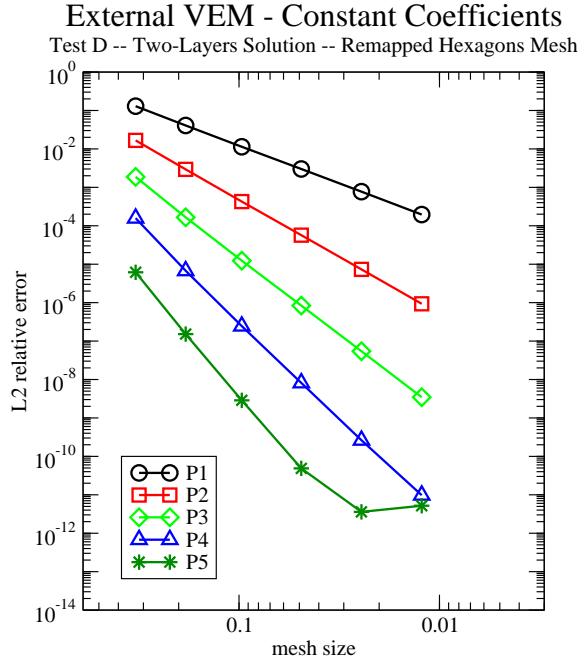


Fig. 70. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of smoothly remapped hexagons.

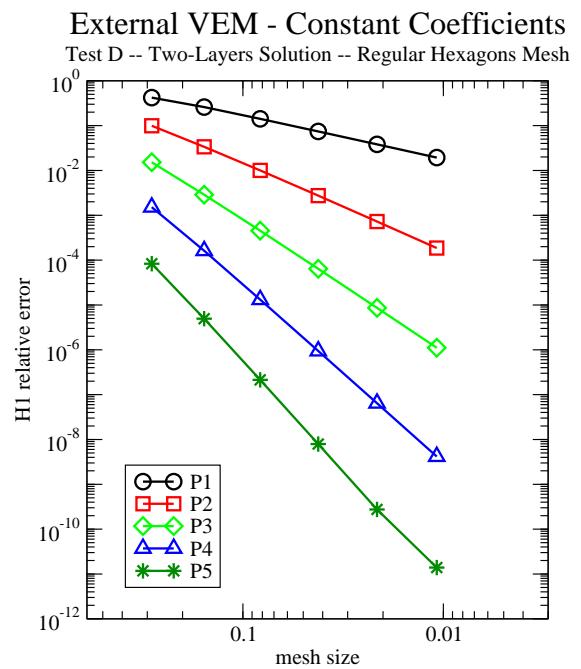
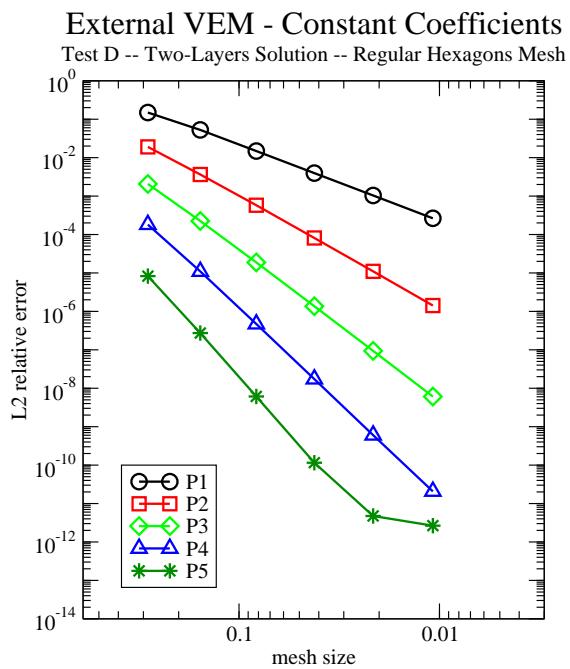


Fig. 71. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular hexagons.

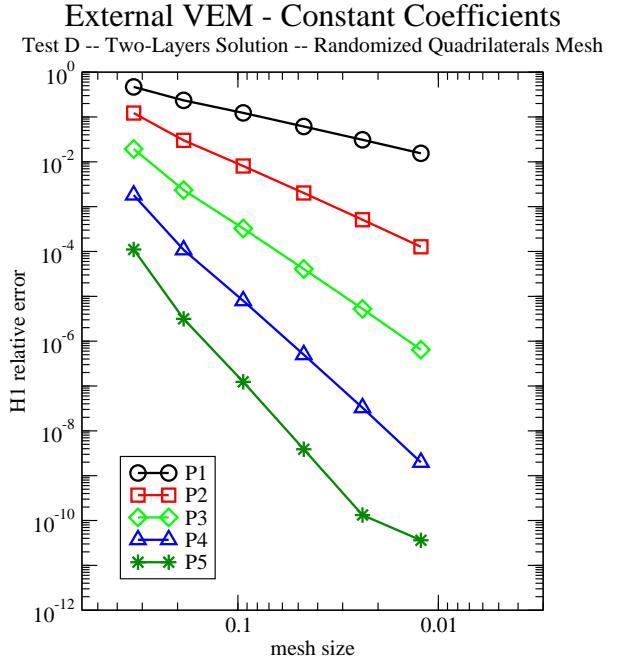
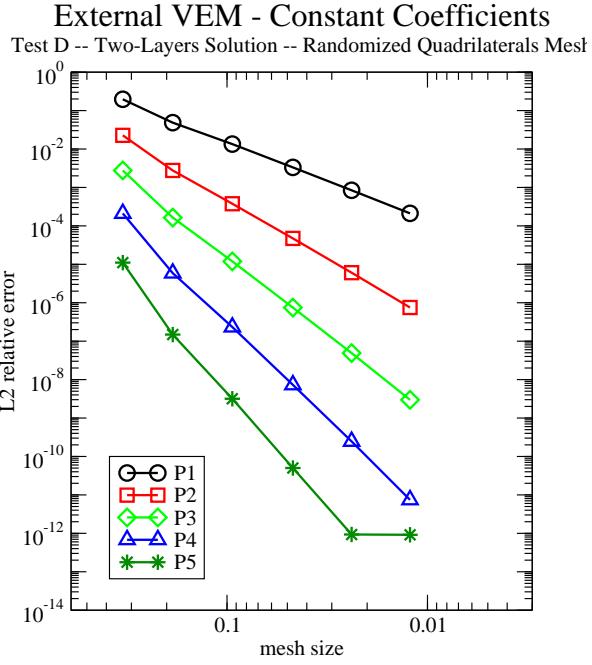


Fig. 72. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of randomized quadrilateral cells.

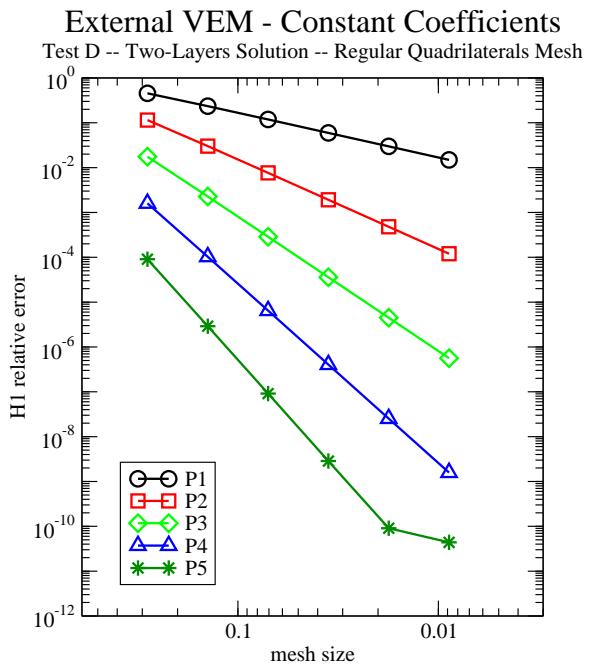
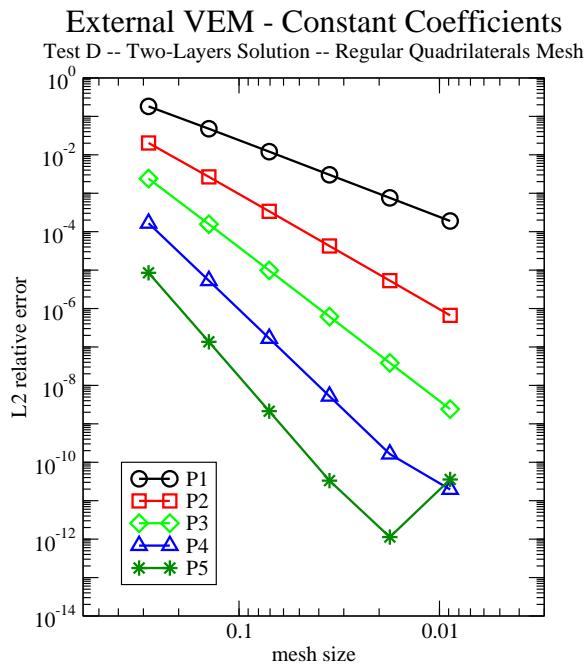


Fig. 73. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular quadrilateral cells (squares).

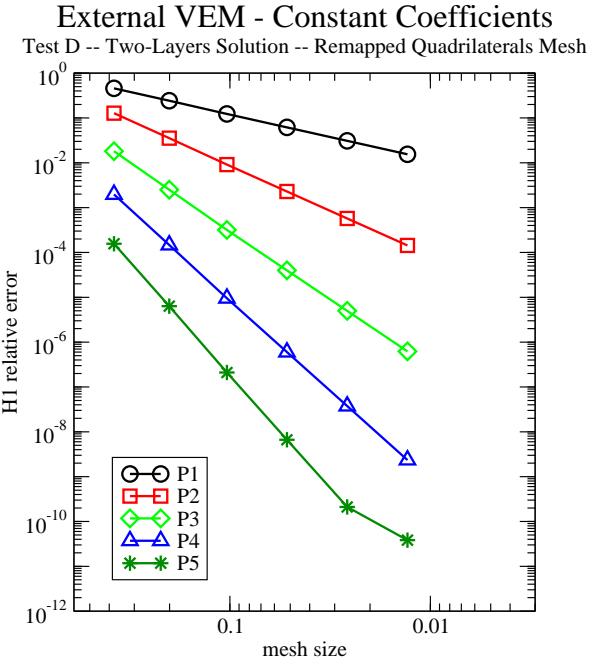
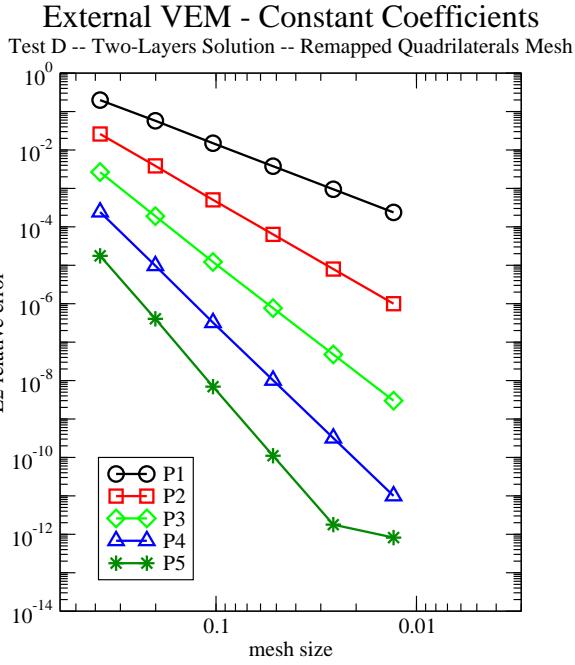


Fig. 74. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

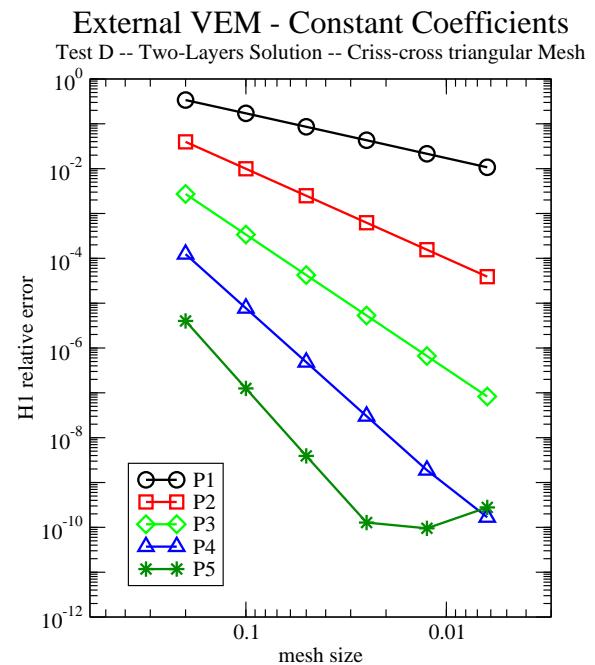
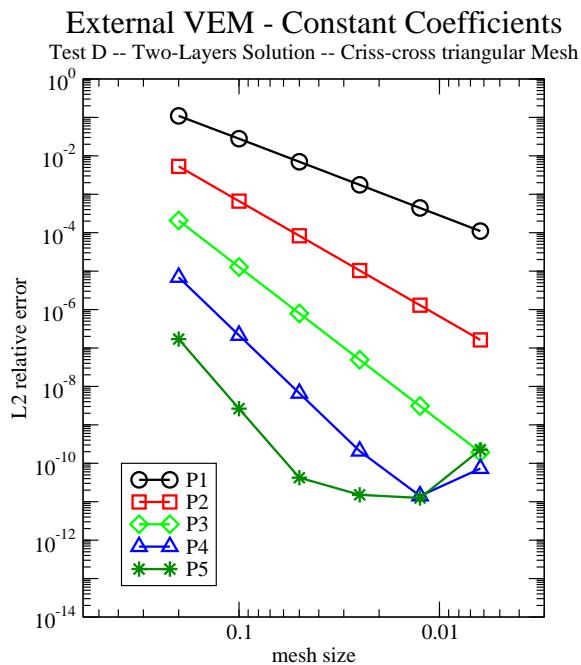


Fig. 75. External VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular triangular cells, (criss-cross).

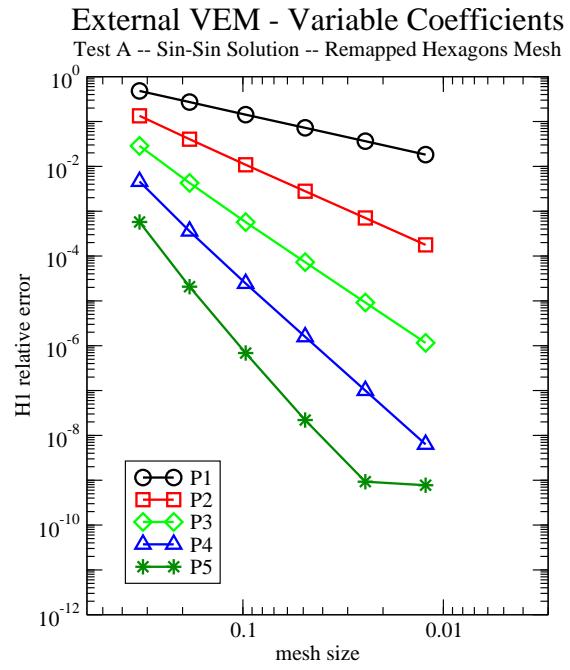
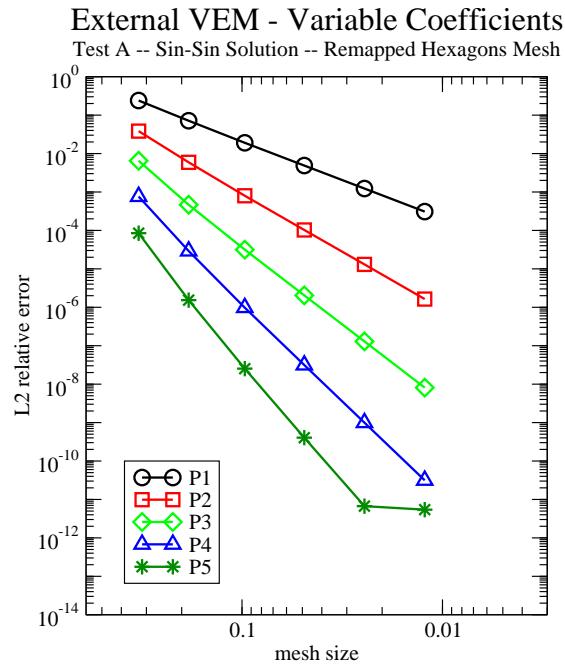


Fig. 76. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

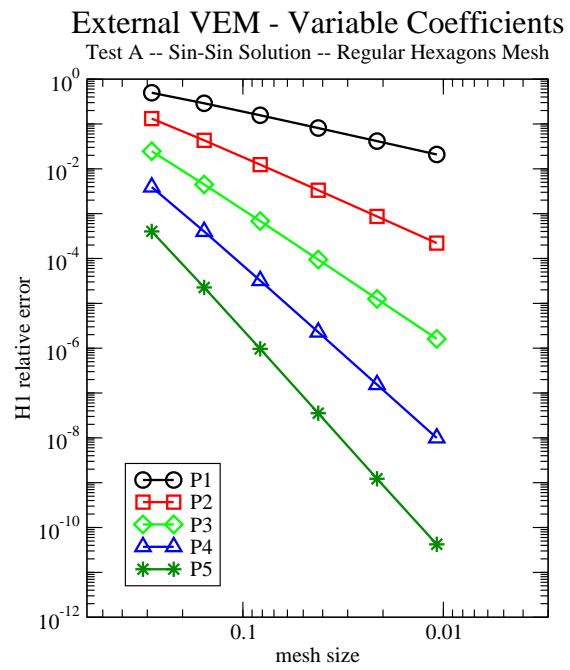
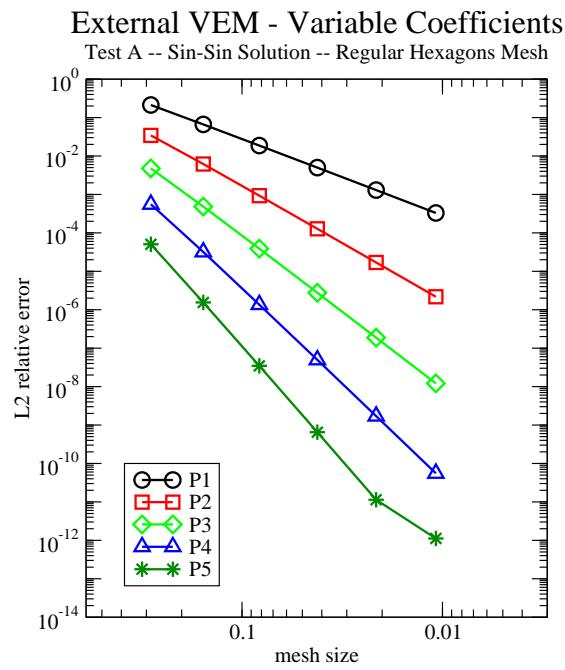


Fig. 77. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

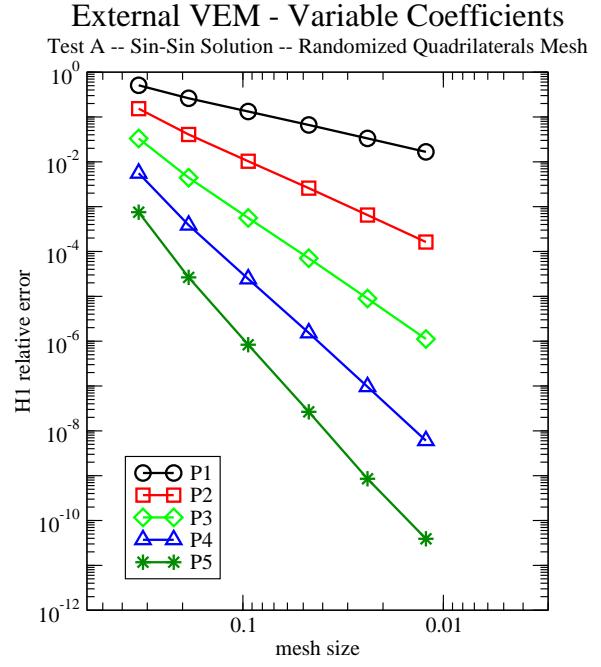
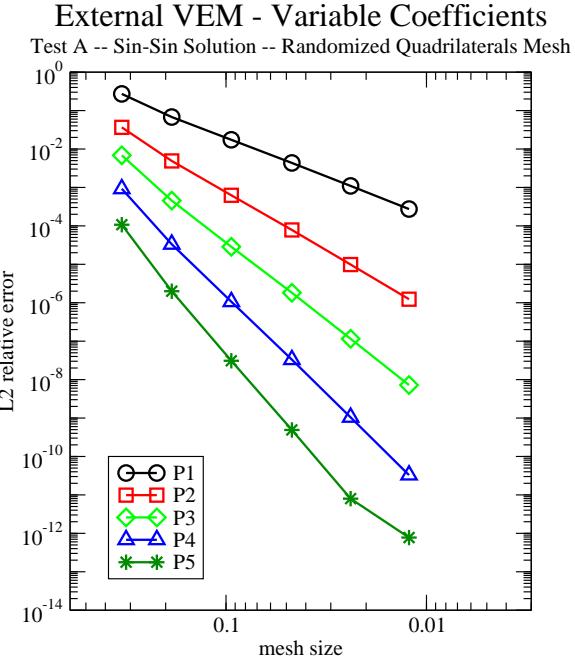


Fig. 78. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

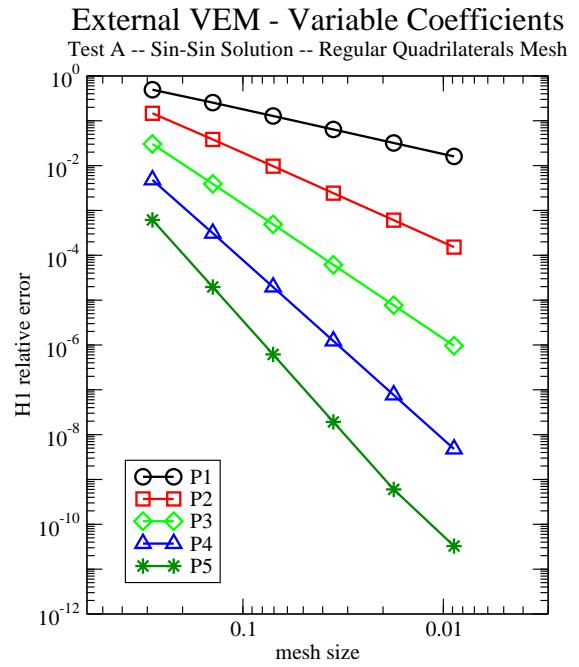
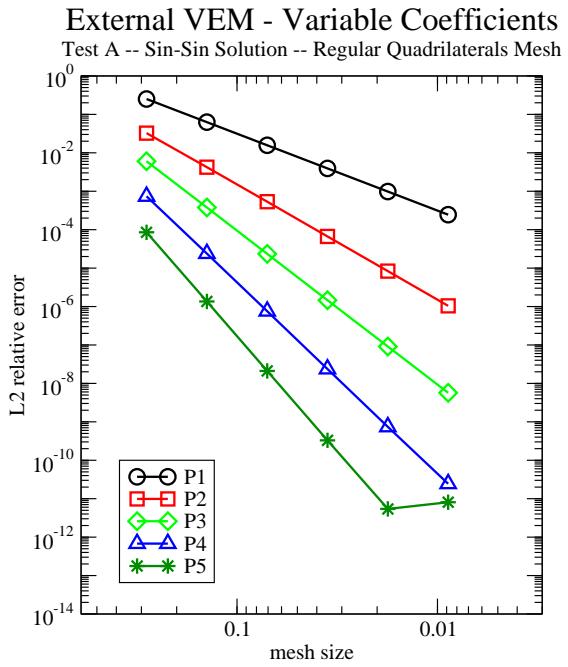


Fig. 79. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

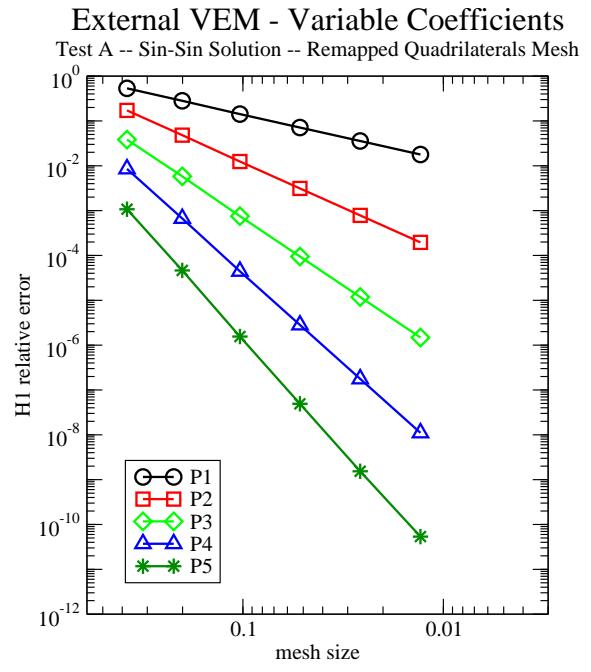
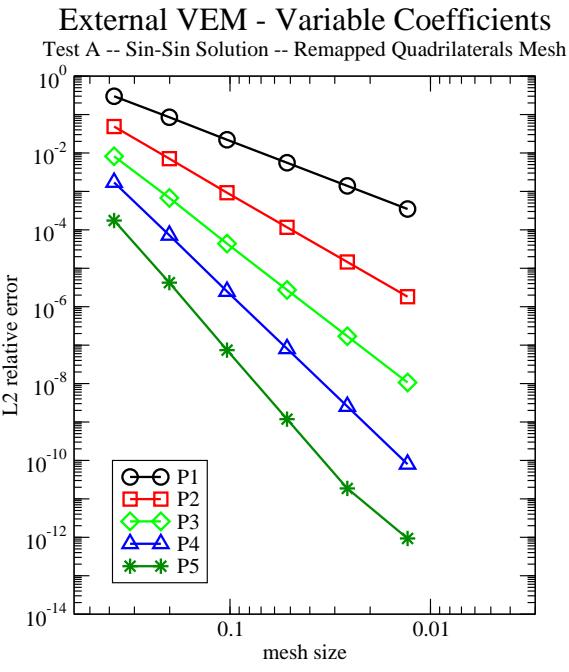


Fig. 80. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

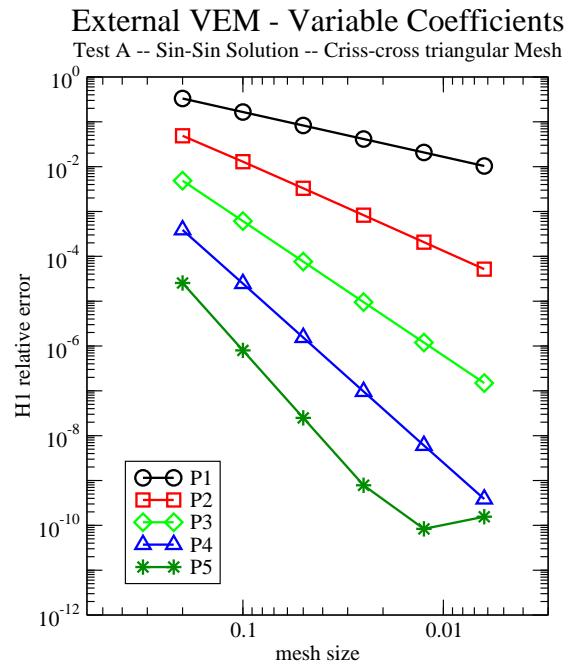
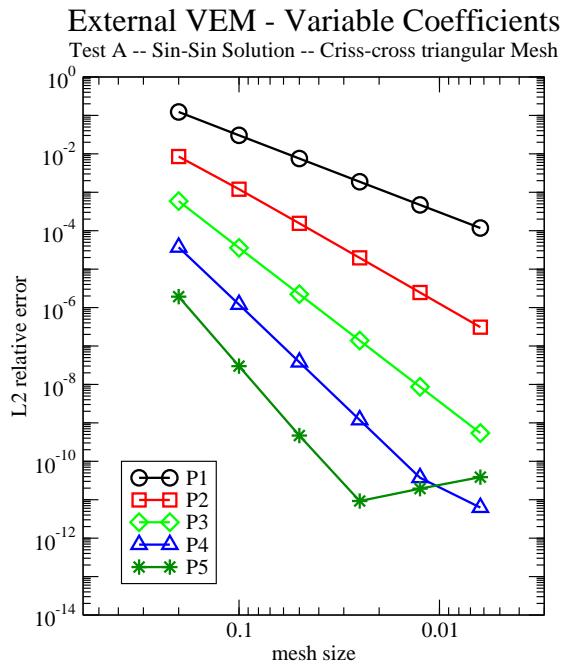


Fig. 81. External VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

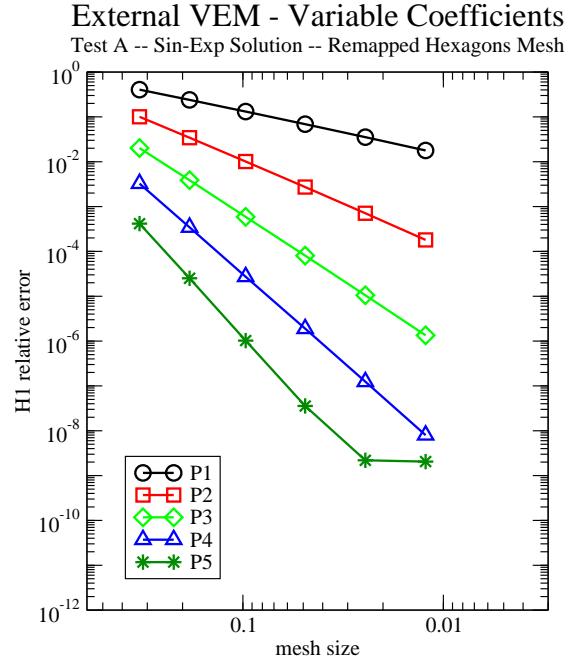
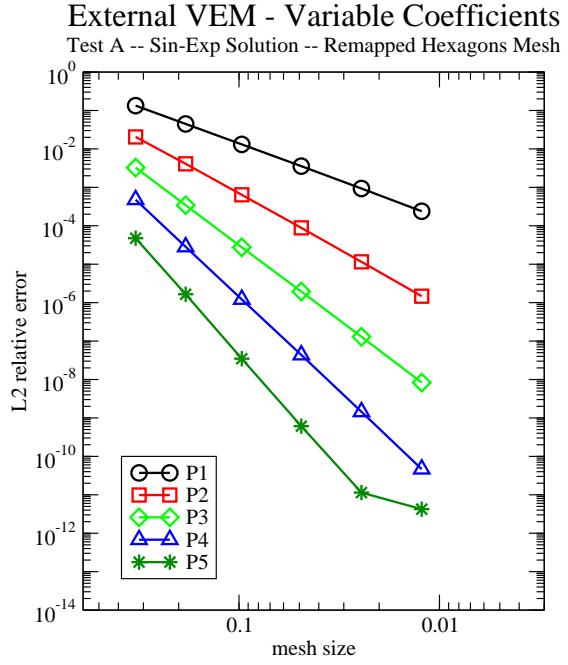


Fig. 82. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

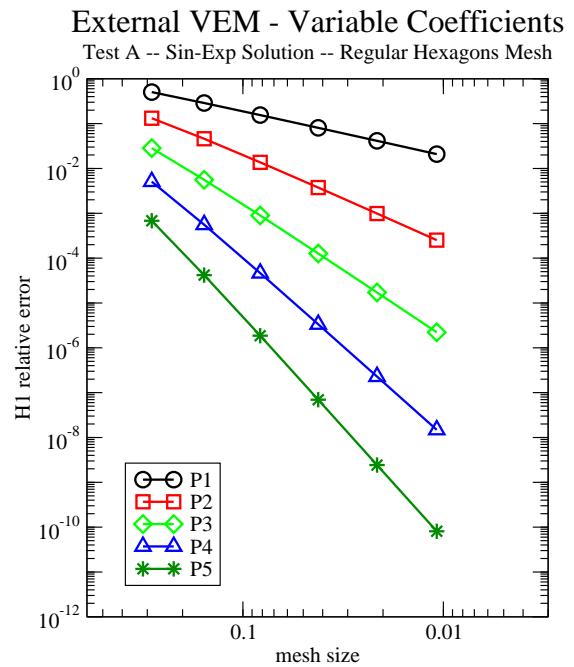
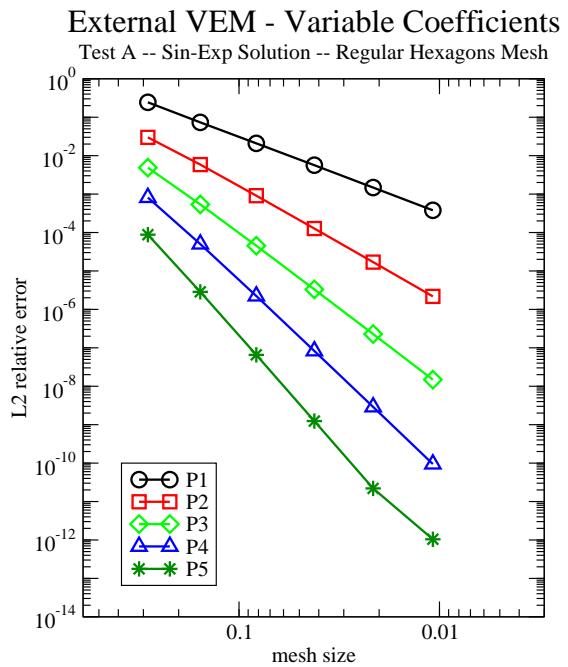


Fig. 83. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

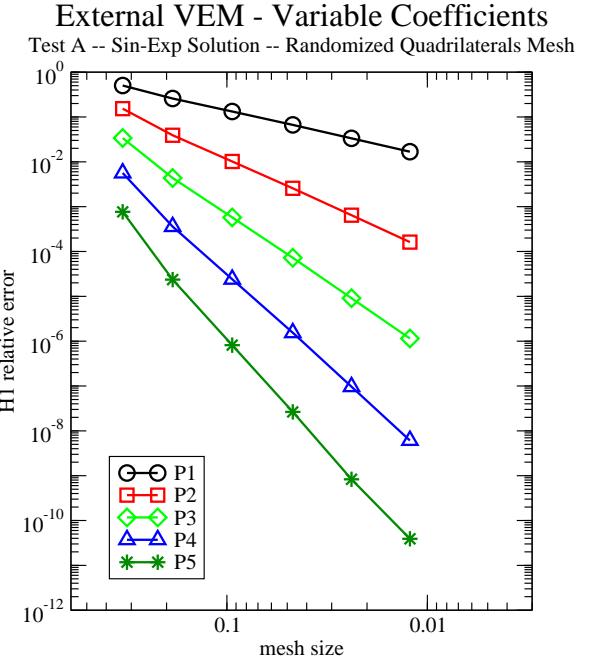
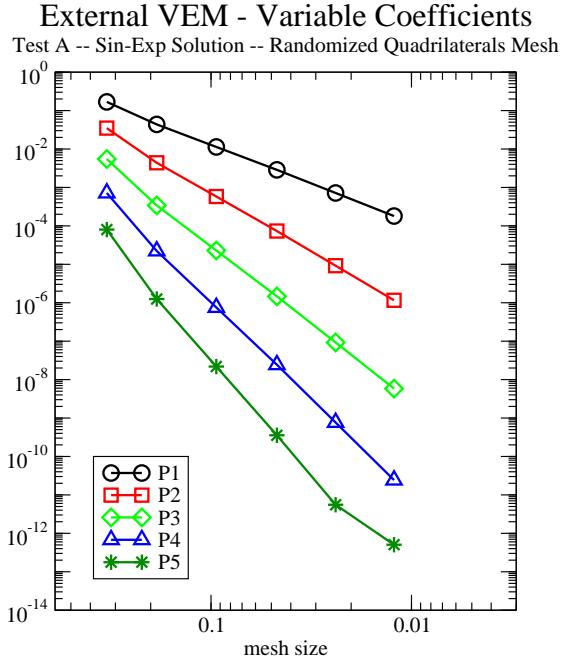


Fig. 84. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

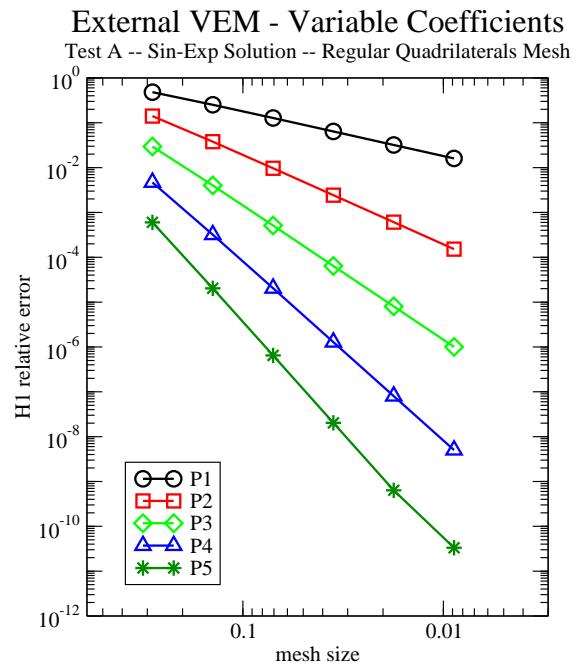
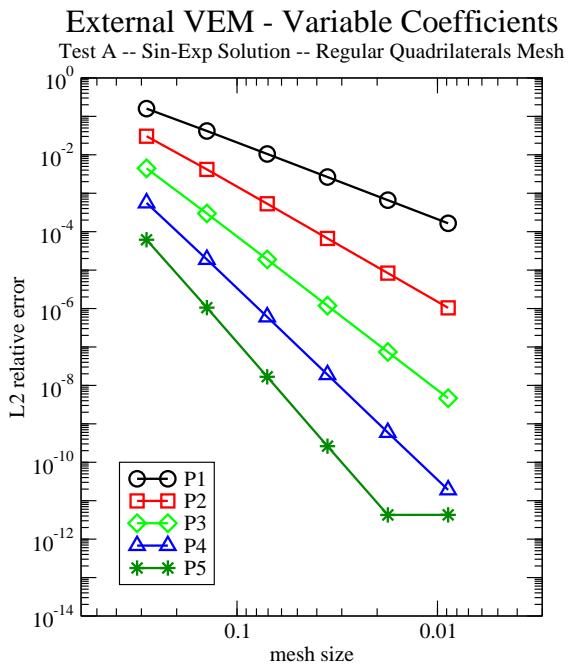


Fig. 85. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

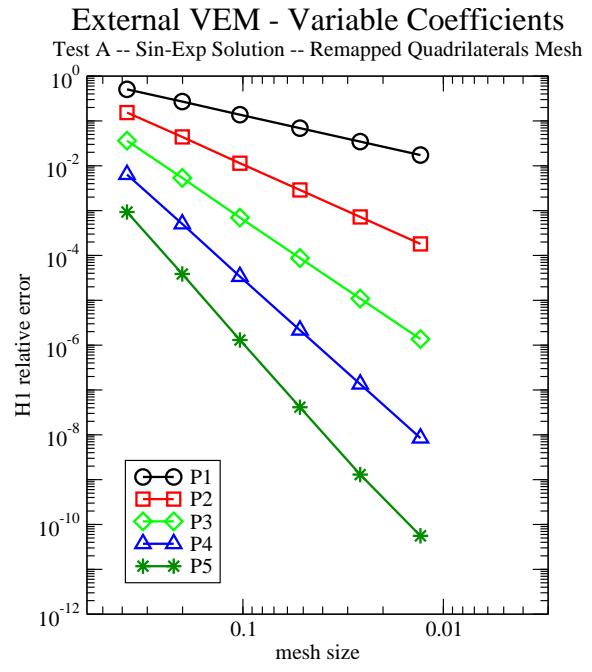
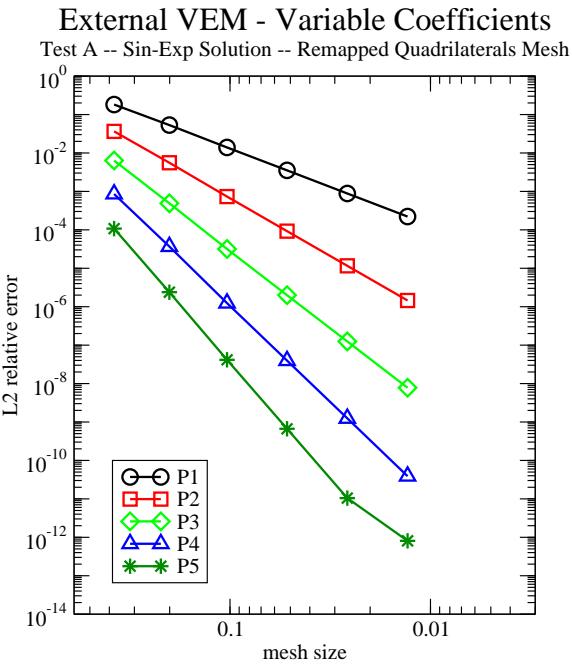


Fig. 86. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

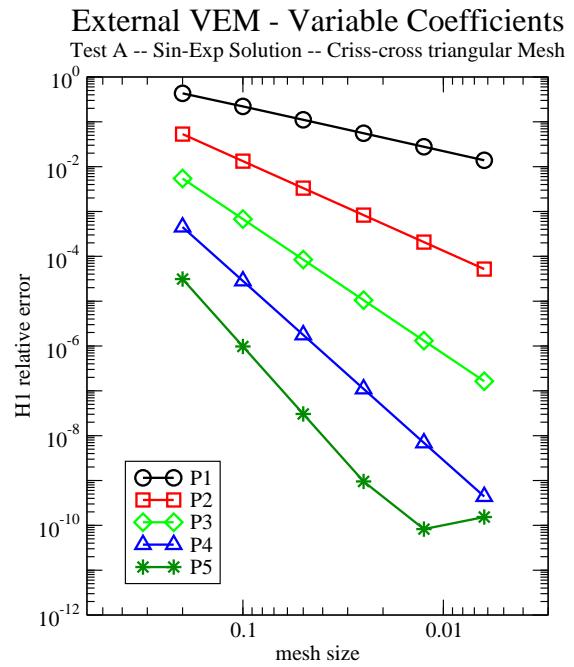
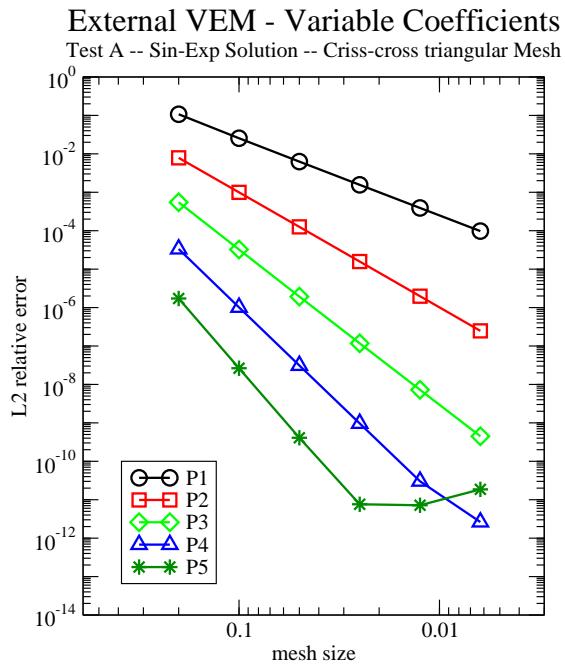


Fig. 87. External VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

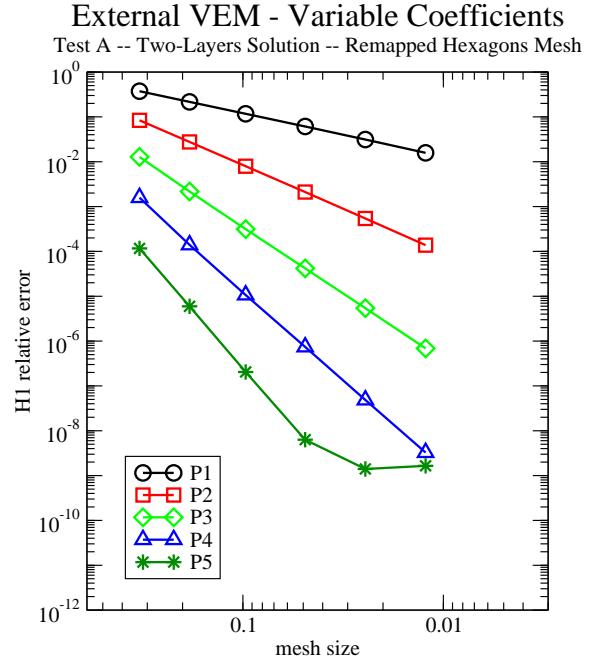
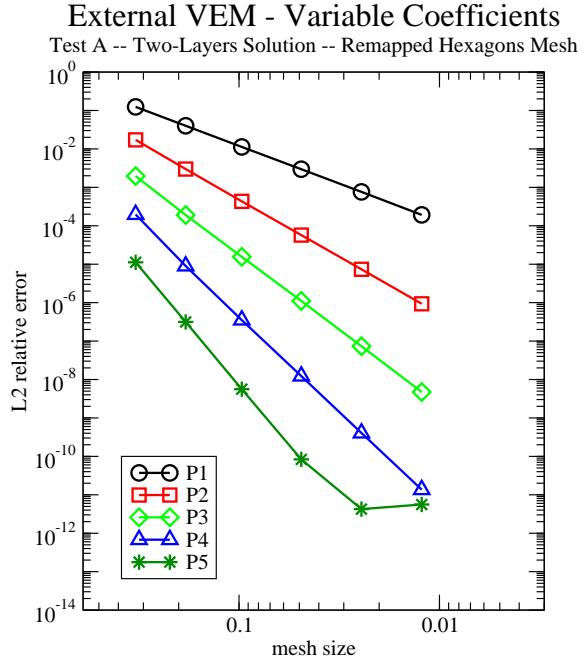


Fig. 88. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

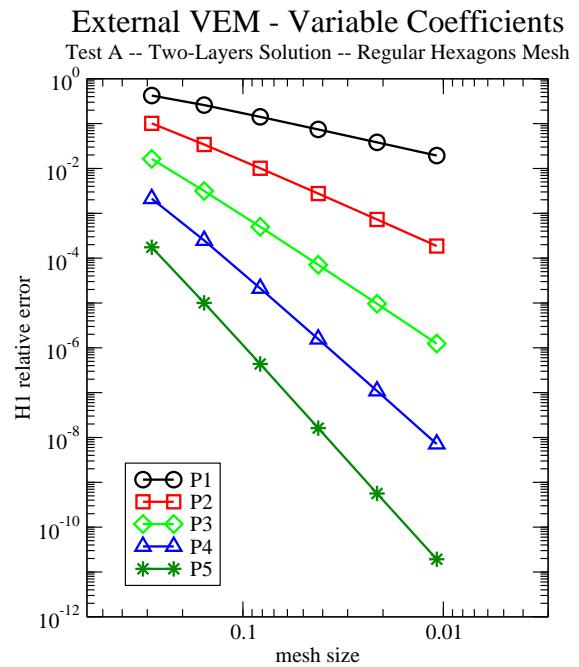
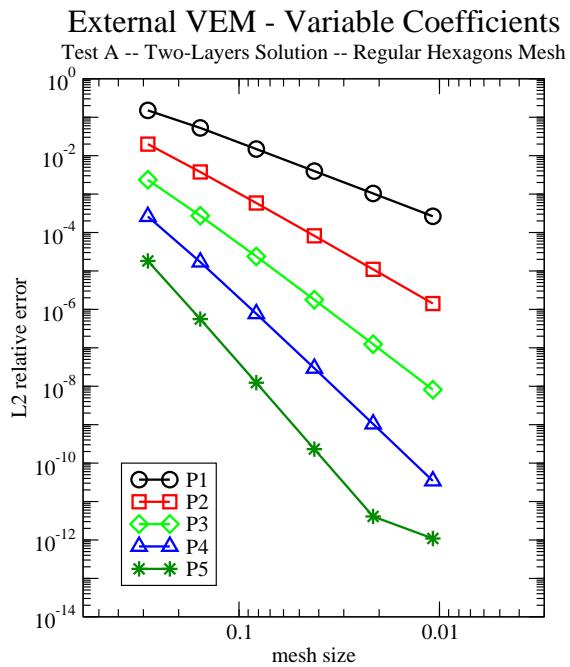


Fig. 89. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular hexagons.

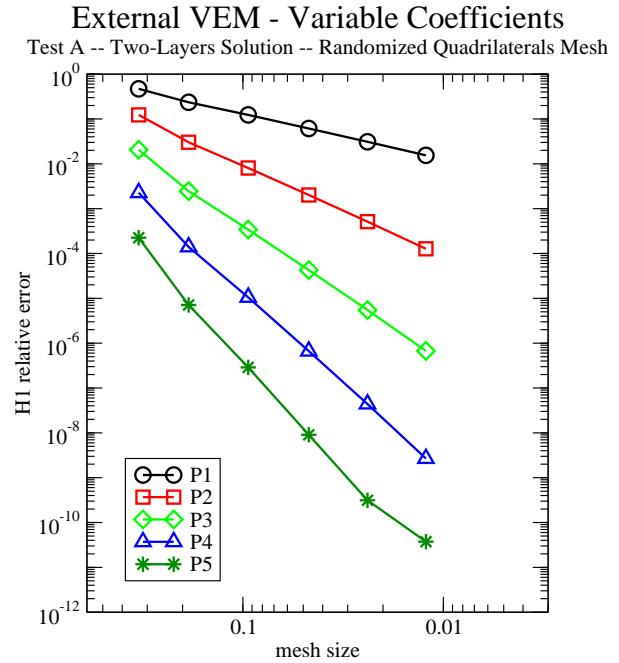
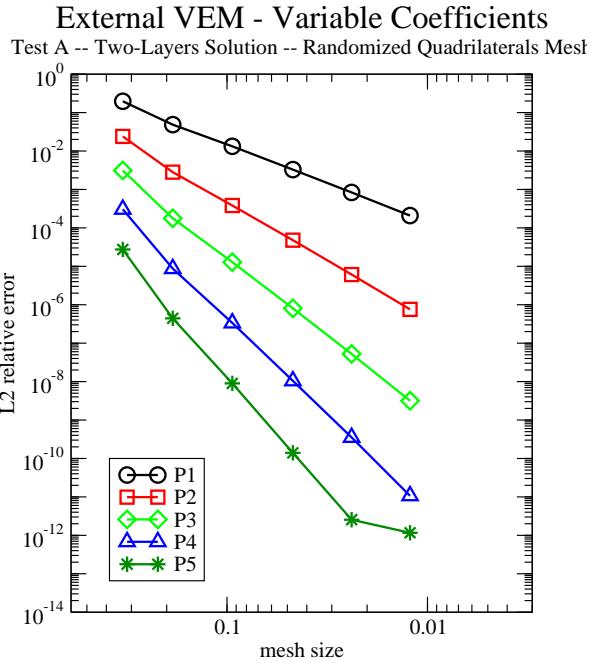


Fig. 90. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

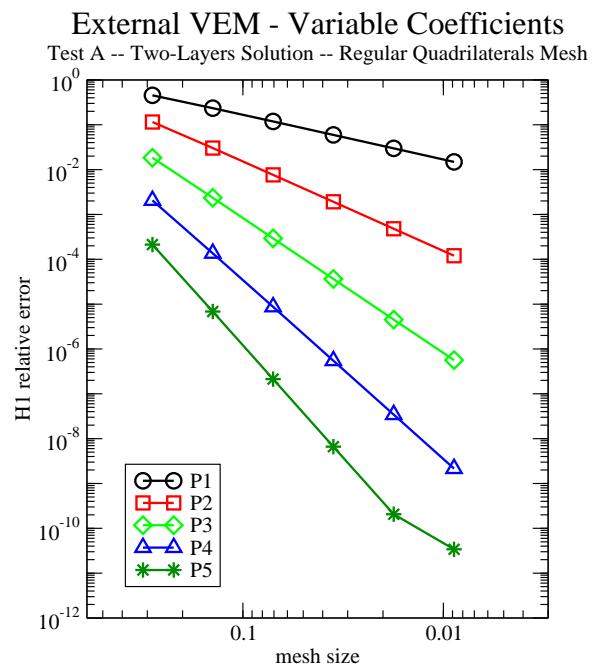
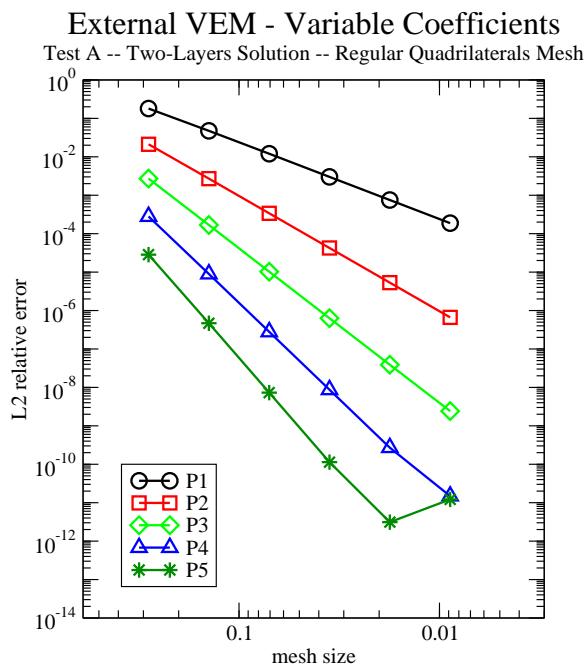


Fig. 91. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

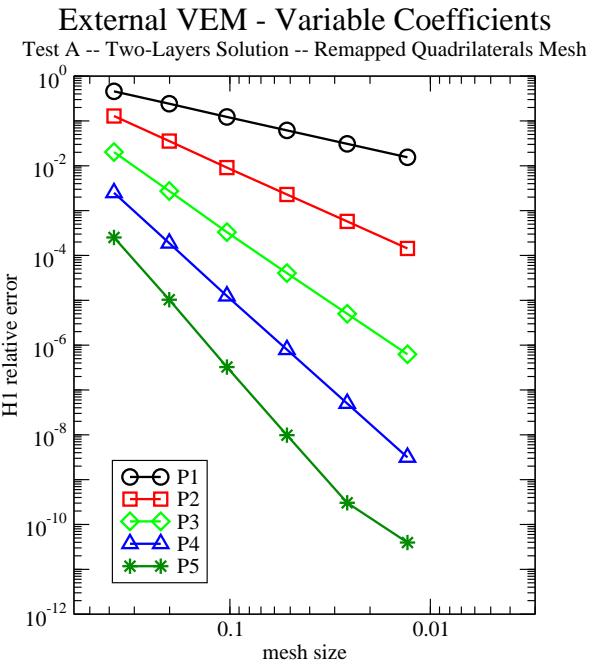
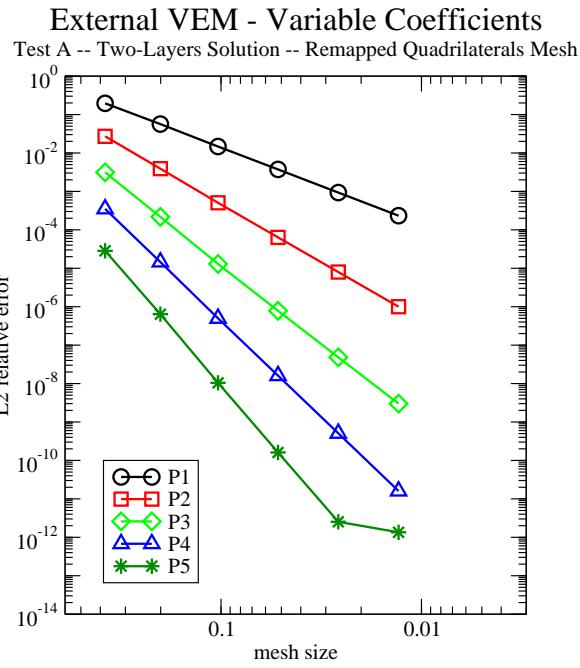


Fig. 92. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

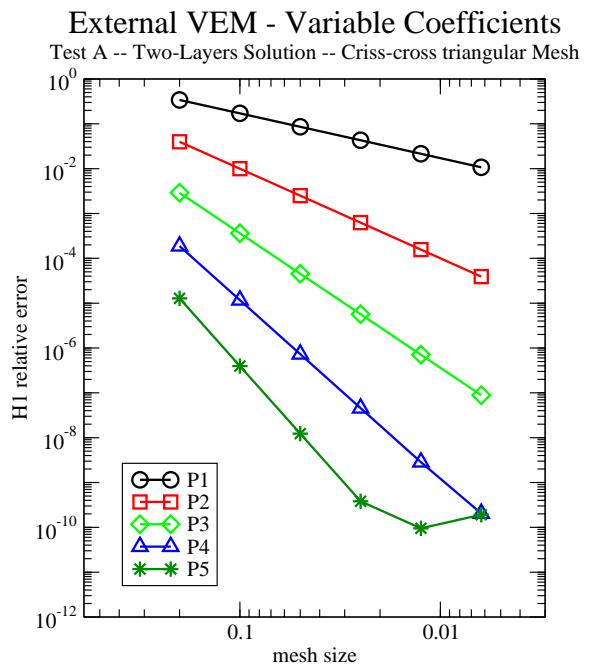
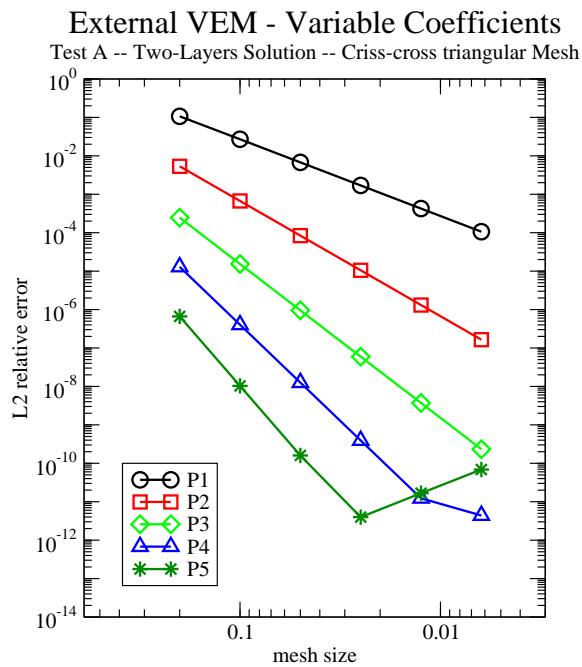


Fig. 93. External VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

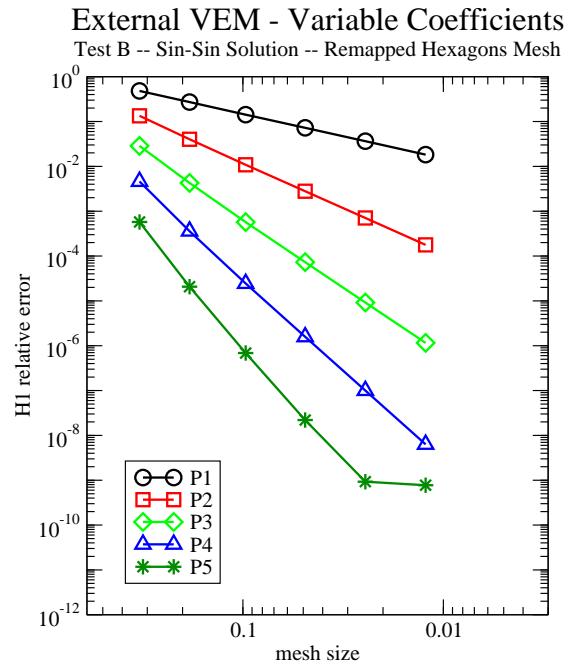
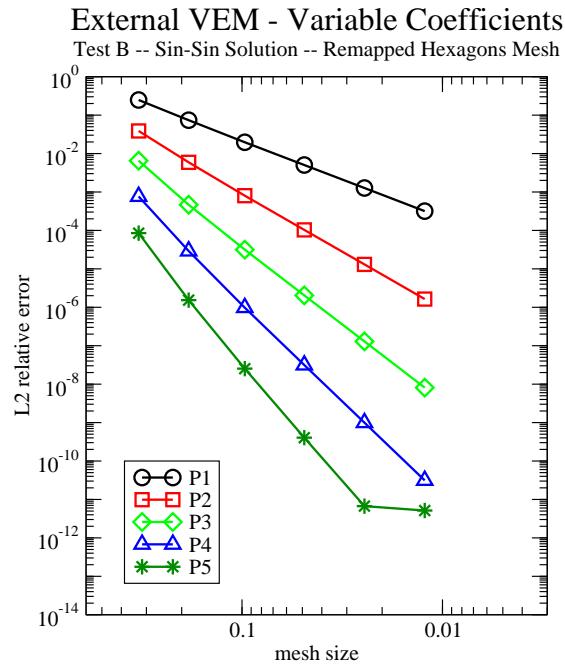


Fig. 94. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of smoothly remapped hexagons.

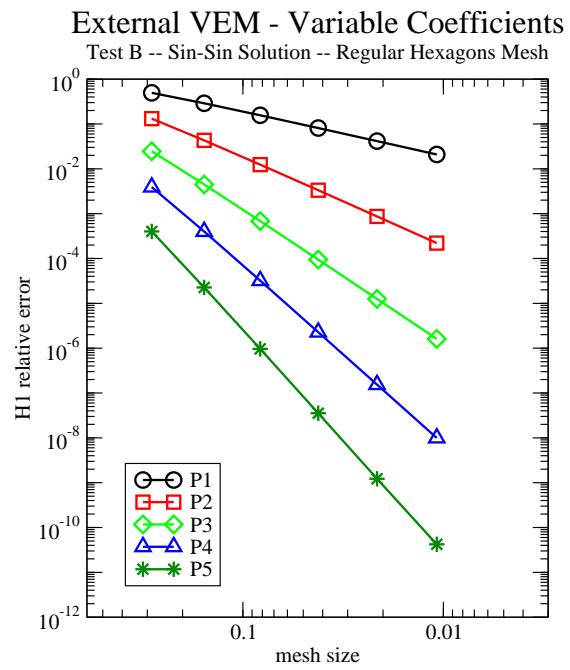
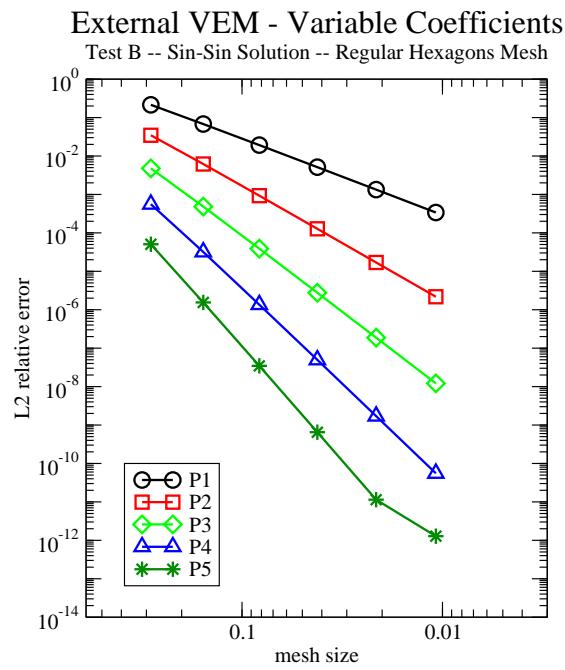


Fig. 95. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular hexagons.

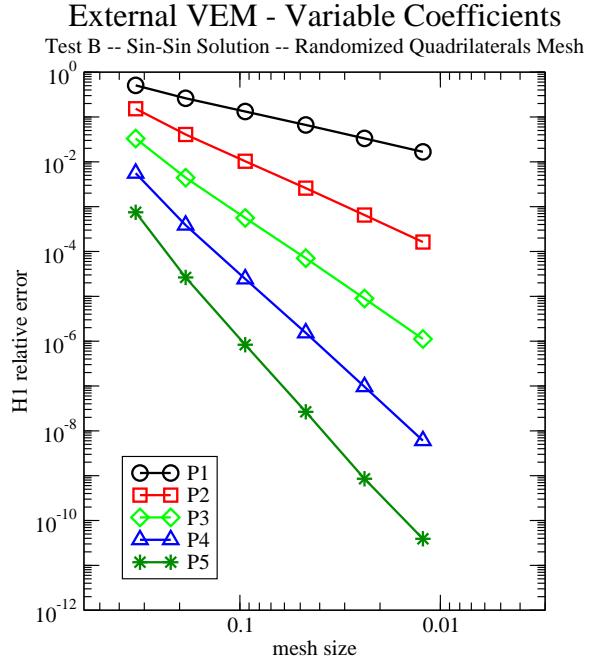
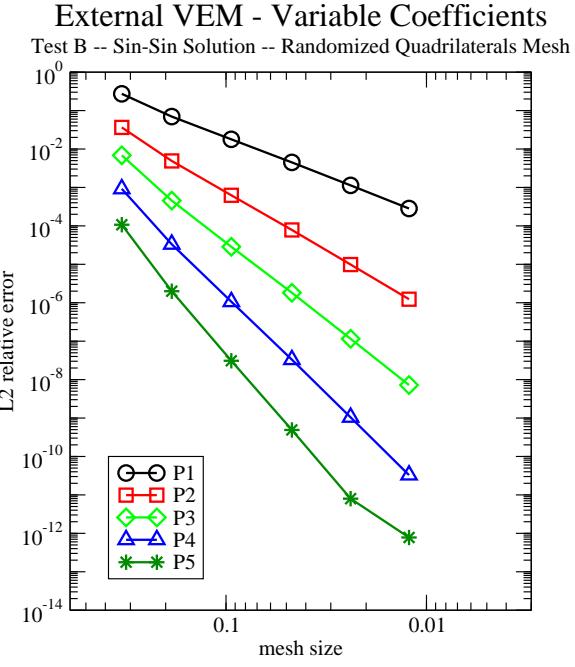


Fig. 96. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of randomized quadrilateral cells.

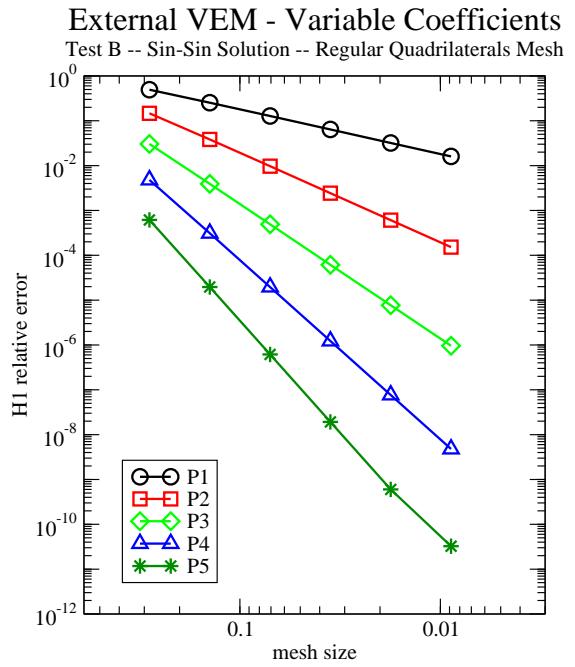
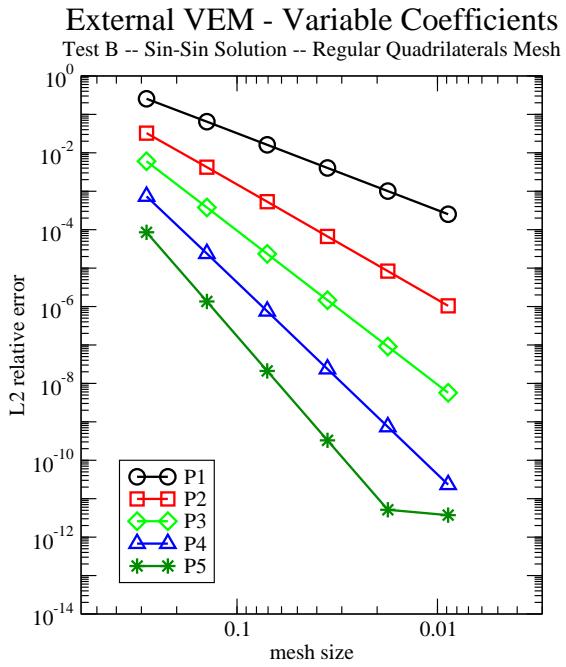


Fig. 97. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular quadrilateral cells (squares).

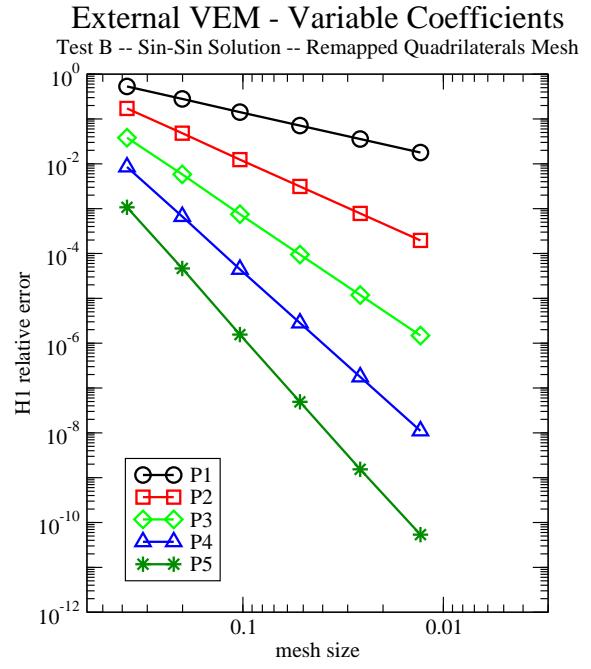
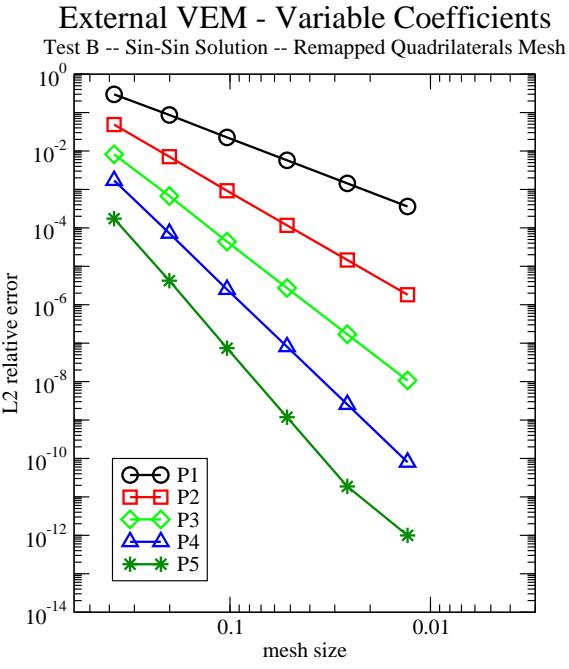


Fig. 98. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

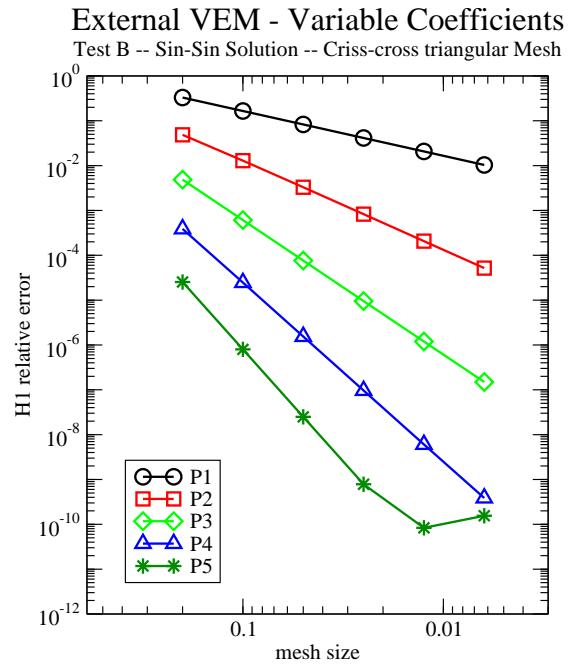
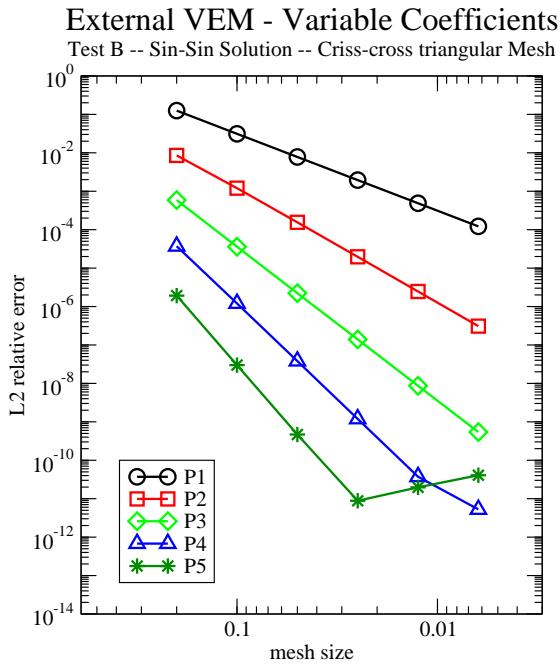


Fig. 99. External VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

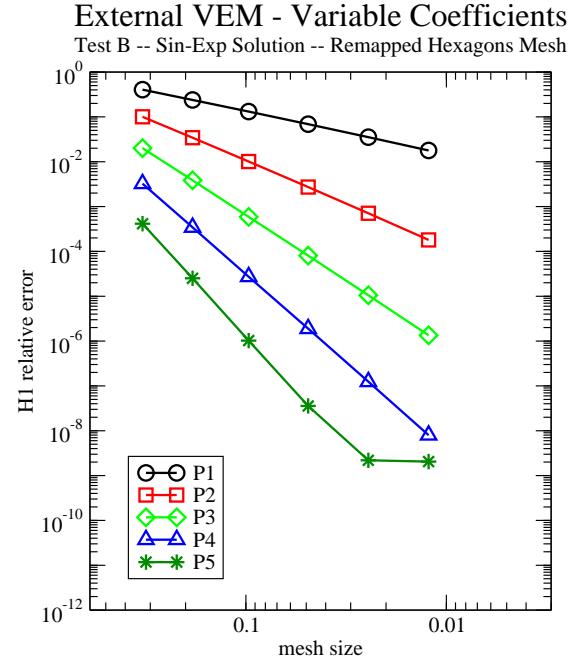
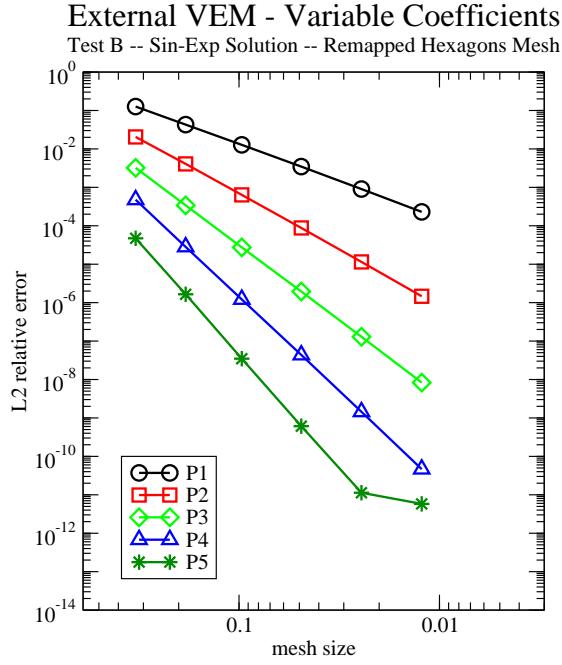


Fig. 100. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of smoothly remapped hexagons.

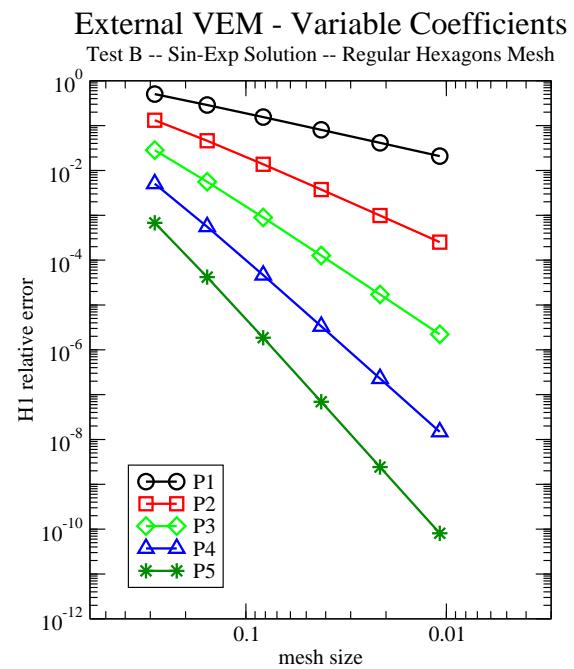
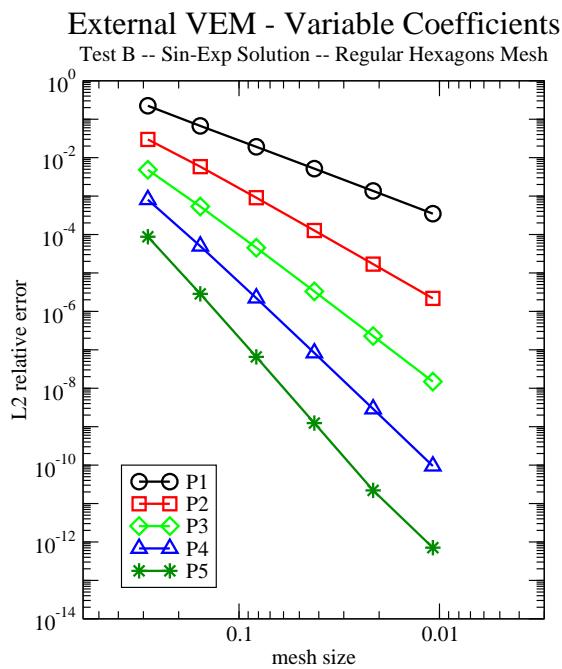


Fig. 101. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular hexagons.

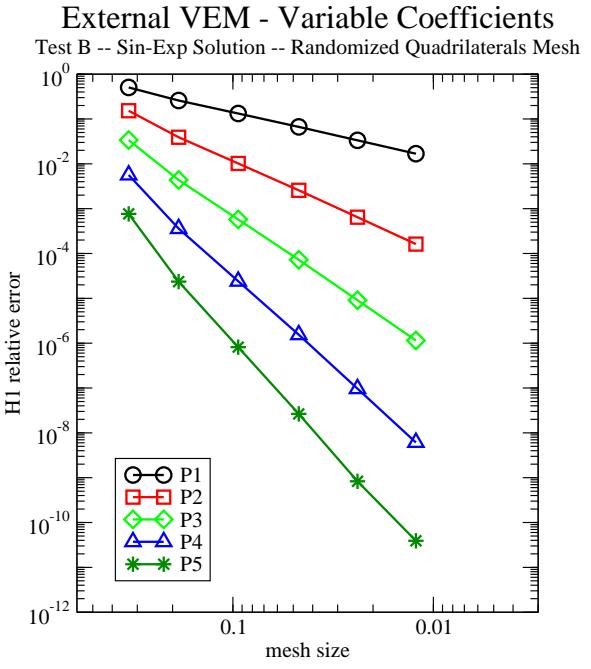
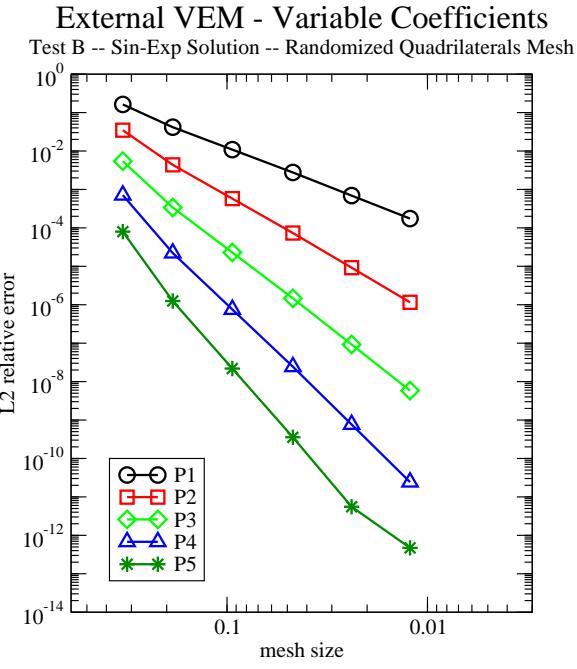


Fig. 102. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of randomized quadrilateral cells.

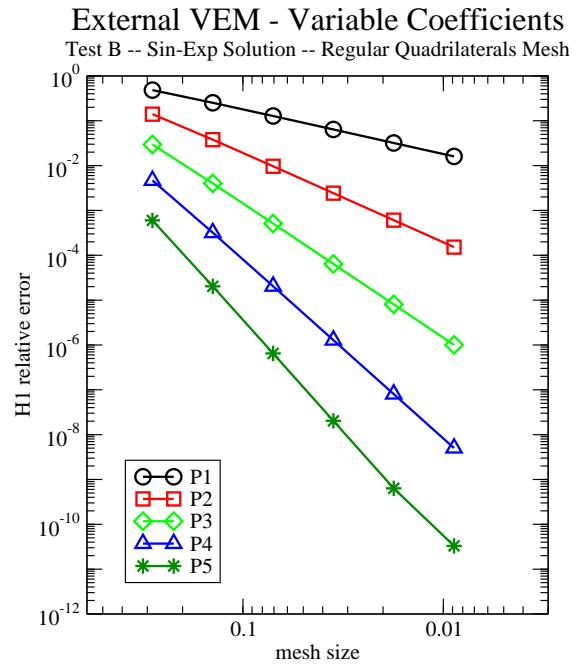
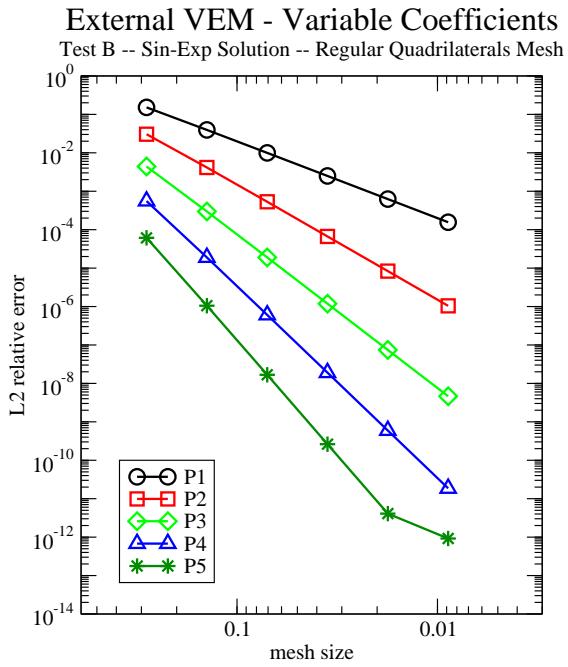


Fig. 103. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular quadrilateral cells (squares).

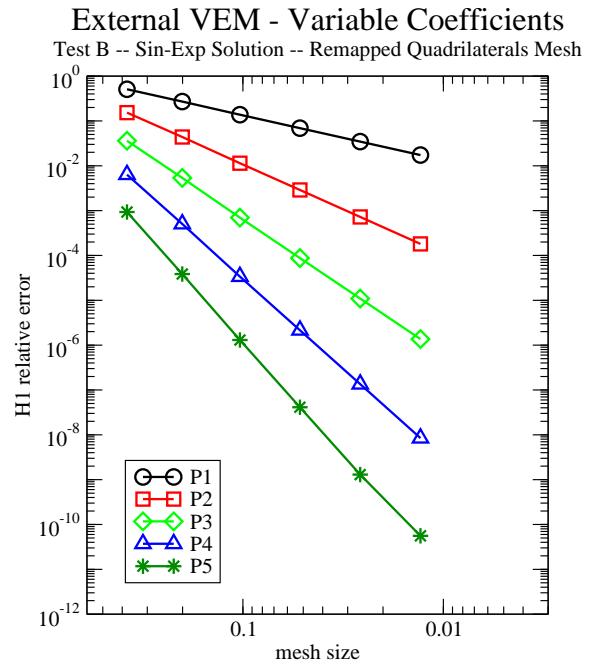
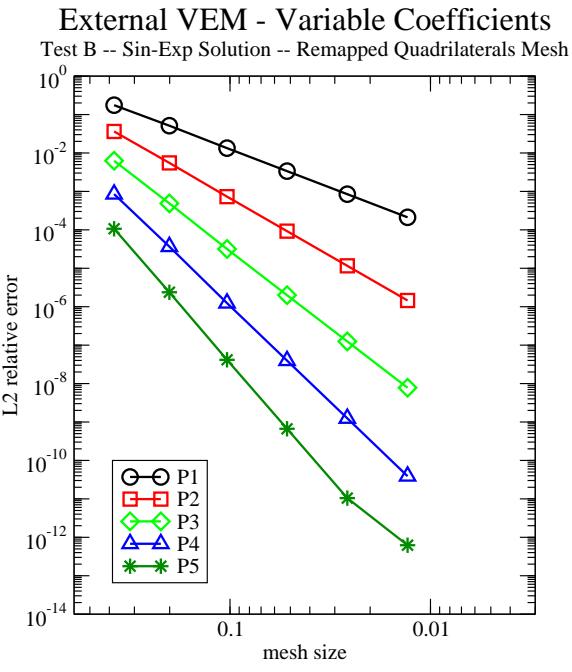


Fig. 104. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

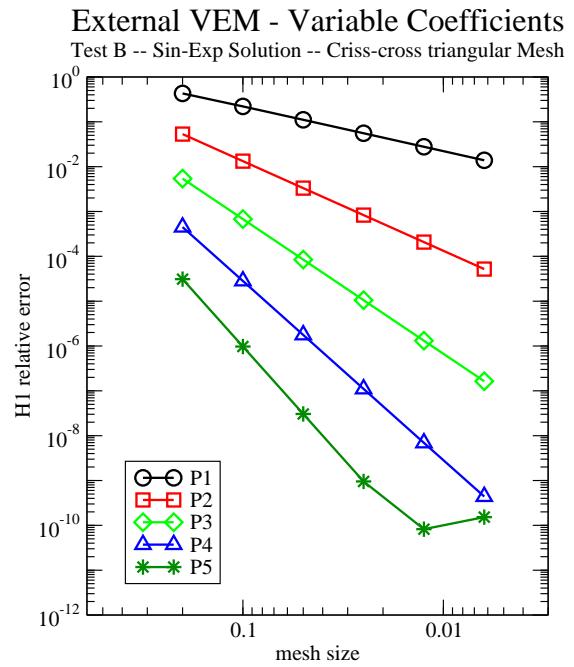
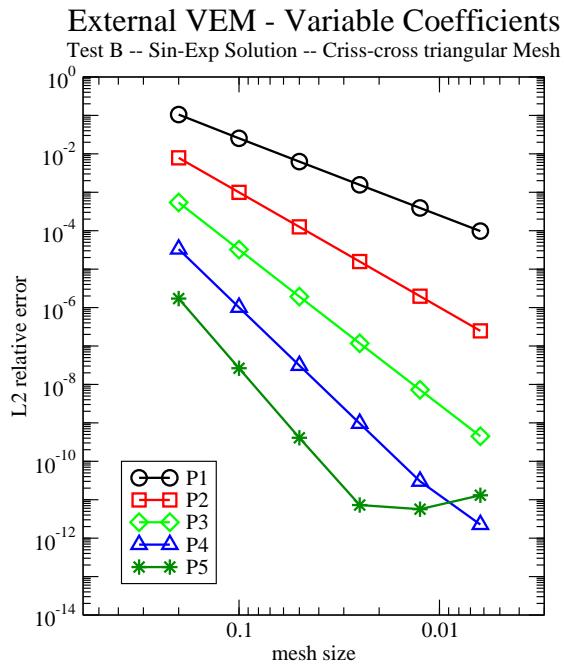


Fig. 105. External VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

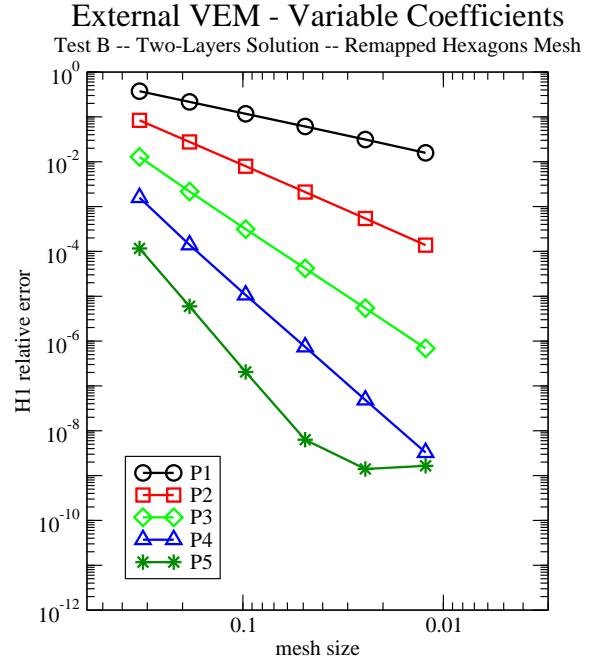
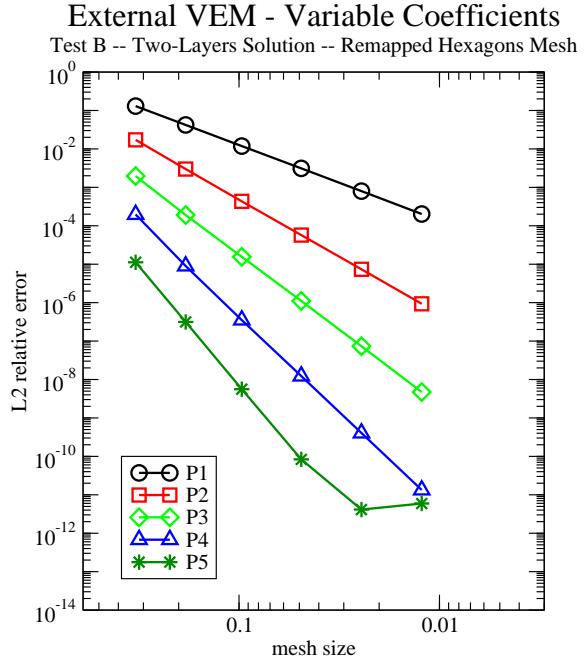


Fig. 106. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of smoothly remapped hexagons.

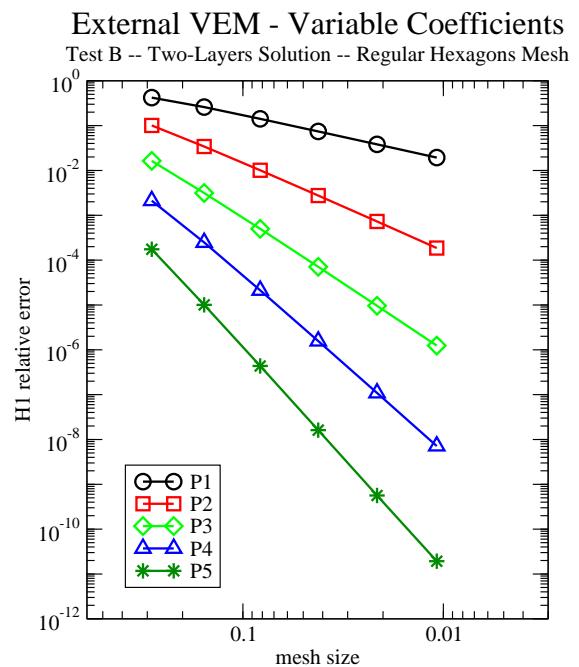
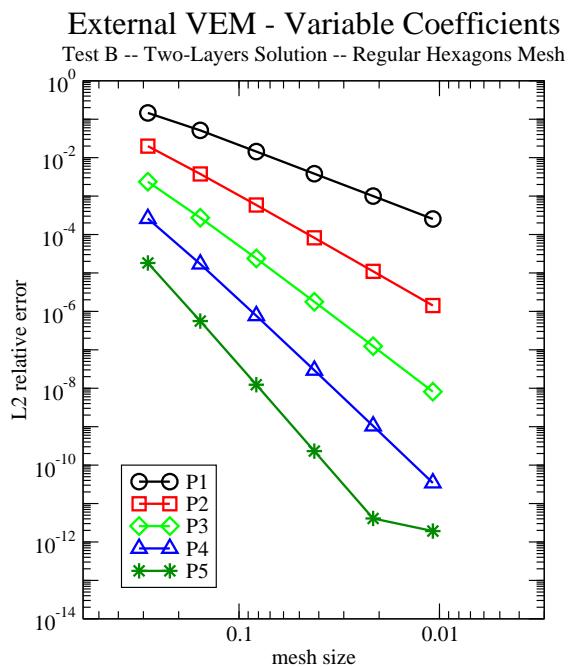


Fig. 107. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular hexagons.

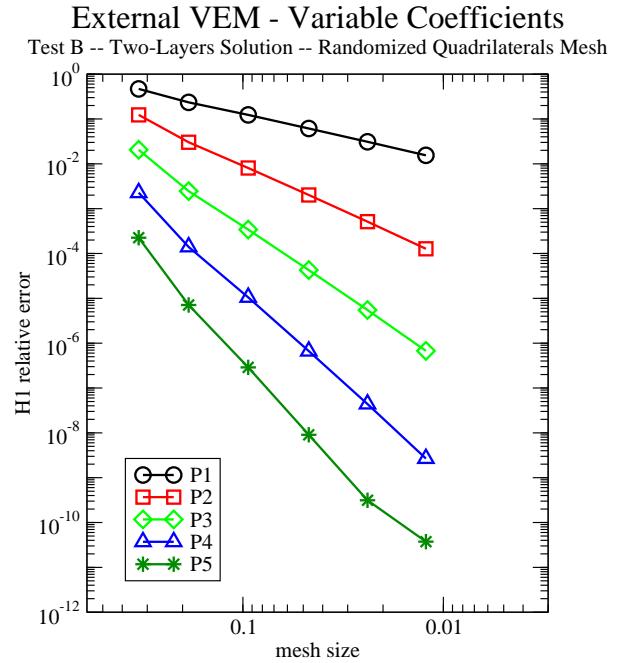
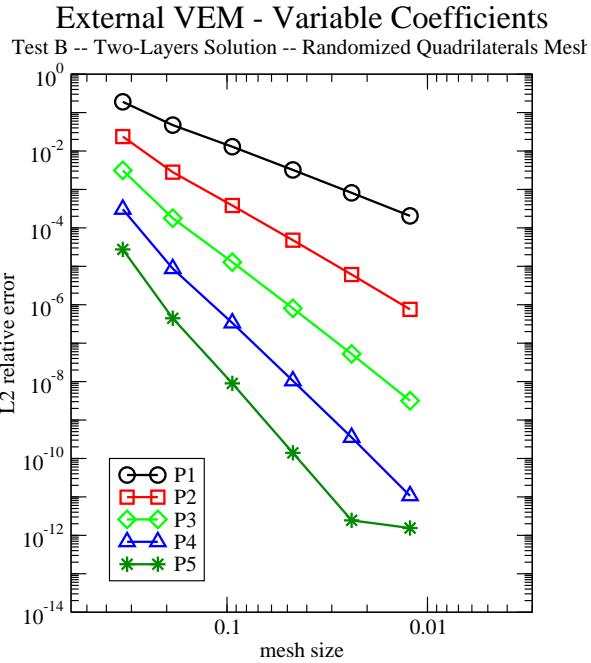


Fig. 108. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of randomized quadrilateral cells.

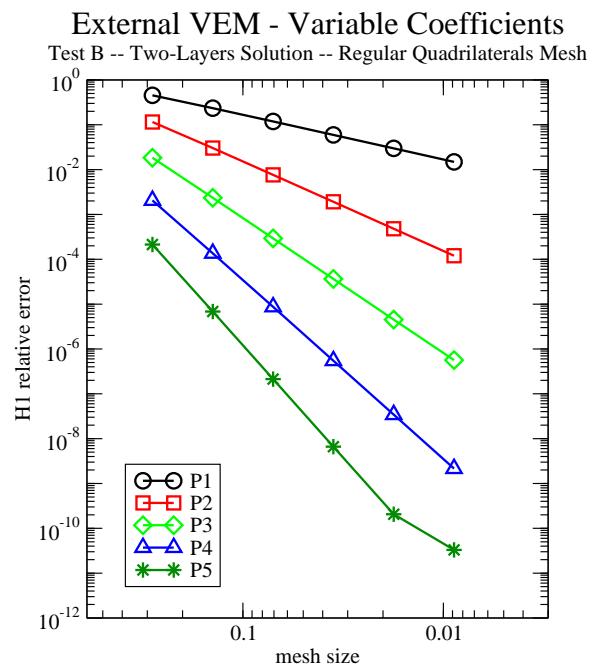
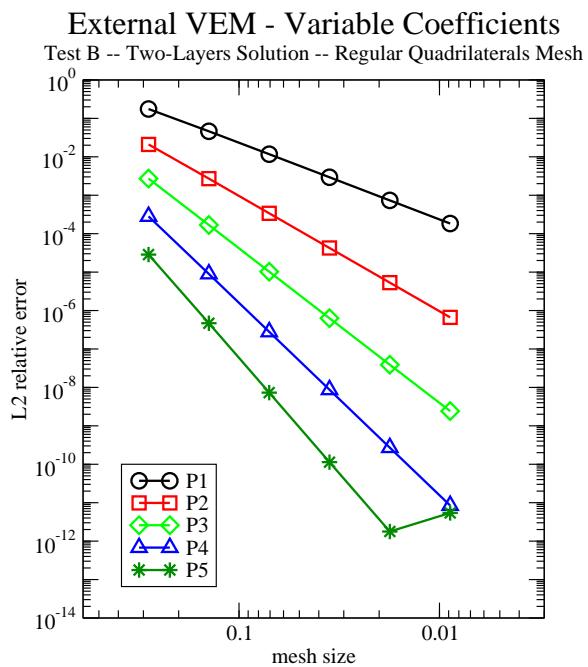


Fig. 109. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular quadrilateral cells (squares).

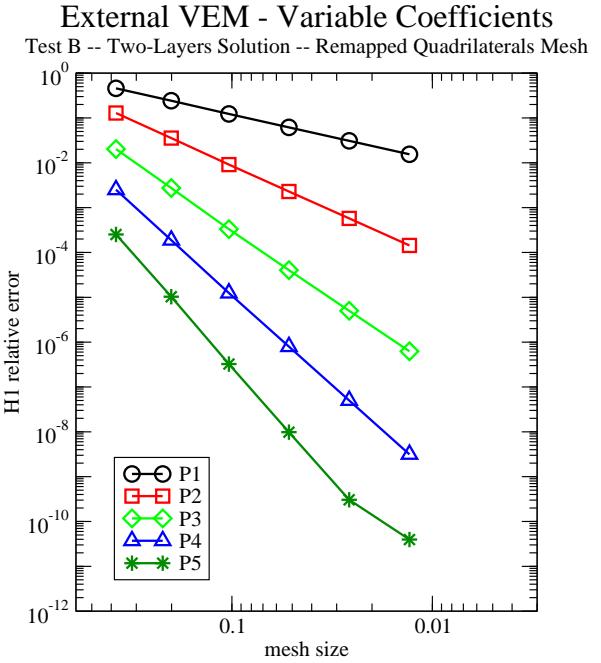
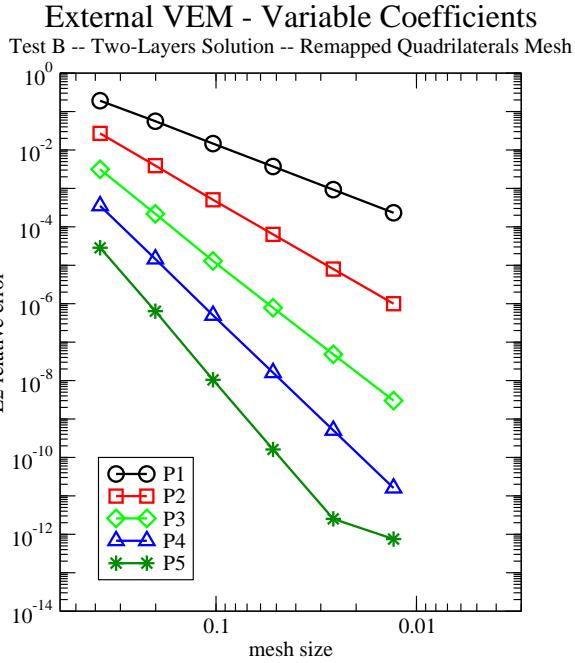


Fig. 110. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

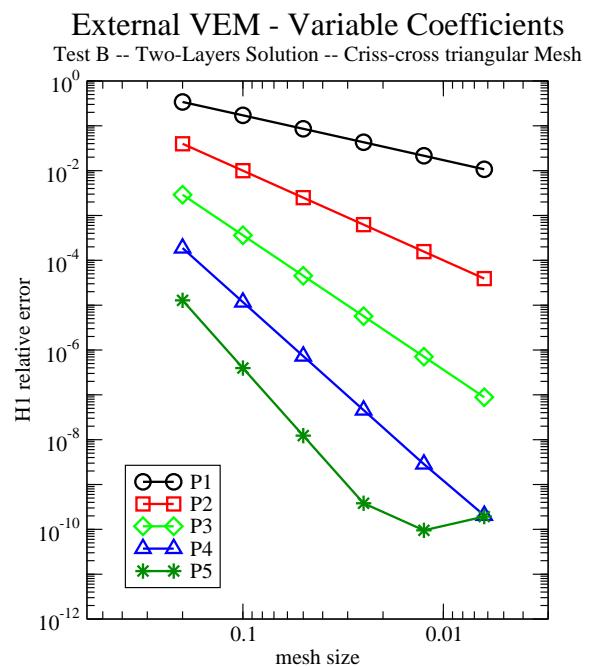
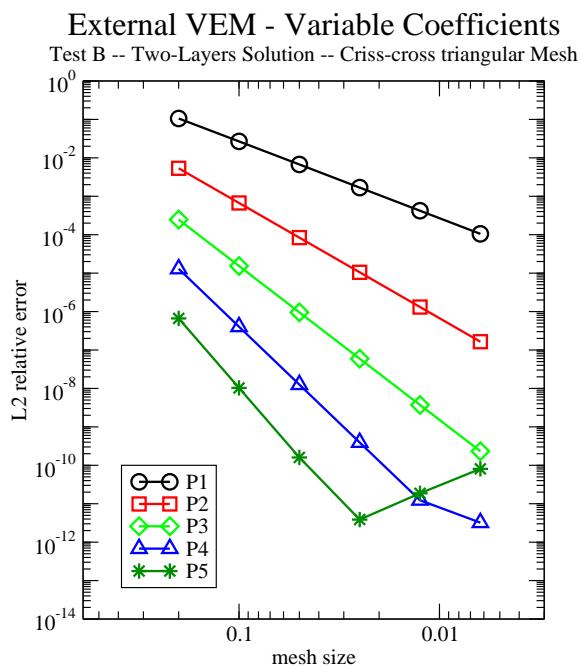


Fig. 111. External VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular triangular cells, (criss-cross).

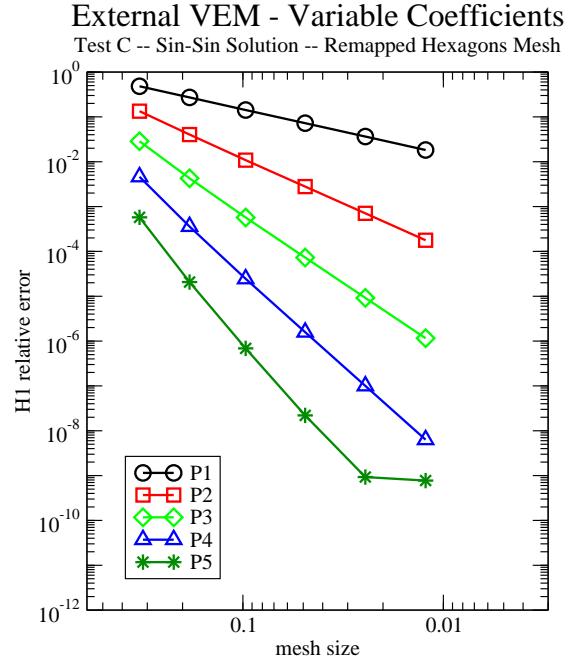
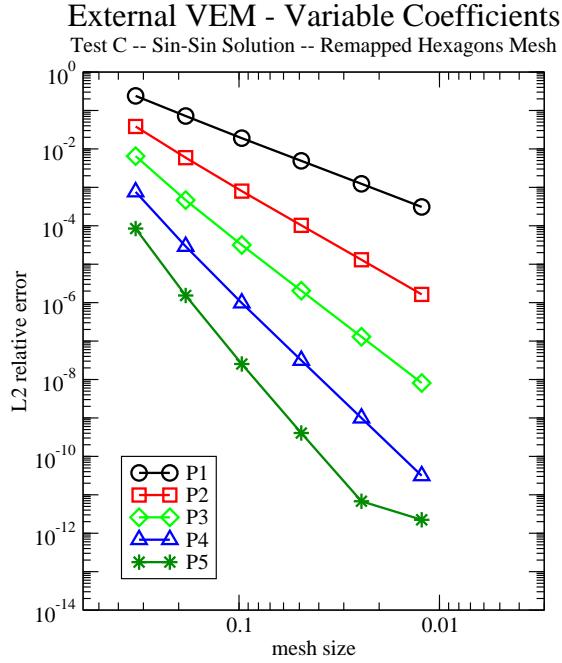


Fig. 112. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

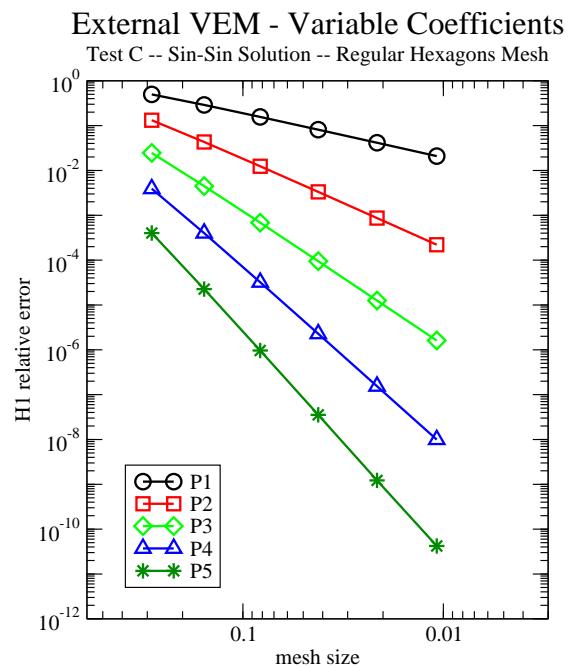
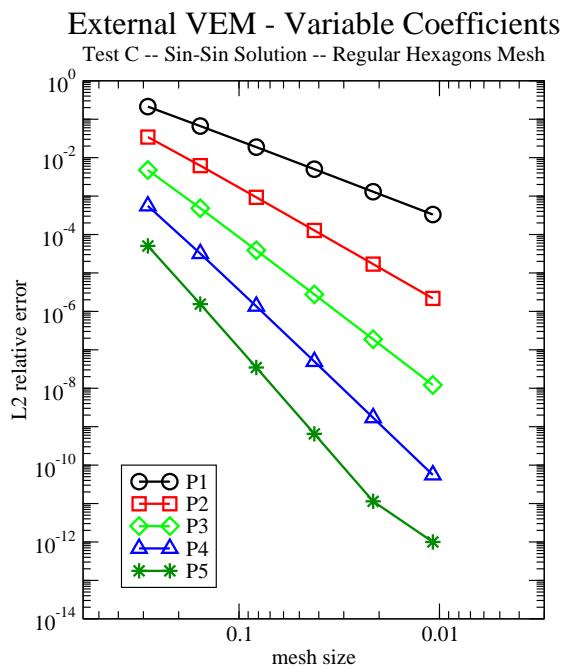


Fig. 113. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

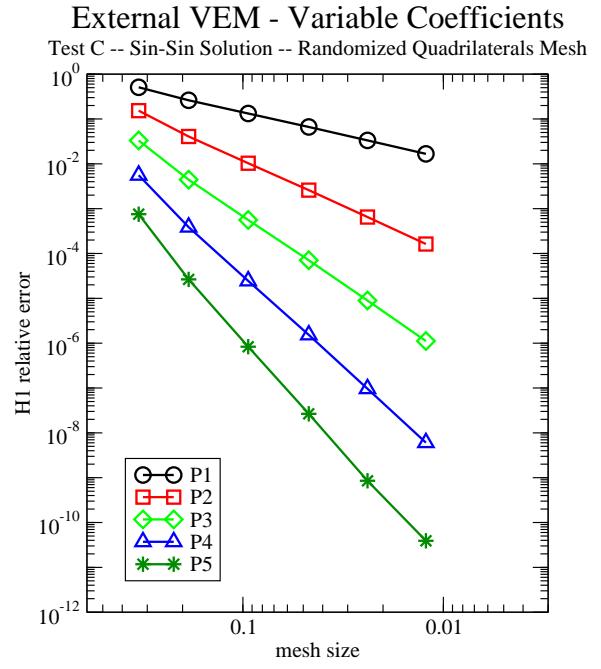
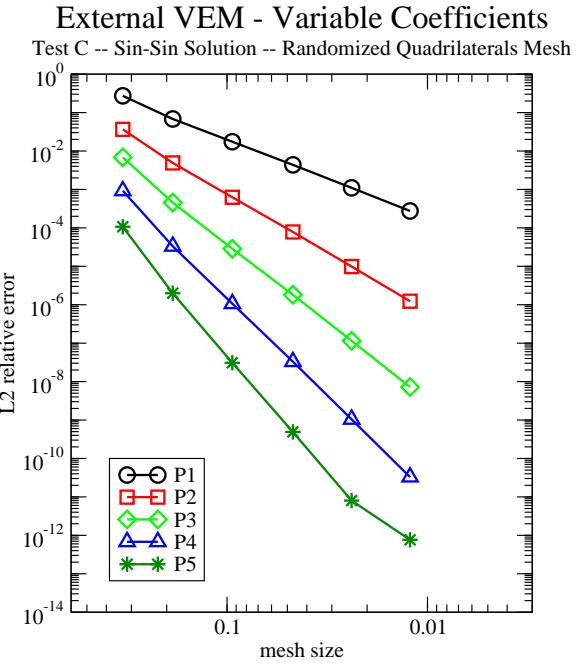


Fig. 114. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

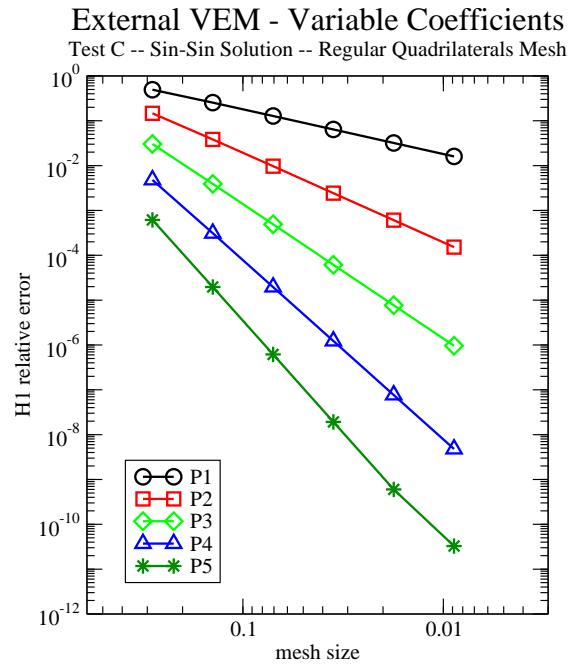
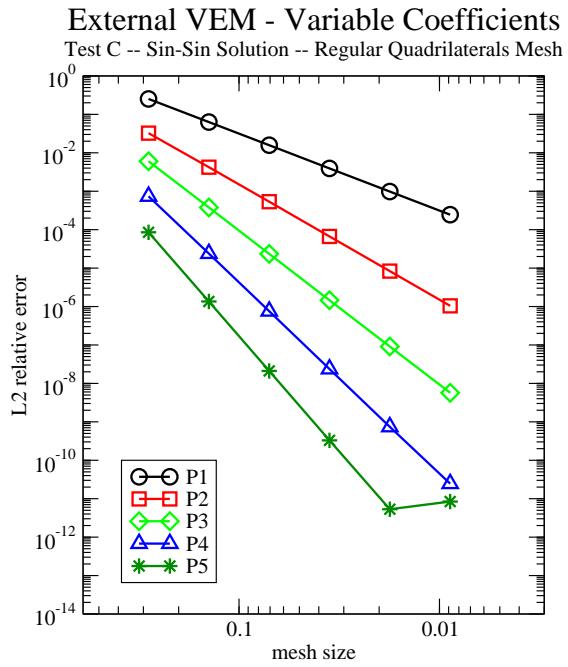


Fig. 115. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

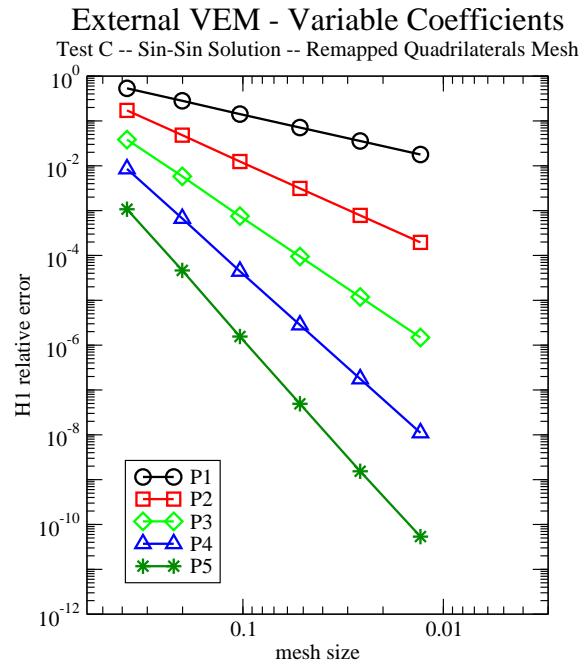
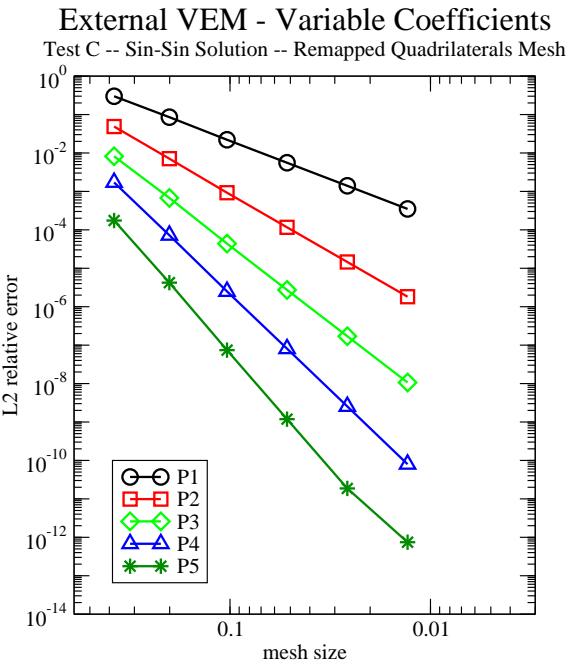


Fig. 116. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

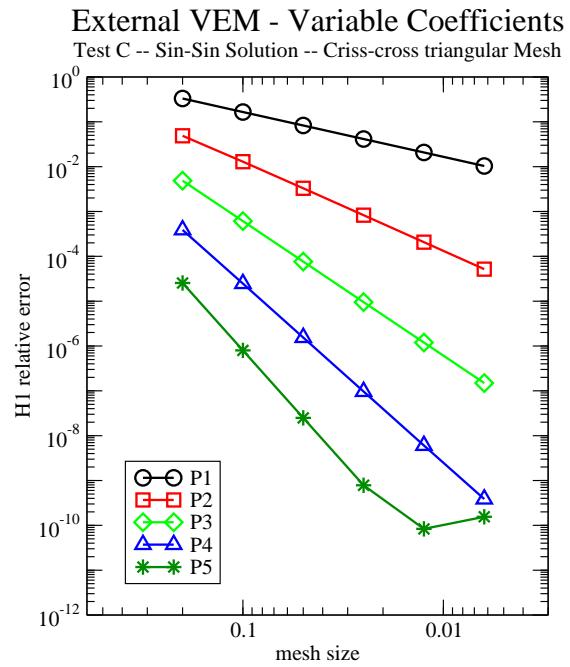
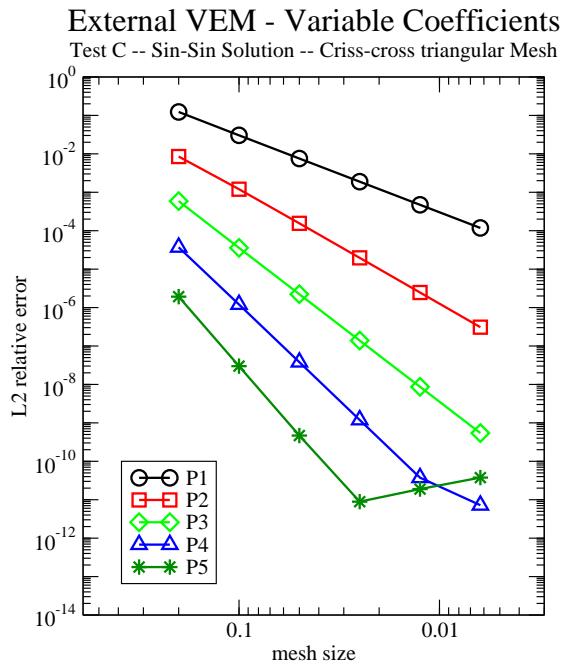


Fig. 117. External VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

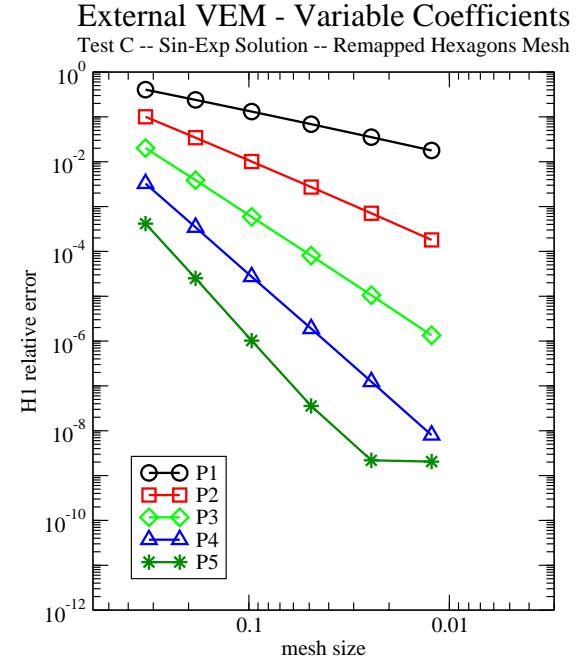
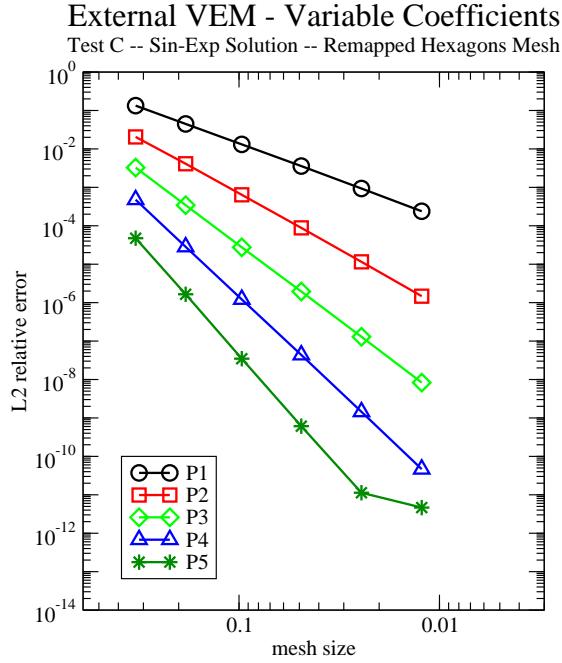


Fig. 118. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

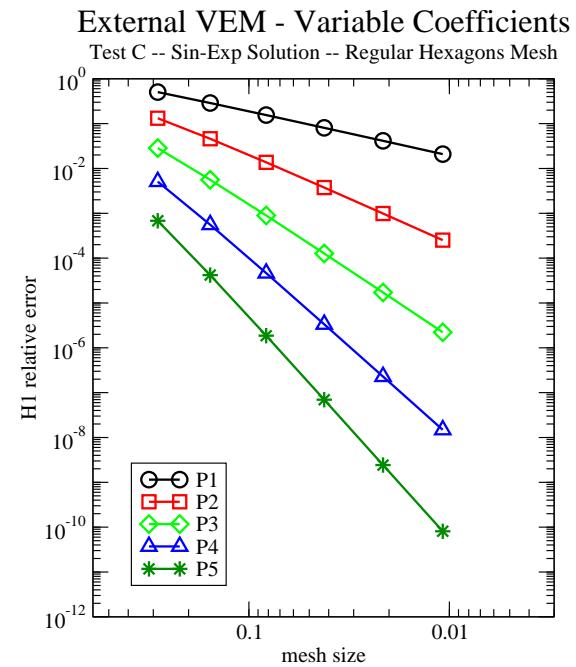
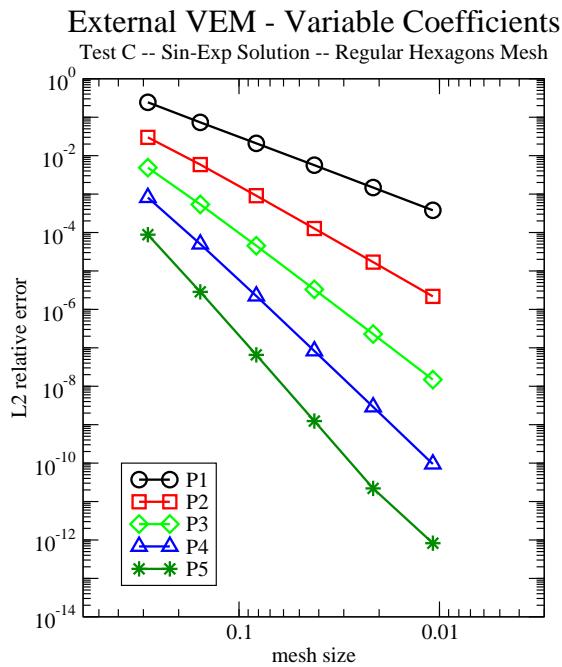


Fig. 119. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

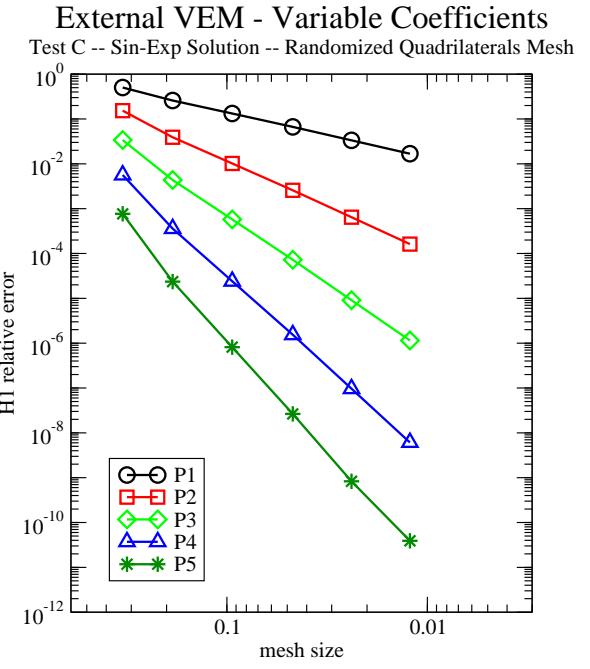
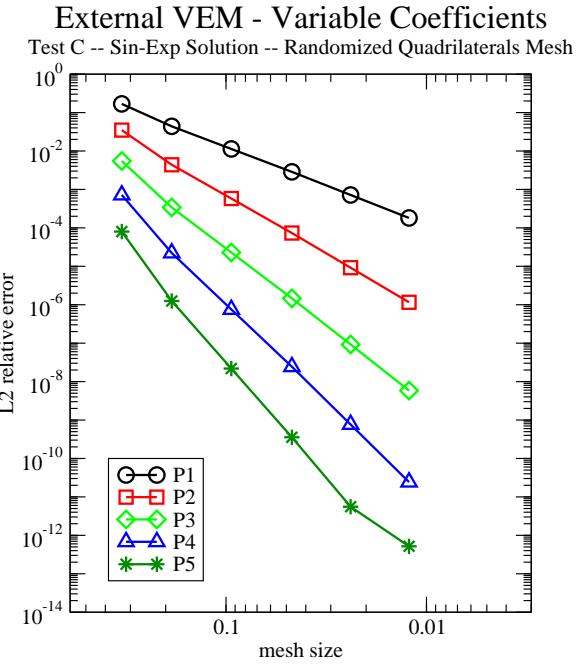


Fig. 120. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

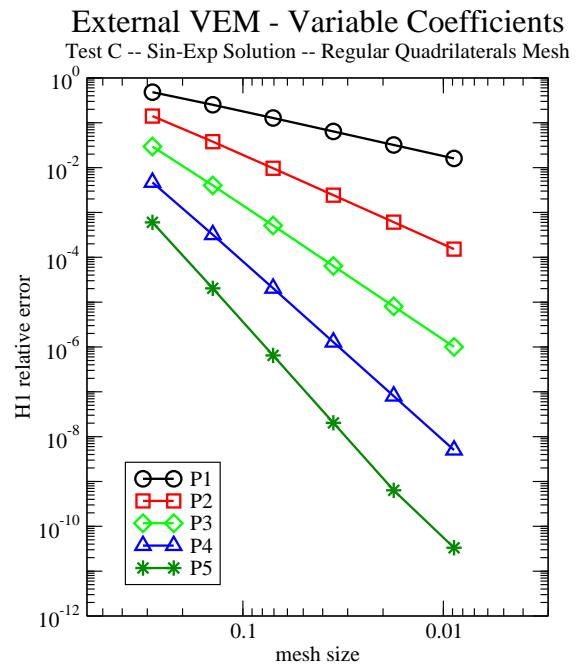
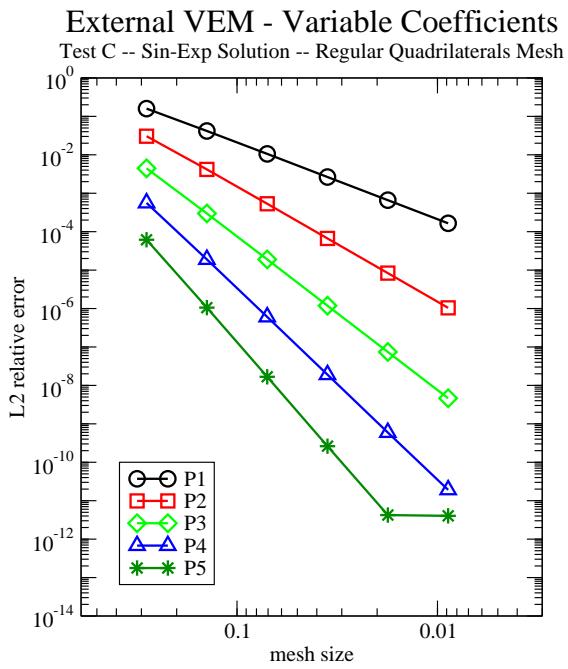


Fig. 121. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

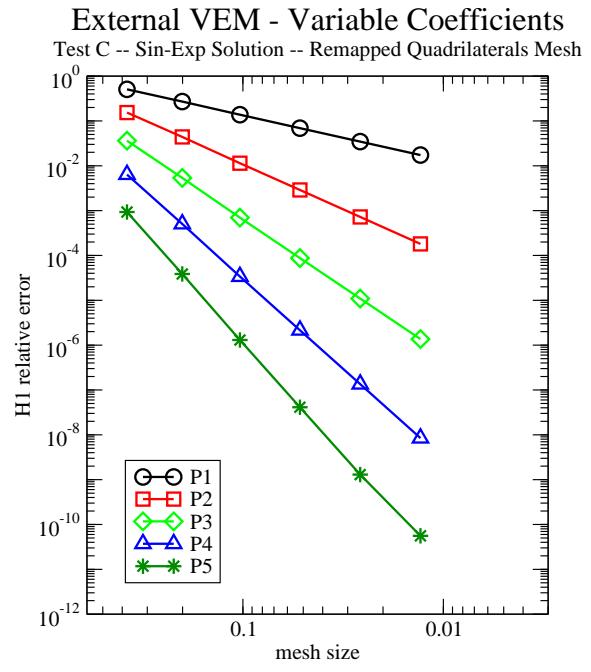
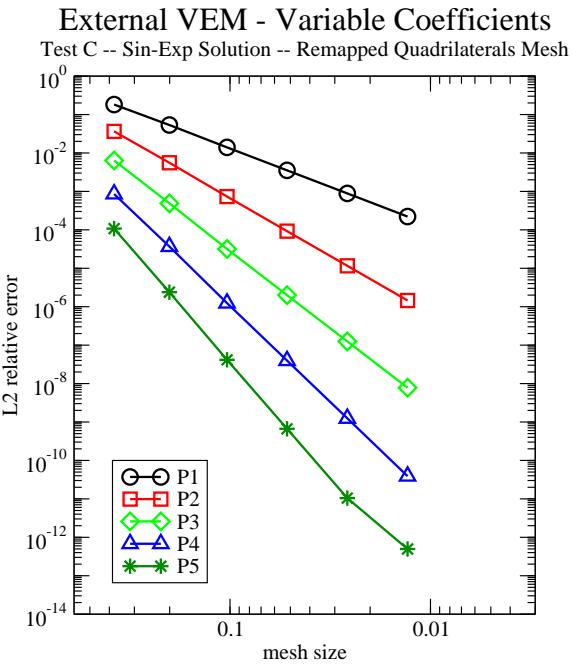


Fig. 122. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

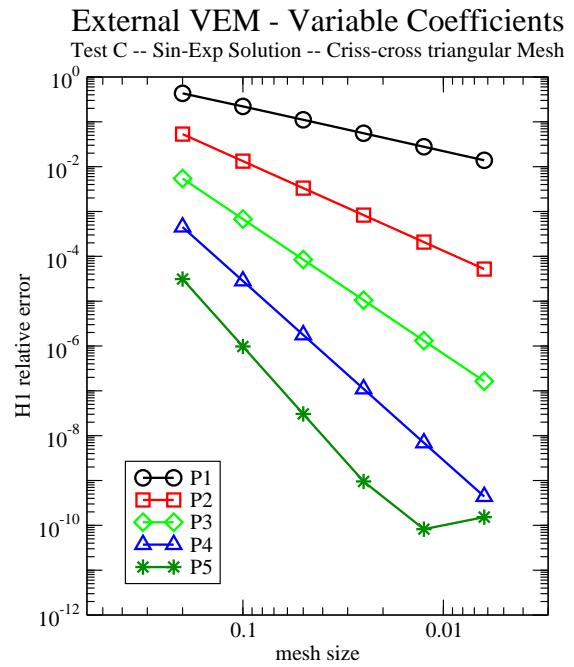
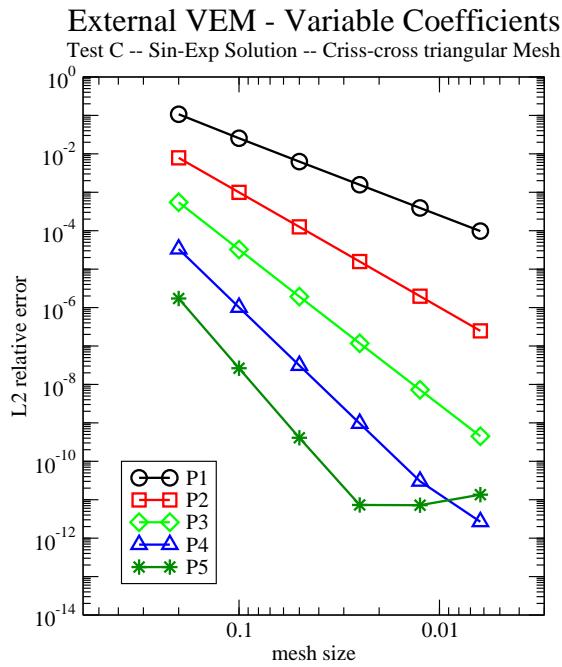


Fig. 123. External VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

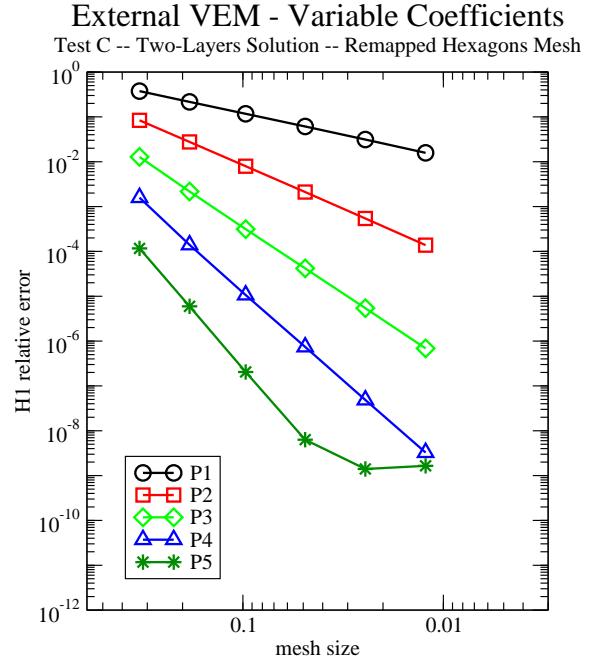
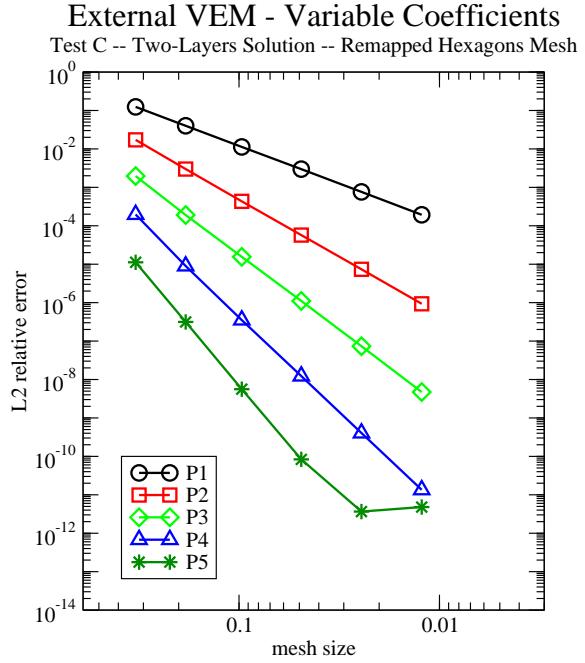


Fig. 124. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

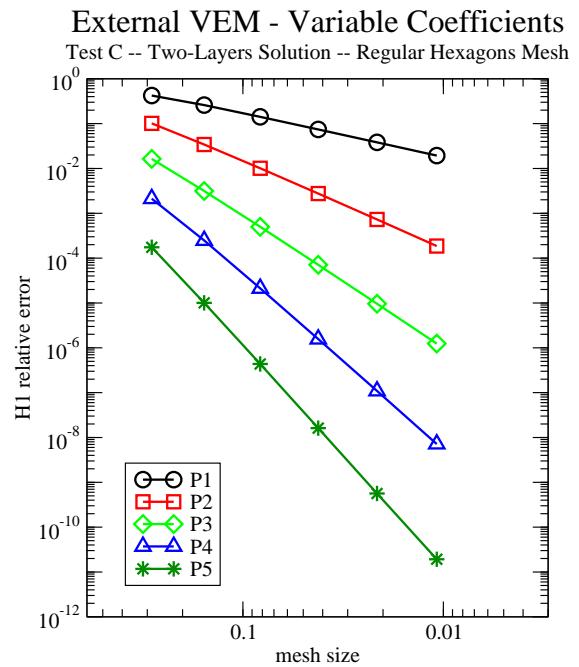
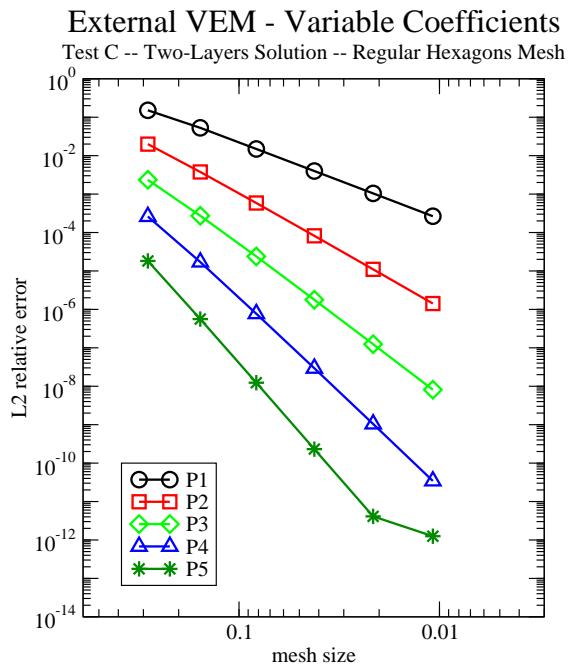


Fig. 125. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular hexagons.

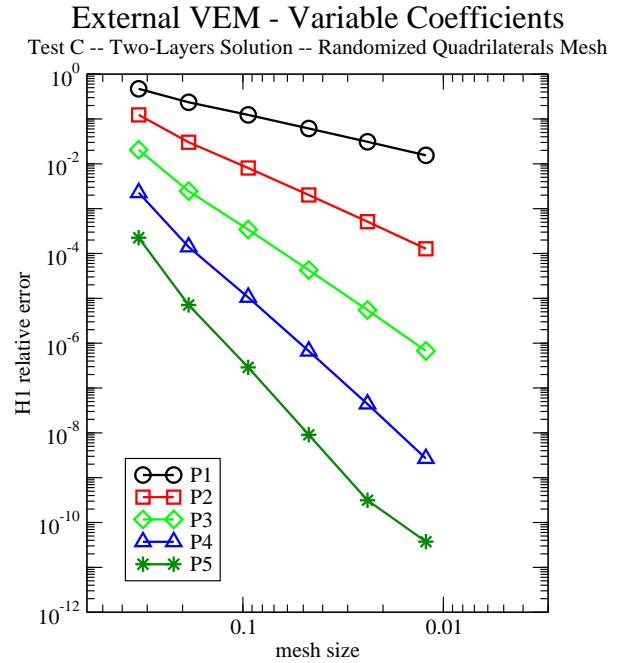
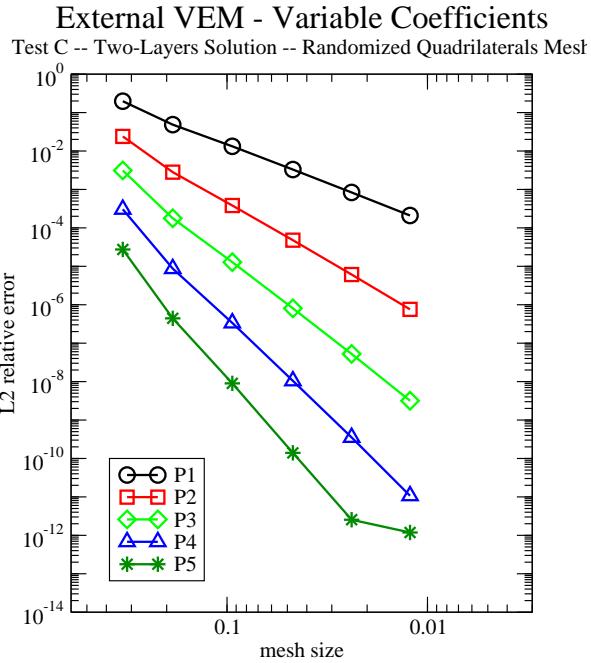


Fig. 126. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

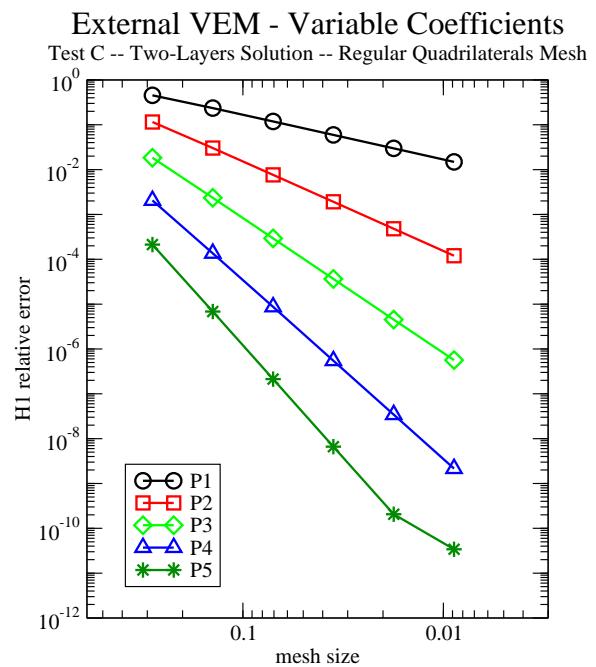
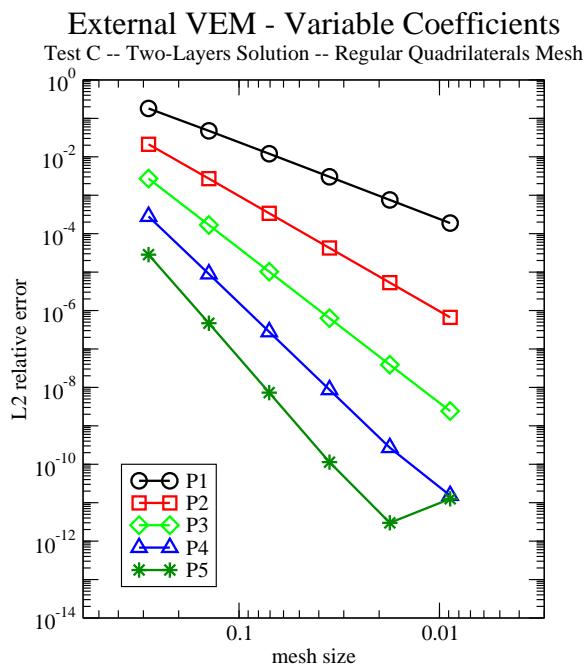


Fig. 127. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

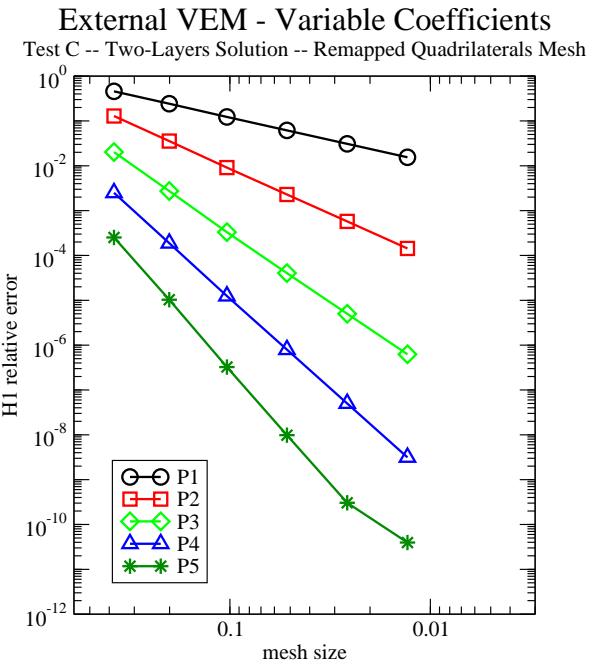
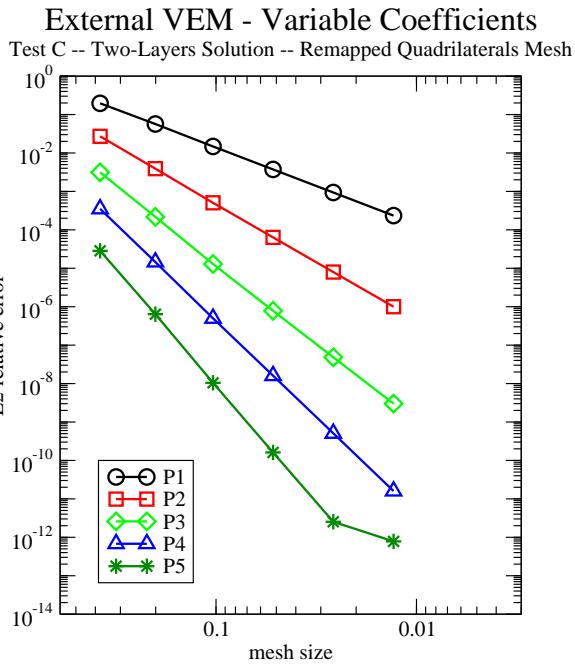


Fig. 128. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

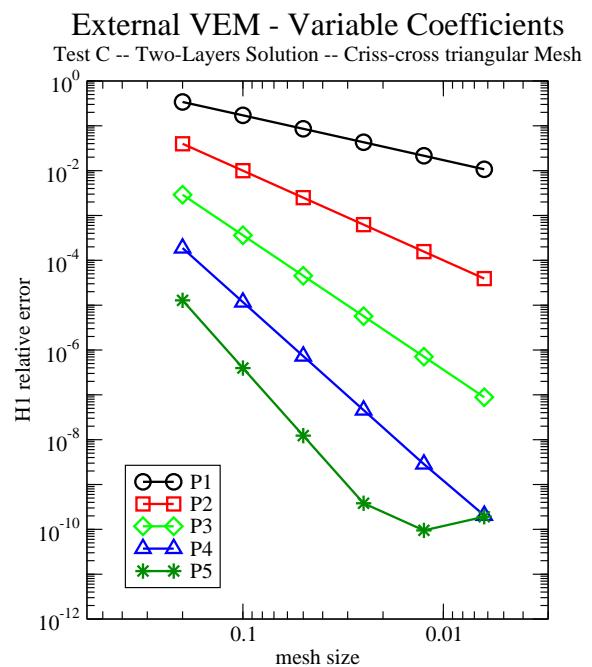
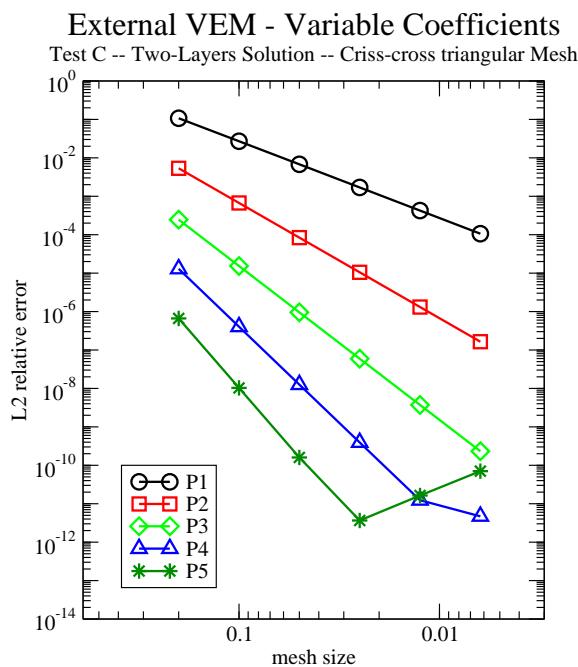


Fig. 129. External VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

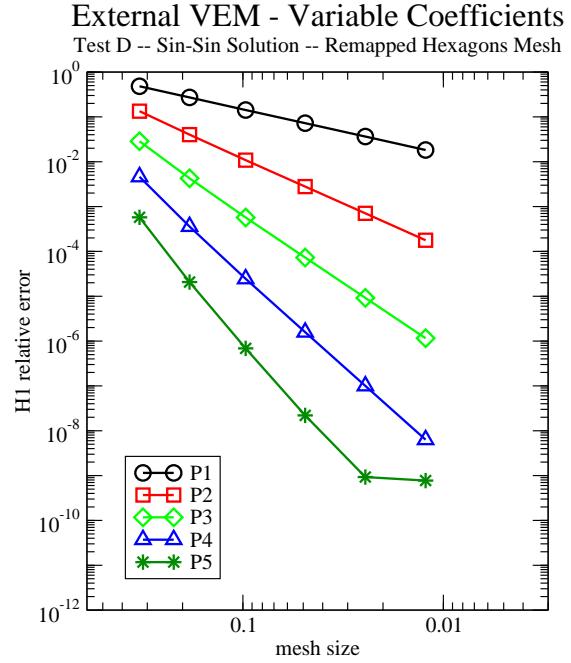
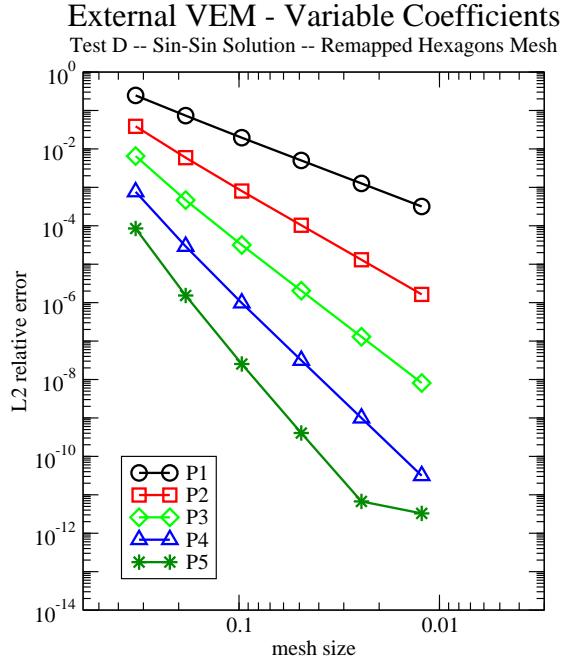


Fig. 130. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of smoothly remapped hexagons.

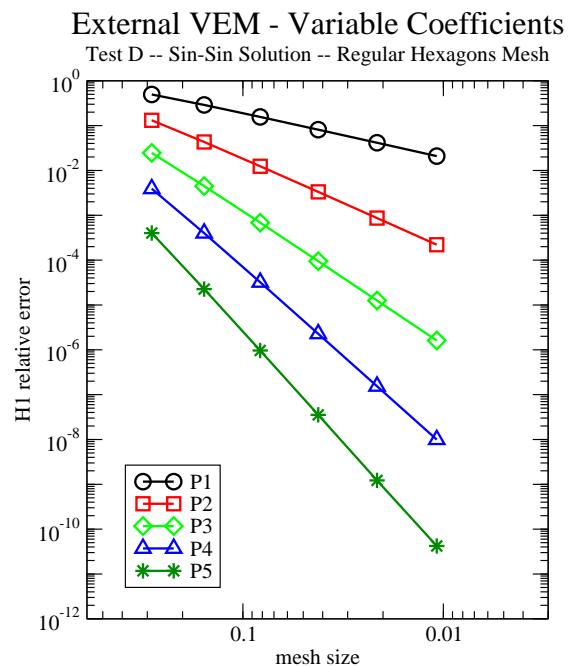
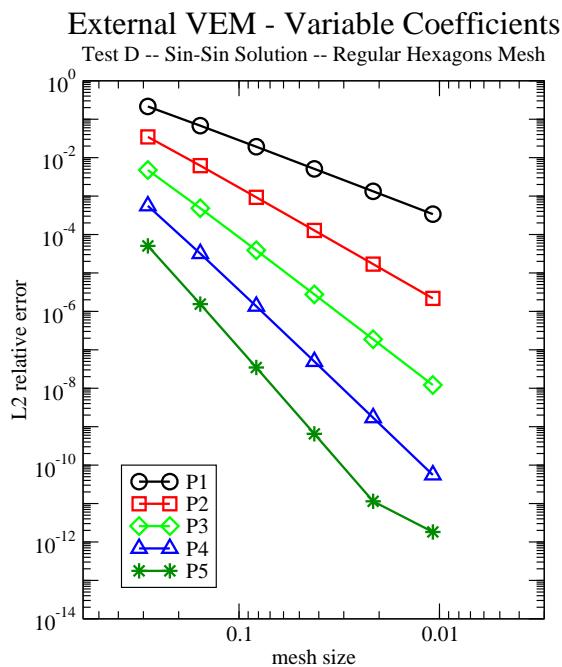


Fig. 131. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular hexagons.

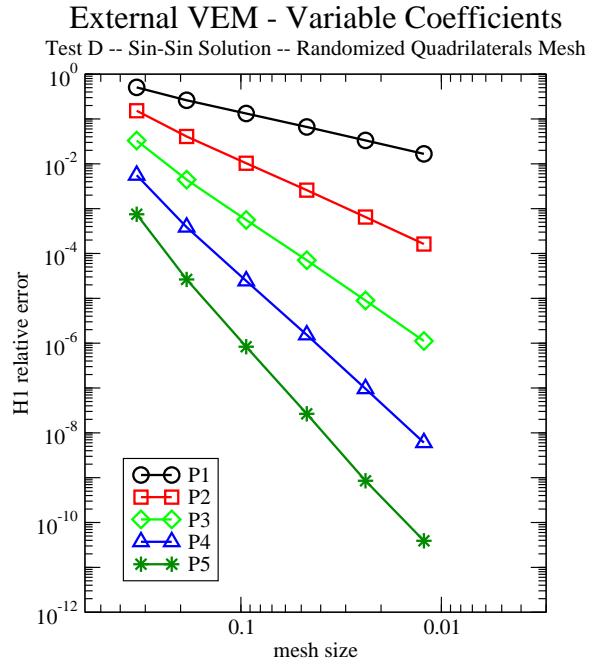
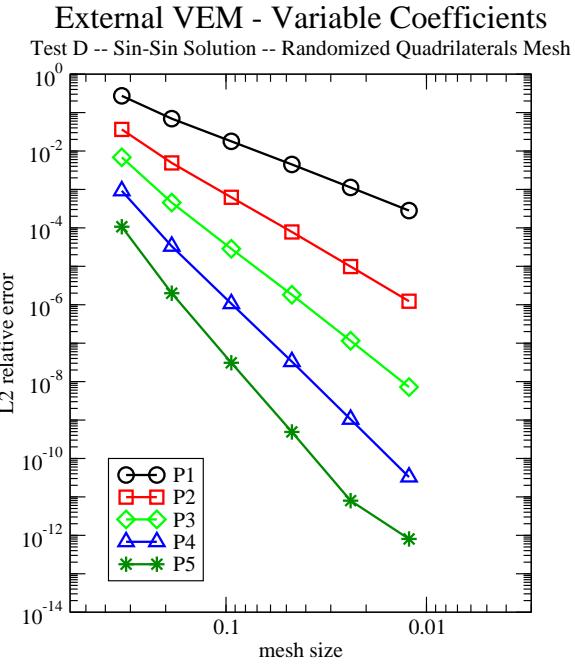


Fig. 132. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of randomized quadrilateral cells.

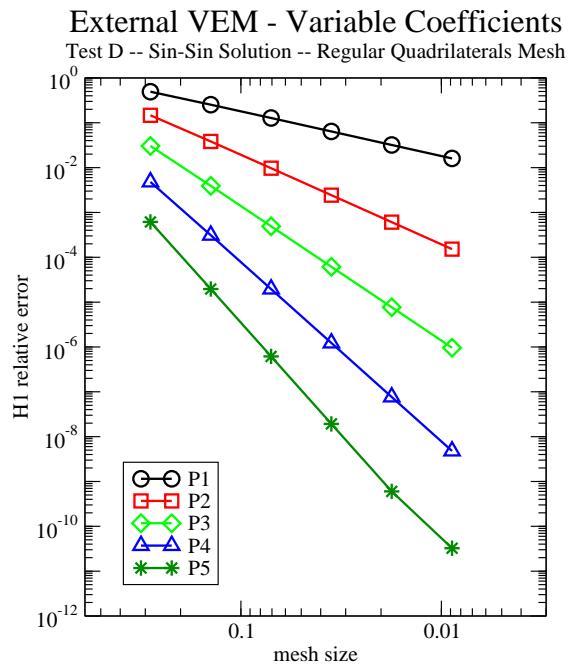
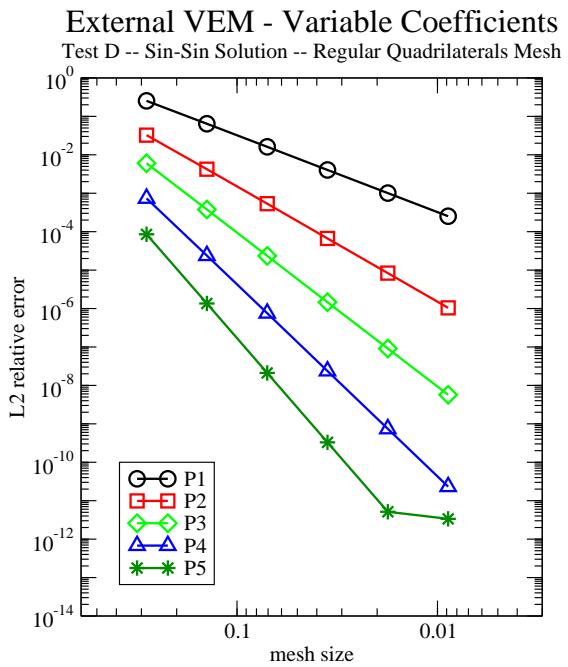


Fig. 133. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular quadrilateral cells (squares).

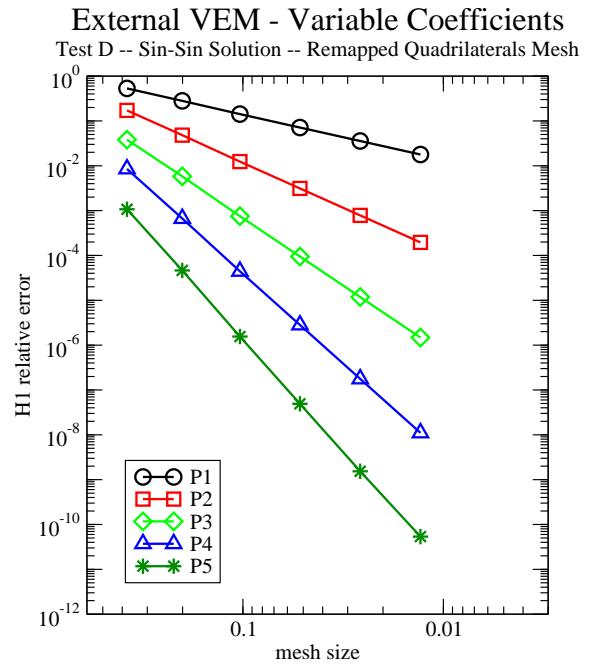
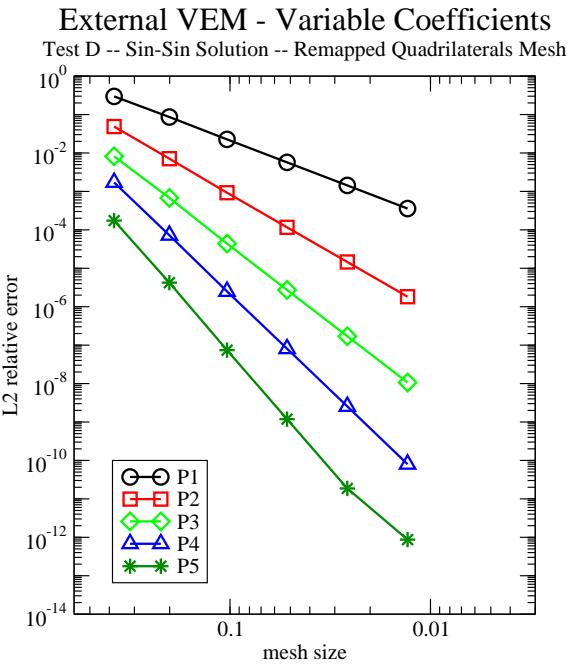


Fig. 134. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

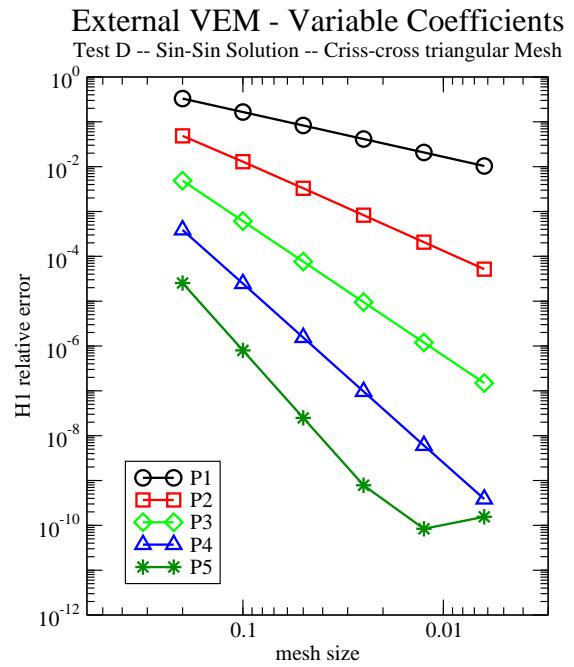
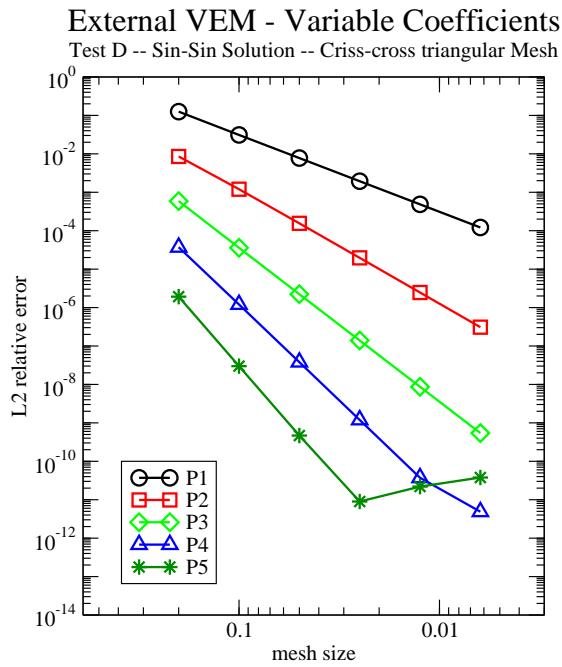


Fig. 135. External VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

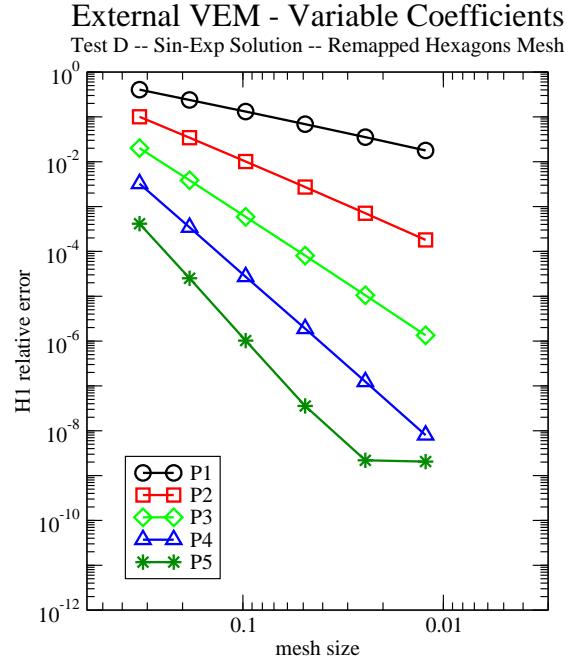
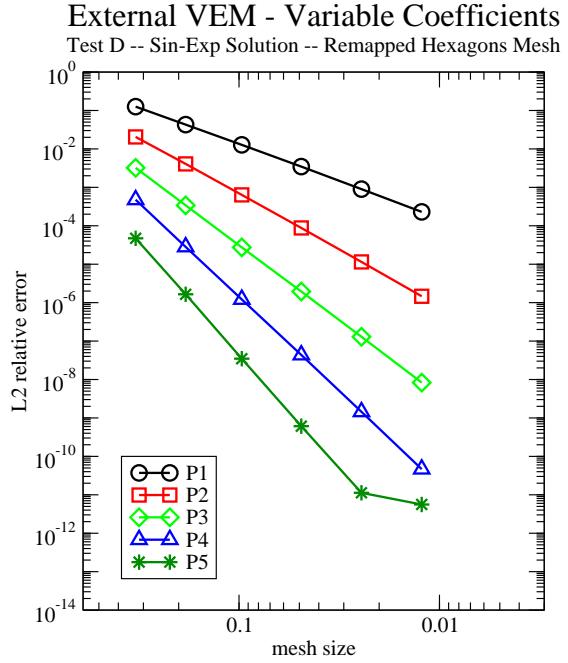


Fig. 136. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of smoothly remapped hexagons.

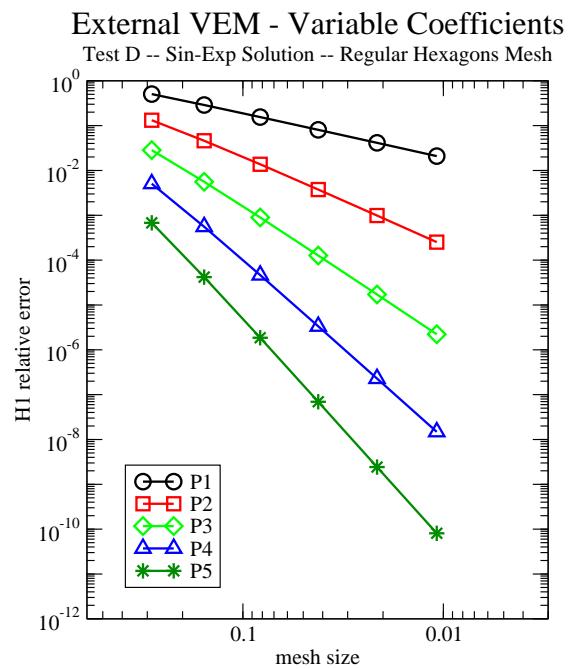
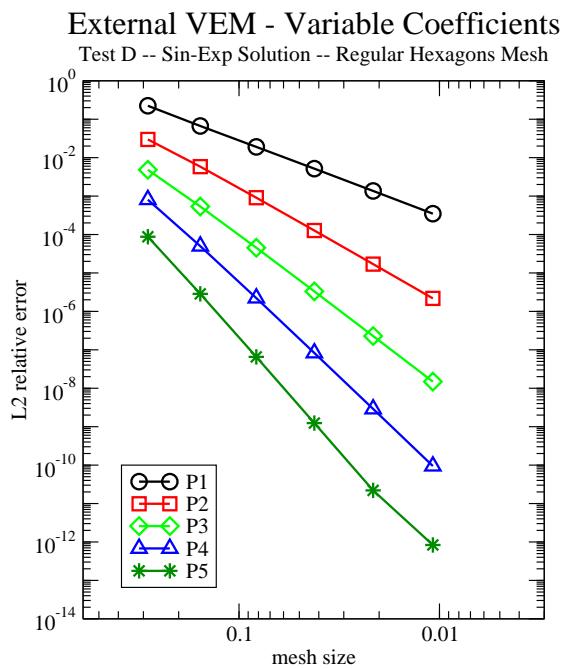


Fig. 137. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular hexagons.

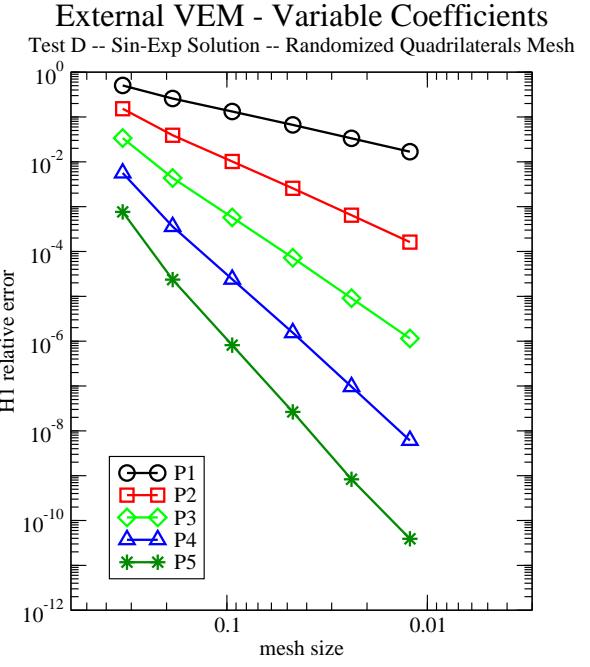
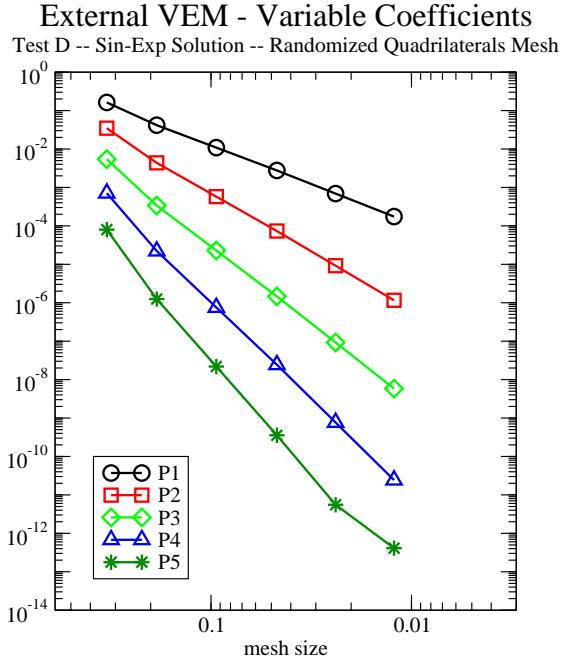


Fig. 138. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of randomized quadrilateral cells.

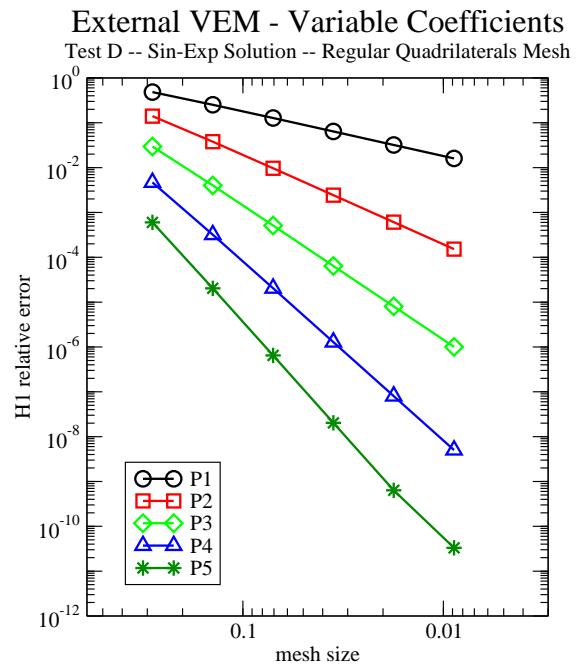
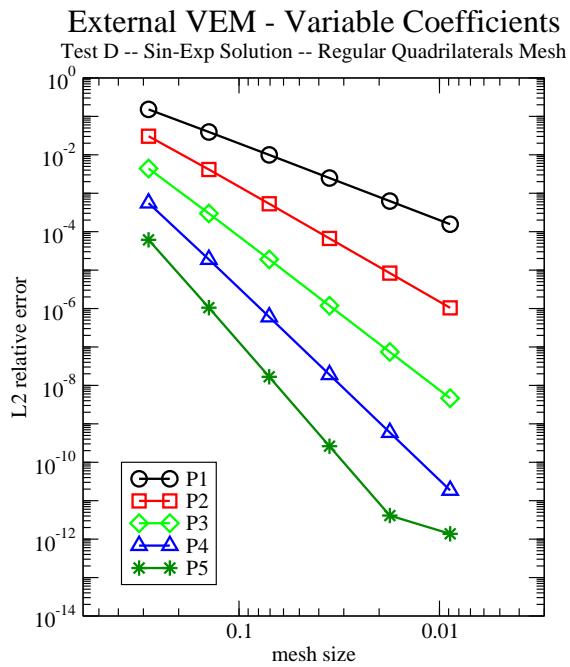


Fig. 139. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular quadrilateral cells (squares).

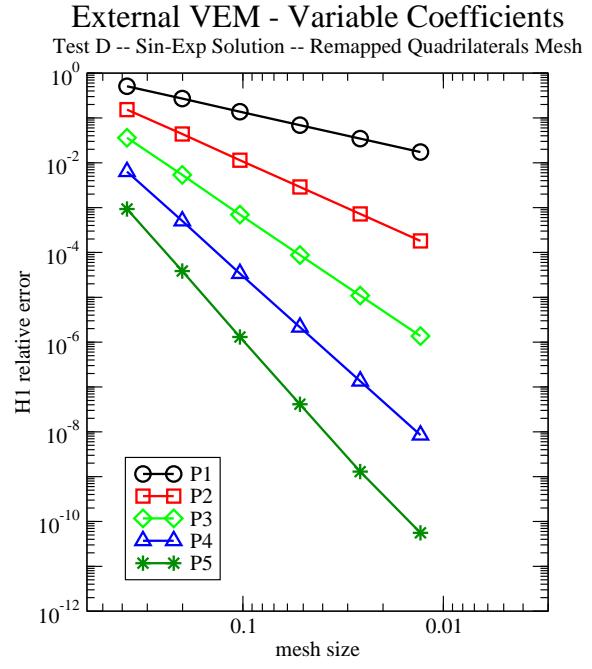
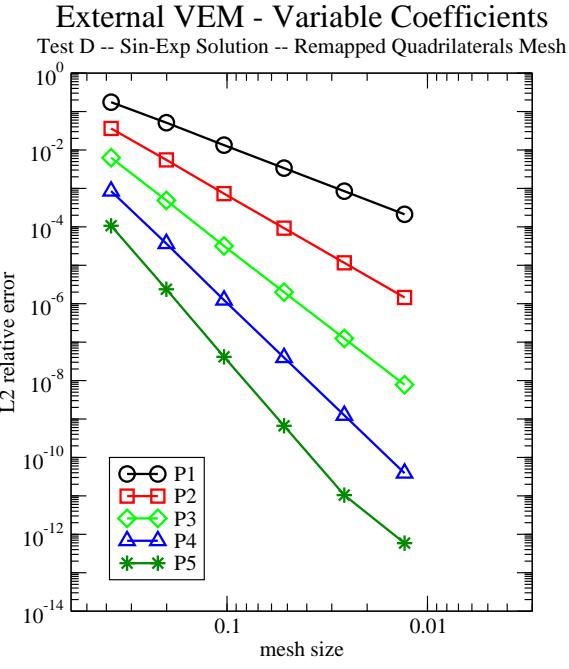


Fig. 140. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

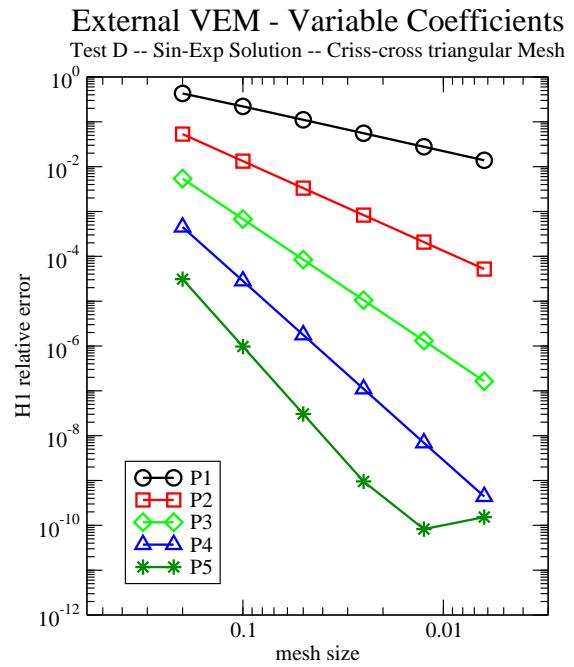
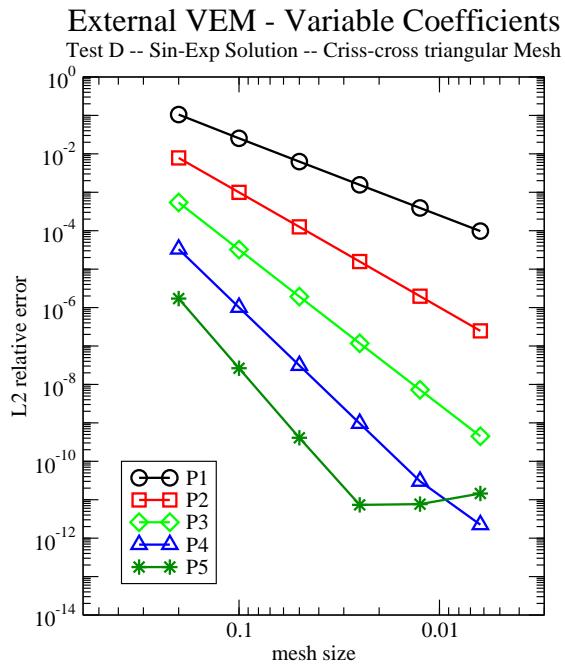


Fig. 141. External VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

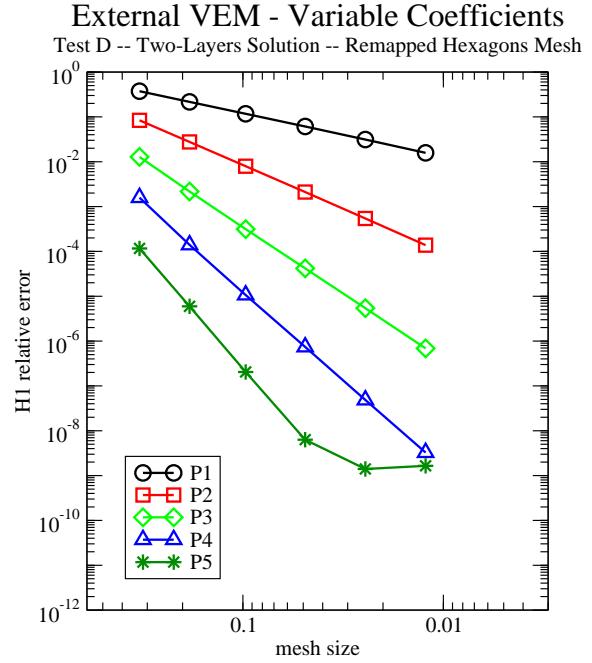
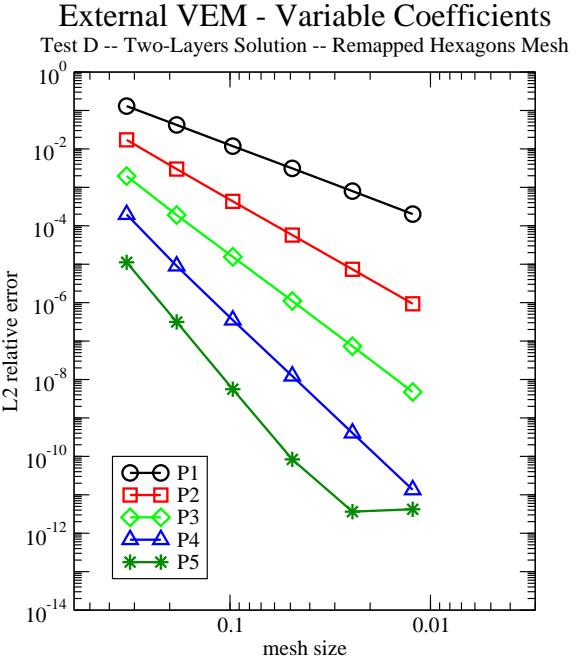


Fig. 142. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of smoothly remapped hexagons.

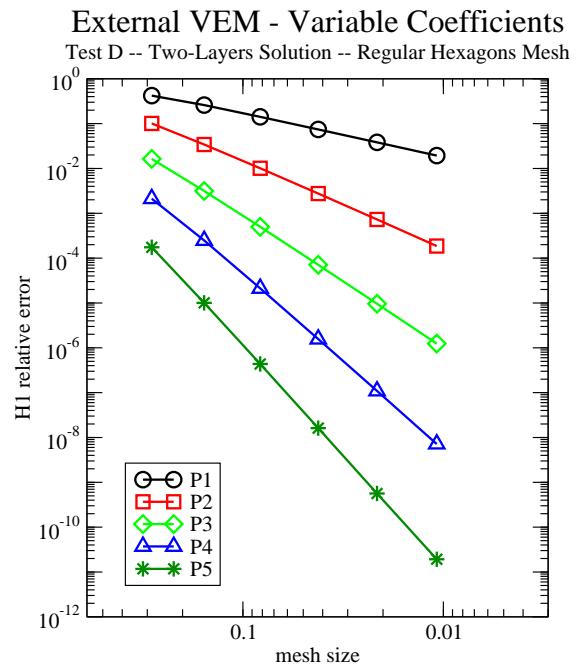
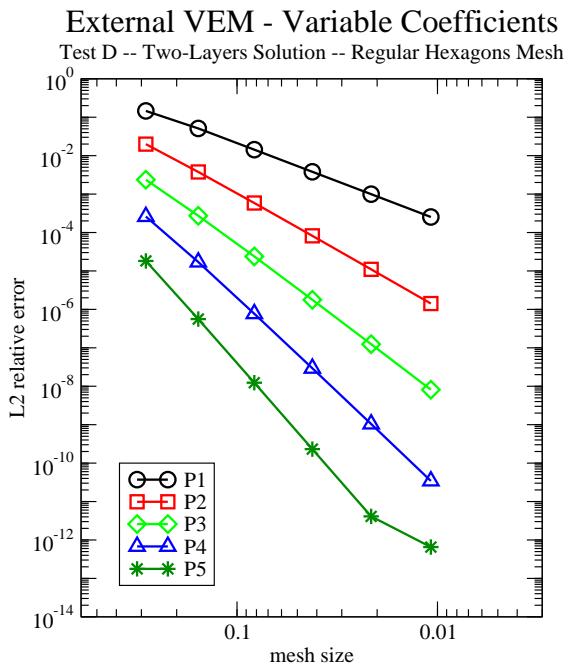


Fig. 143. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular hexagons.

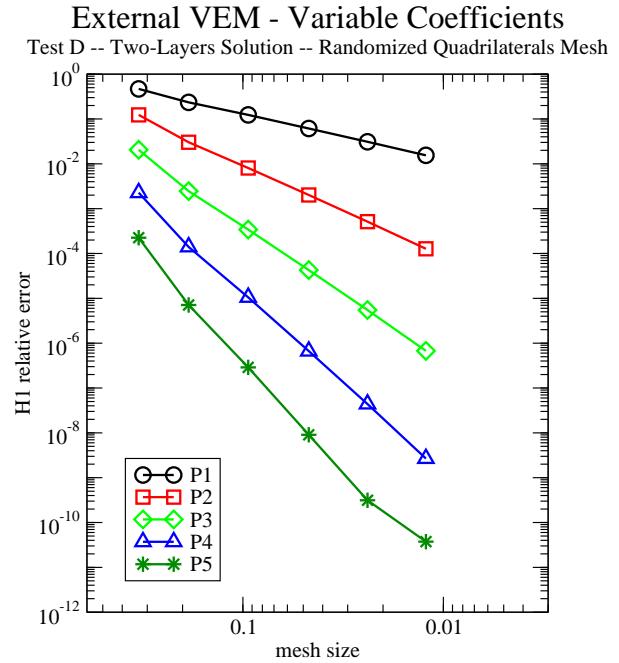
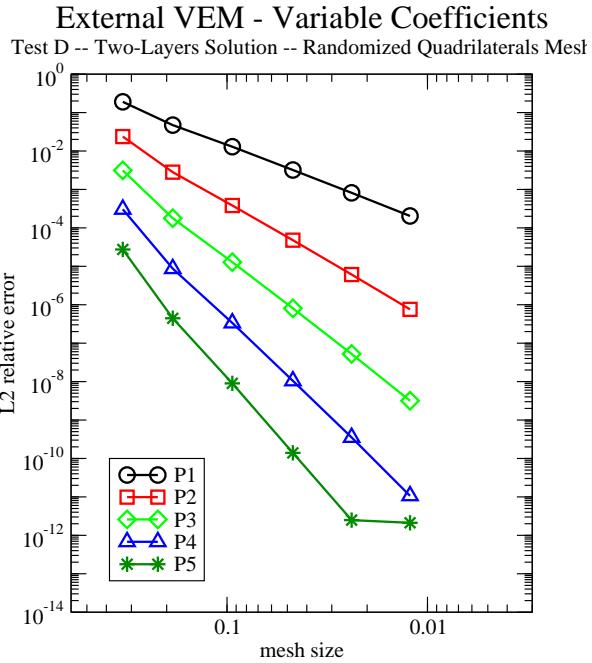


Fig. 144. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of randomized quadrilateral cells.

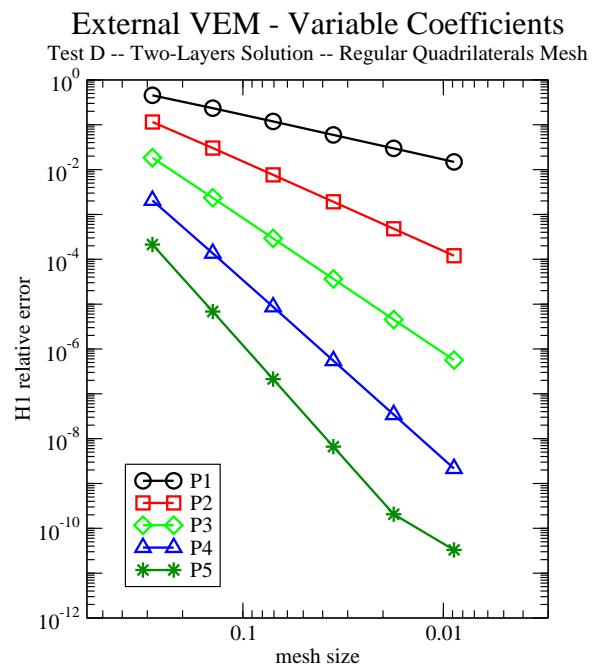
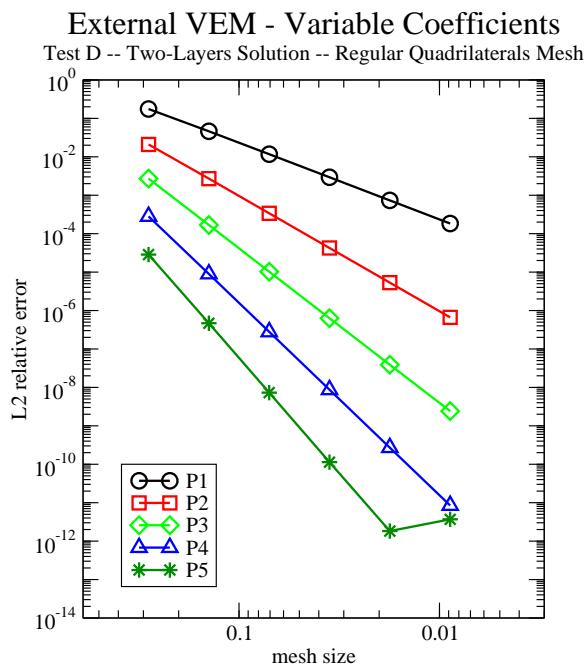


Fig. 145. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular quadrilateral cells (squares).

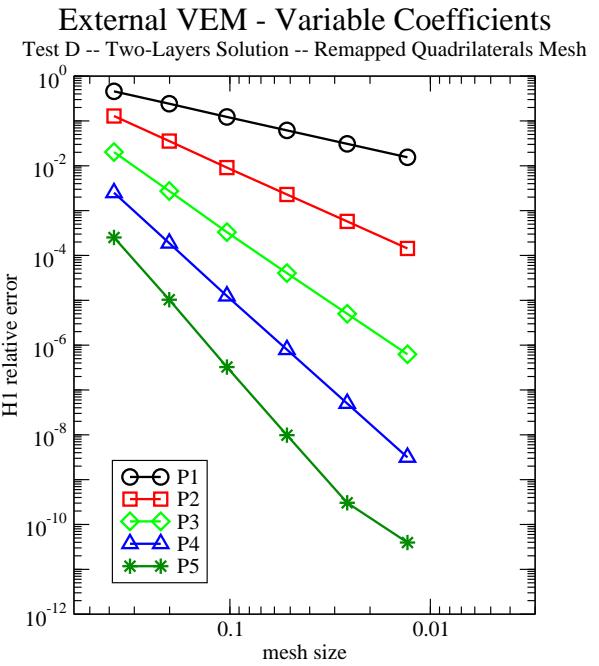
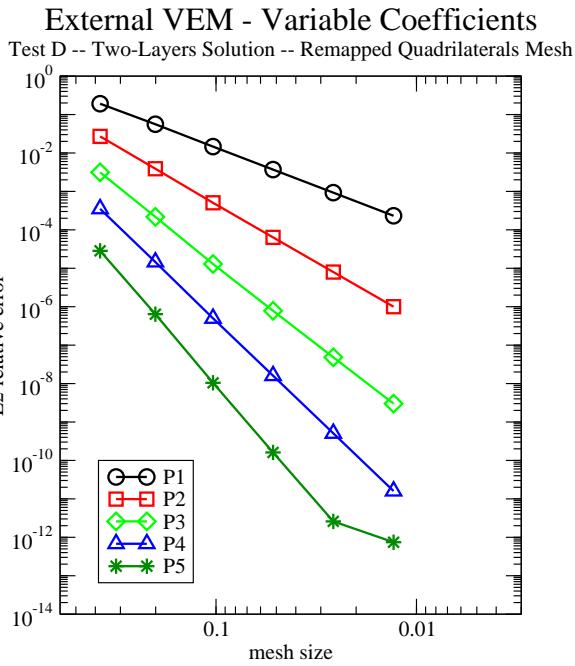


Fig. 146. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

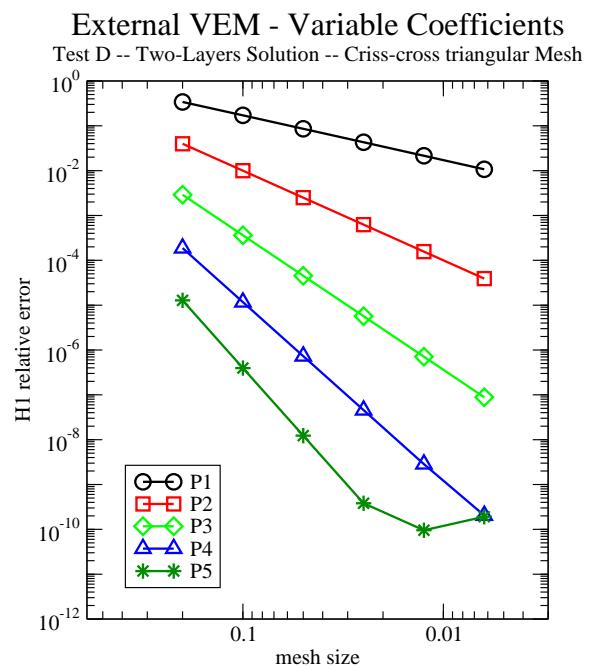
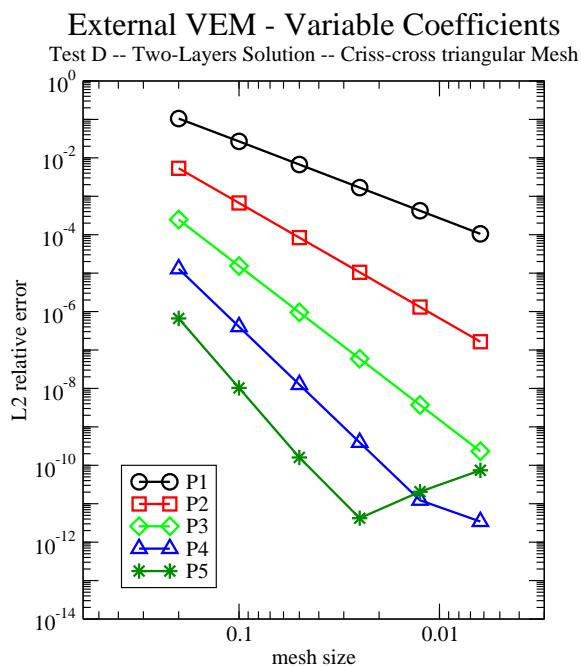
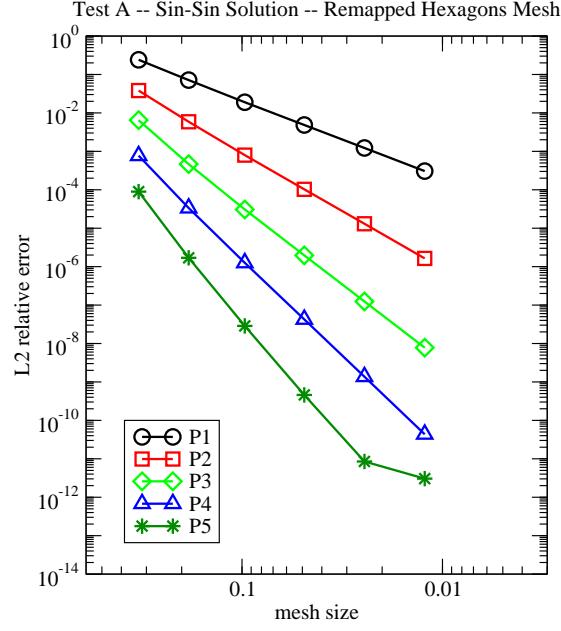


Fig. 147. External VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

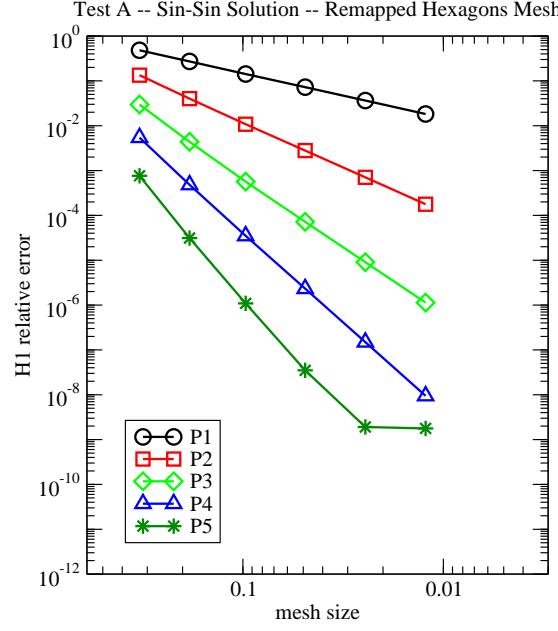
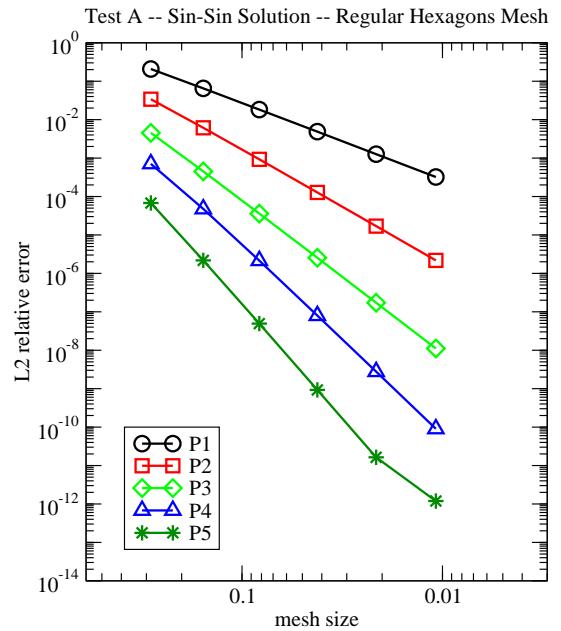


Fig. 148. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

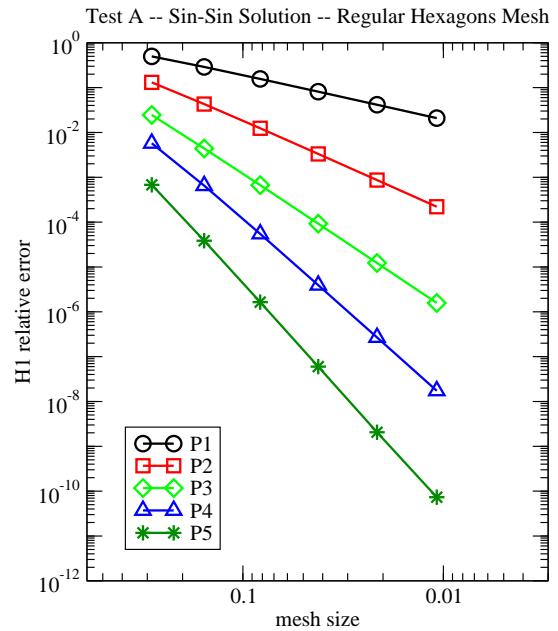
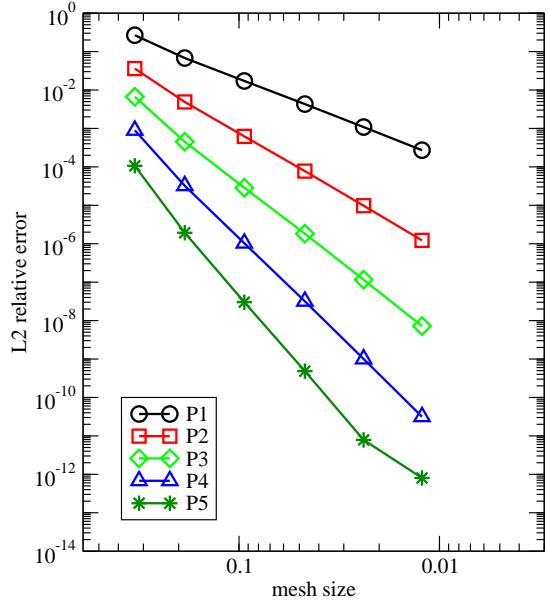


Fig. 149. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

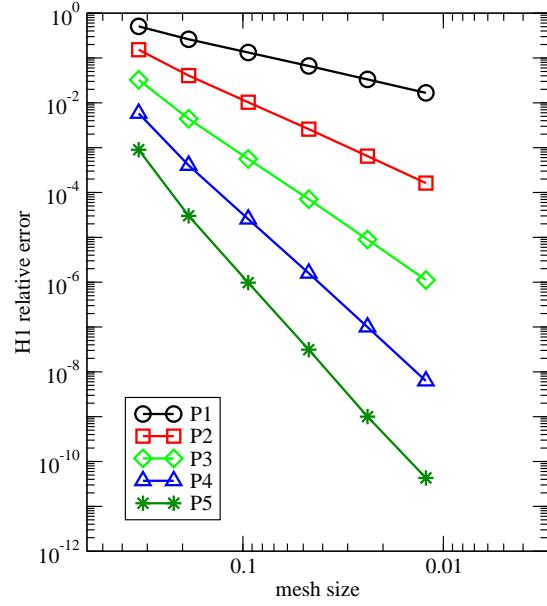
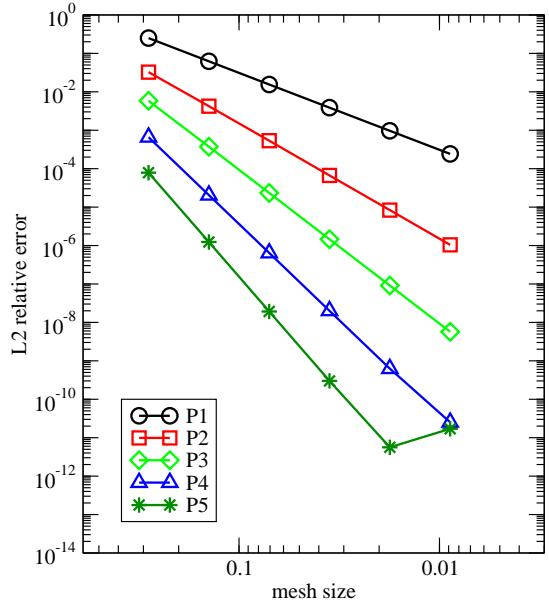


Fig. 150. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

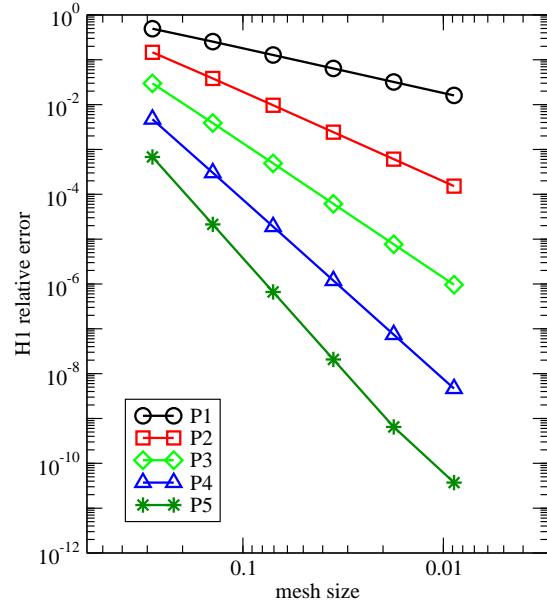
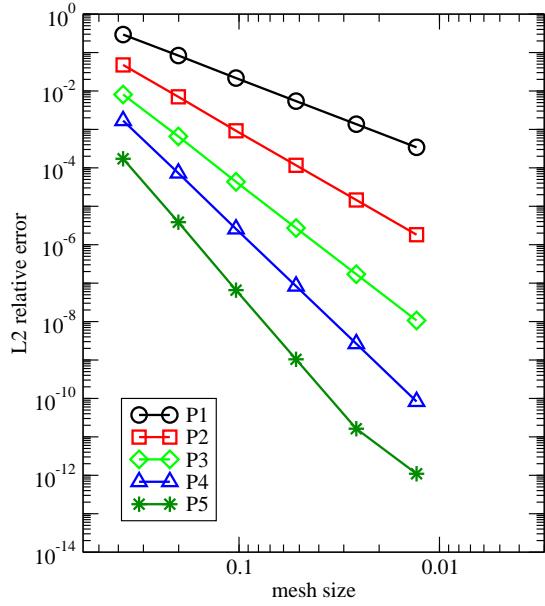


Fig. 151. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

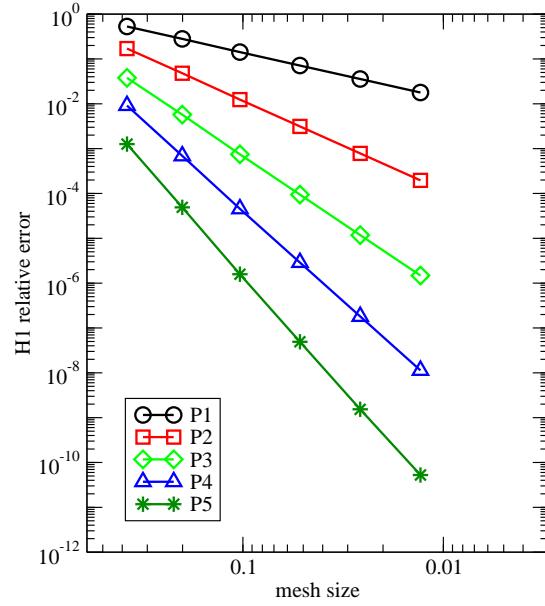
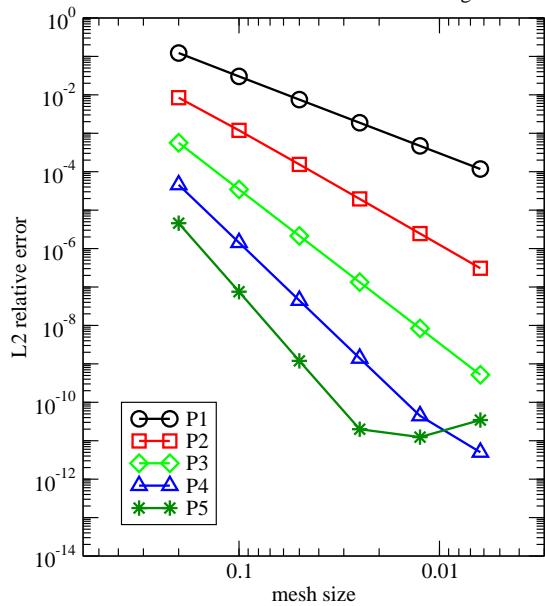


Fig. 152. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Sin Solution -- Criss-cross triangular Mesh

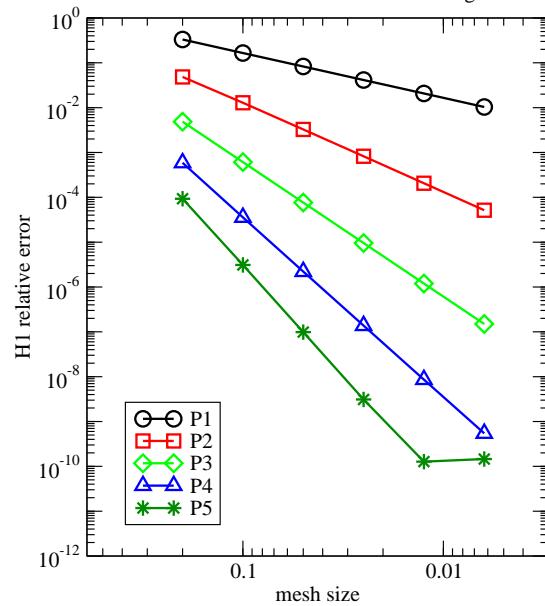
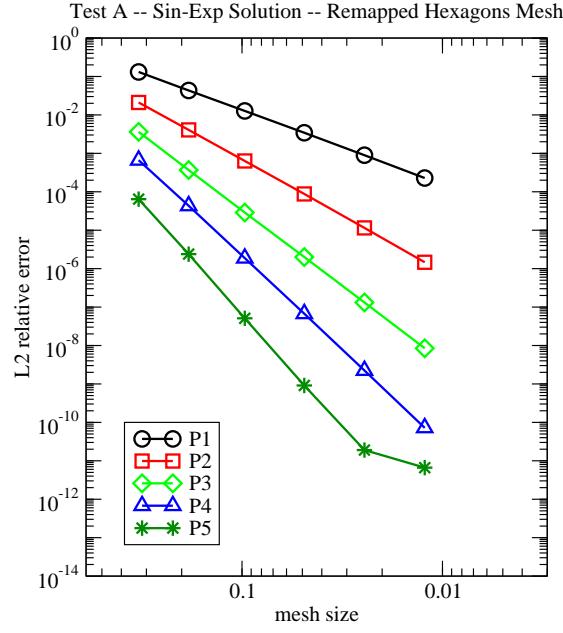


Fig. 153. Internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

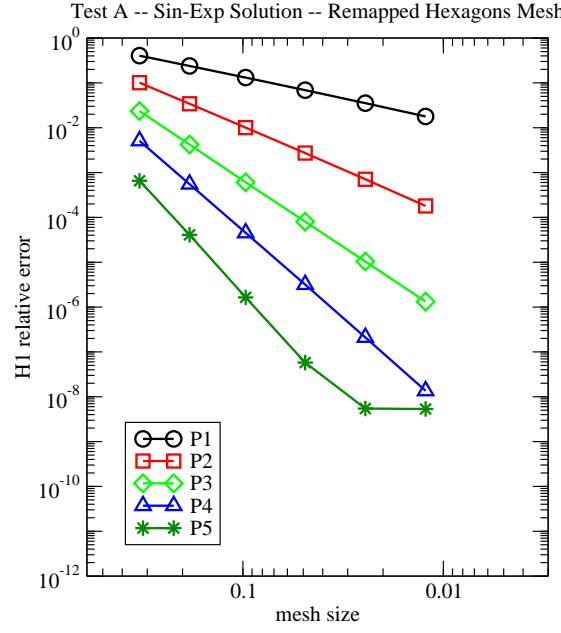
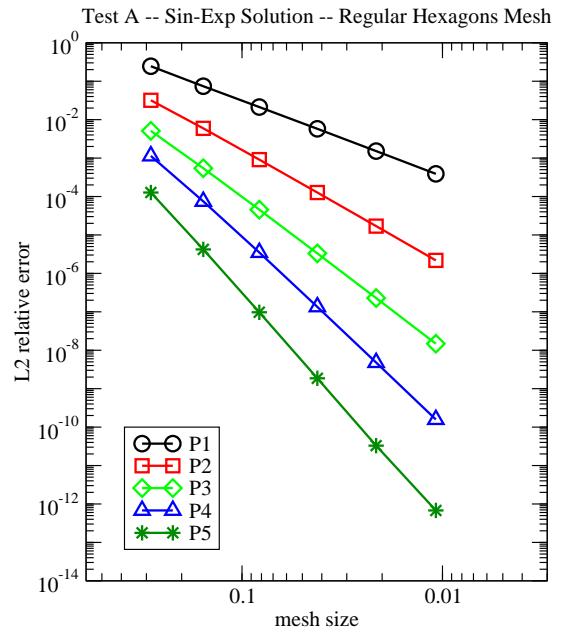


Fig. 154. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

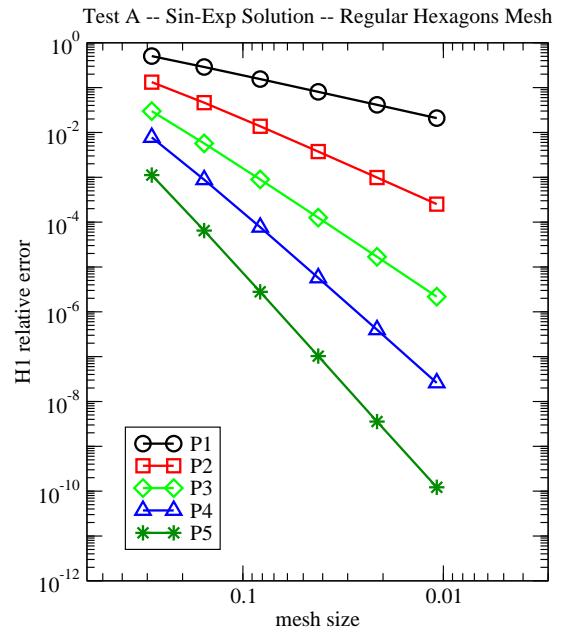
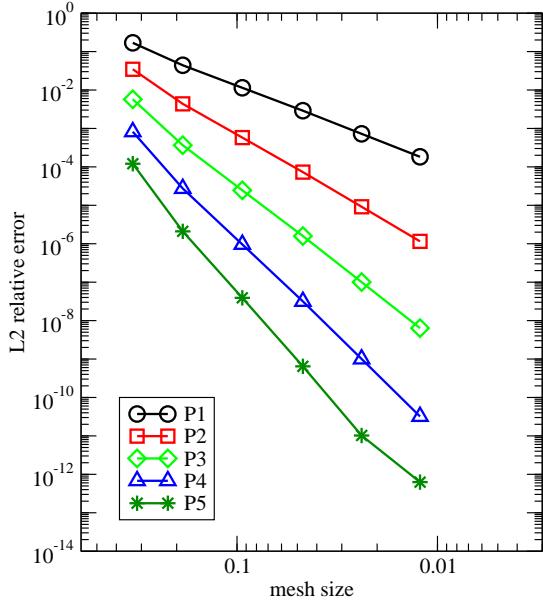


Fig. 155. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

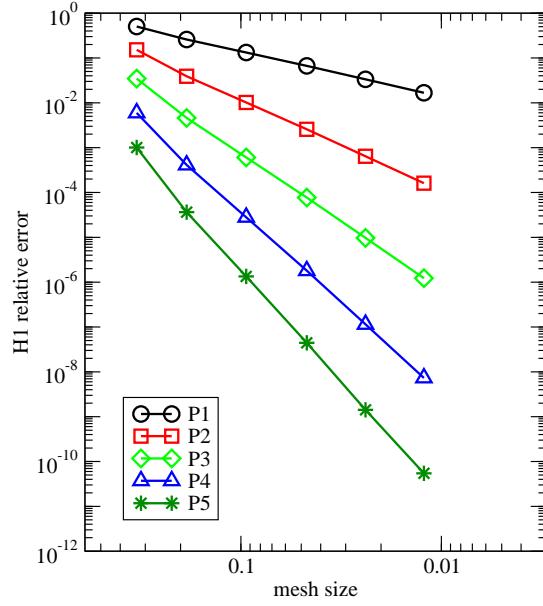
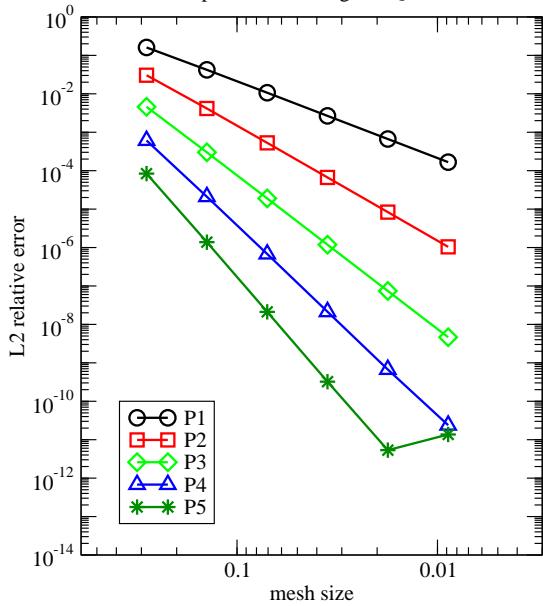


Fig. 156. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

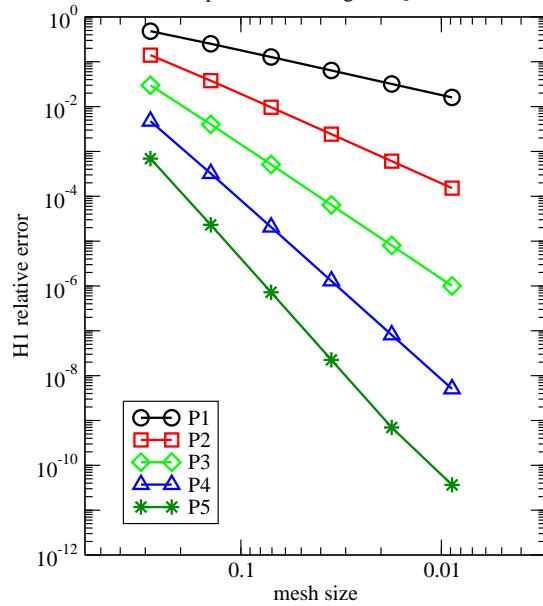
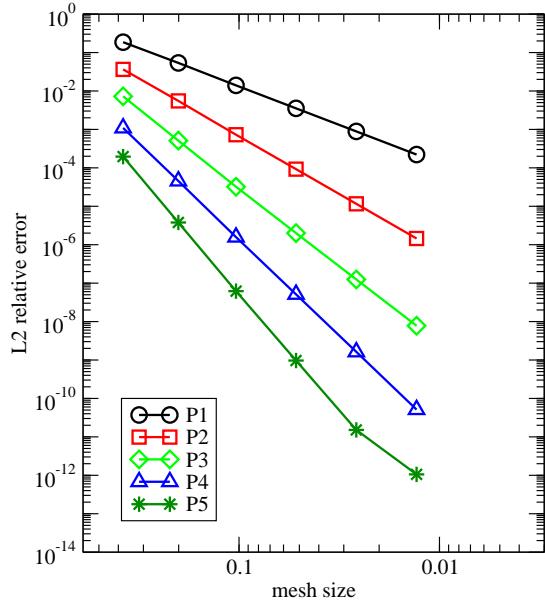


Fig. 157. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

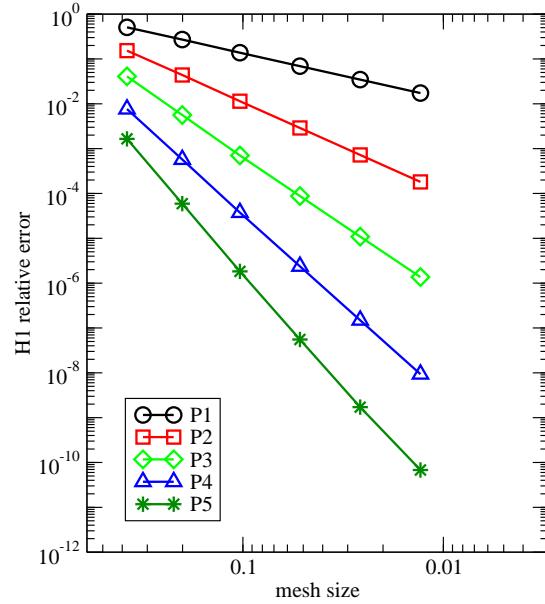
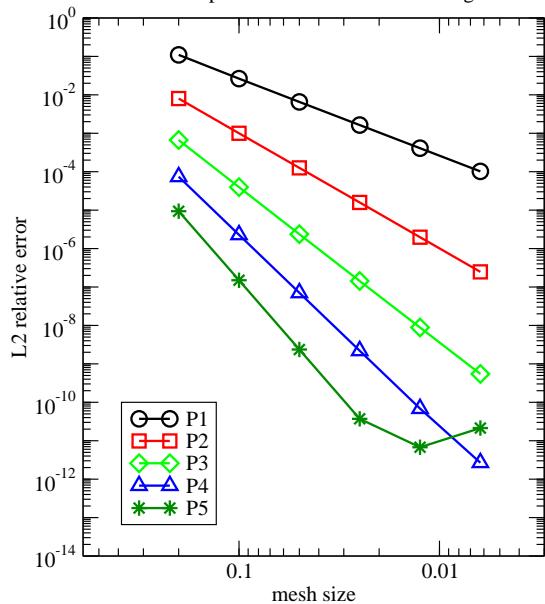


Fig. 158. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Sin-Exp Solution -- Criss-cross triangular Mesh

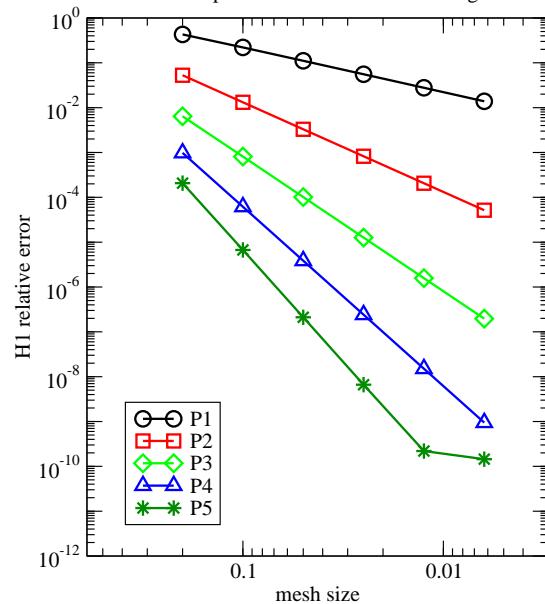
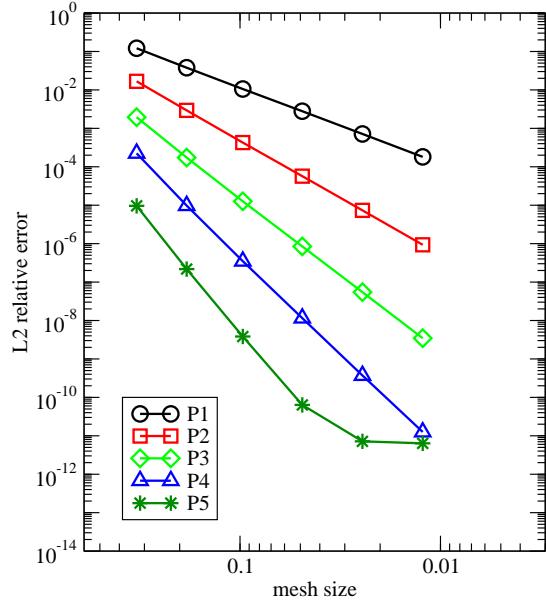


Fig. 159. Internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Remapped Hexagons Mesh

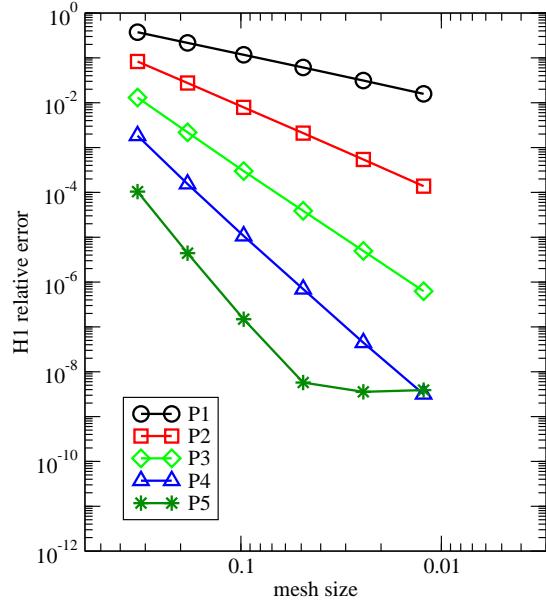
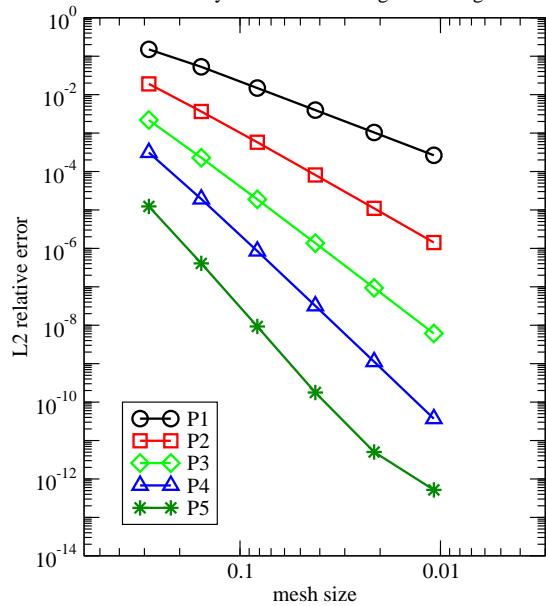


Fig. 160. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Regular Hexagons Mesh

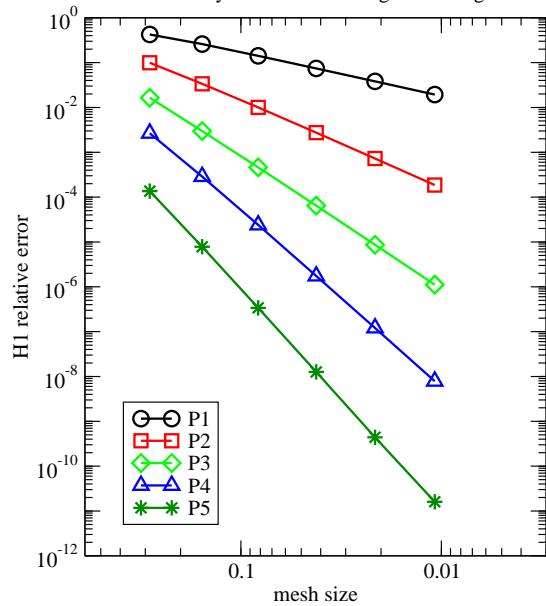
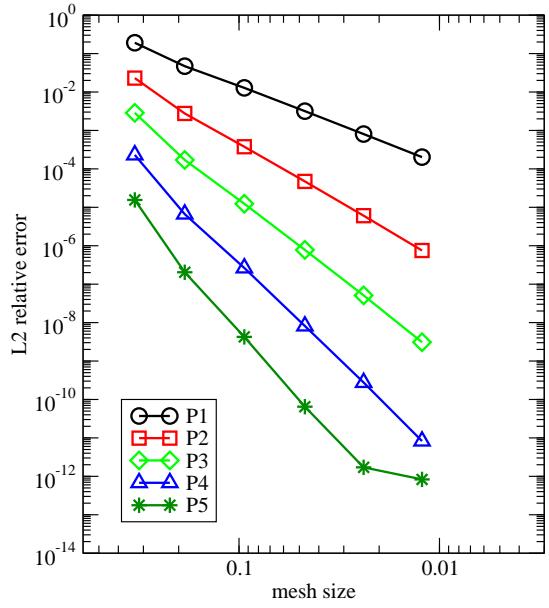


Fig. 161. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Randomized Quadrilaterals Mesh

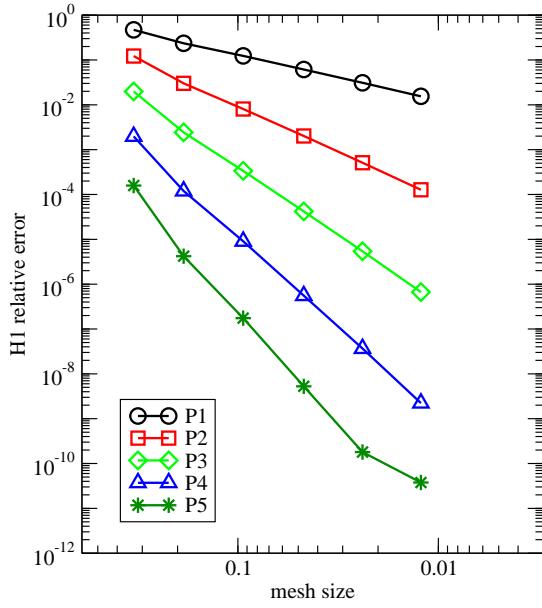
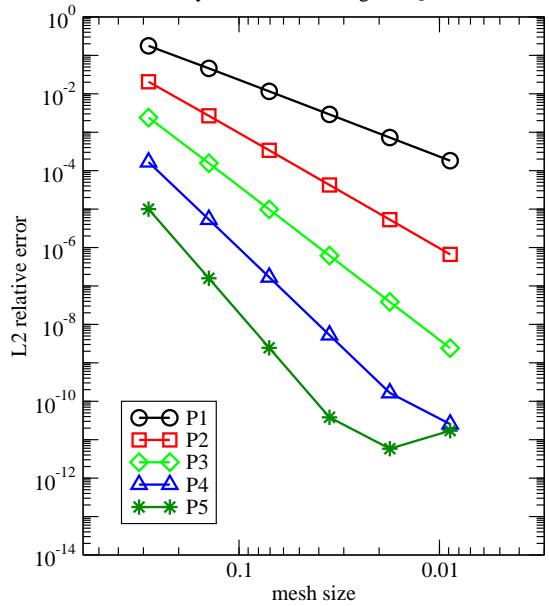


Fig. 162. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test A -- Two-Layers Solution -- Regular Quadrilaterals Mesh

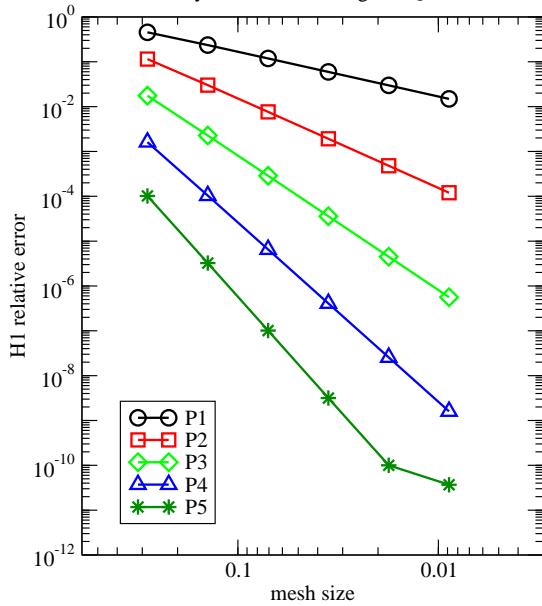
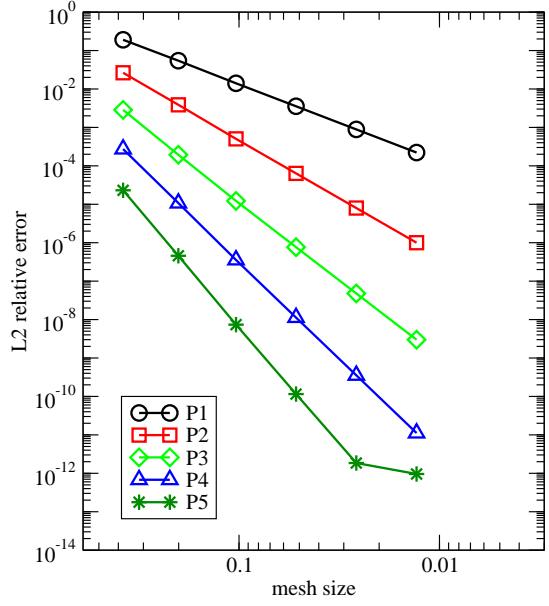


Fig. 163. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

**Internal/External VEM - Constant Coeffs.**  
 Test A -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test A -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

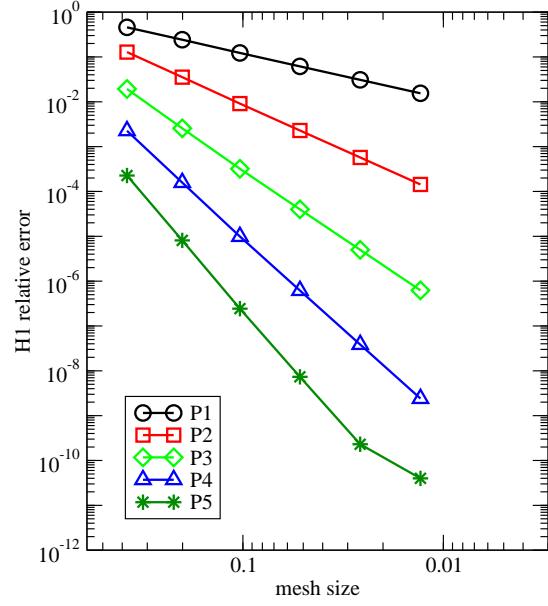
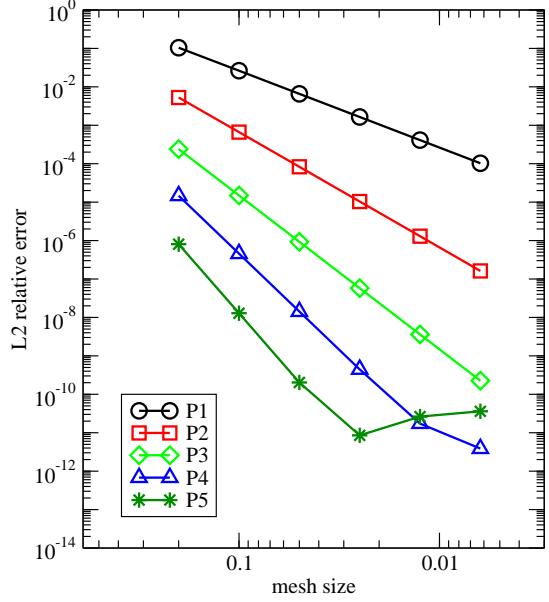


Fig. 164. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

**Internal/External VEM - Constant Coeffs.**  
 Test A -- Two-Layers Solution -- Criss-cross triangular Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test A -- Two-Layers Solution -- Criss-cross triangular Mesh

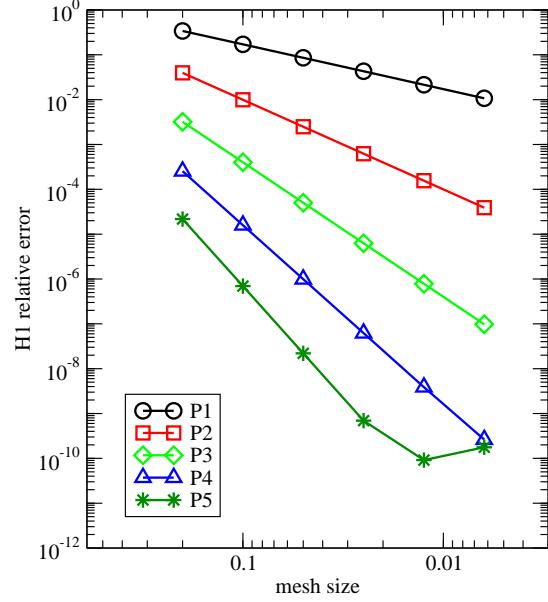
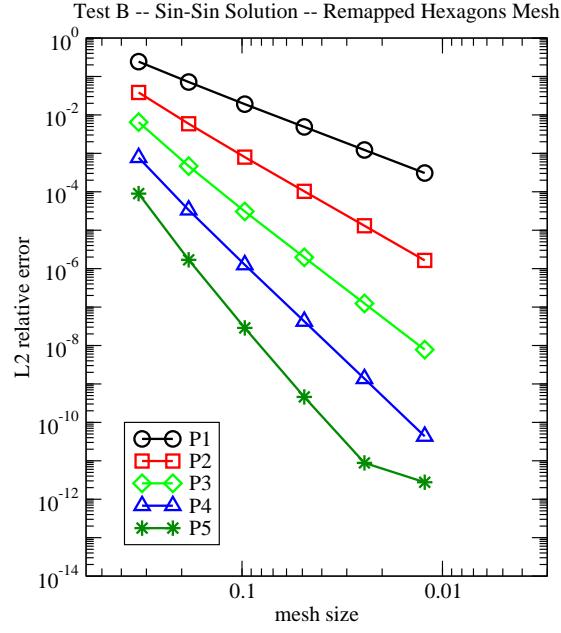


Fig. 165. Internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

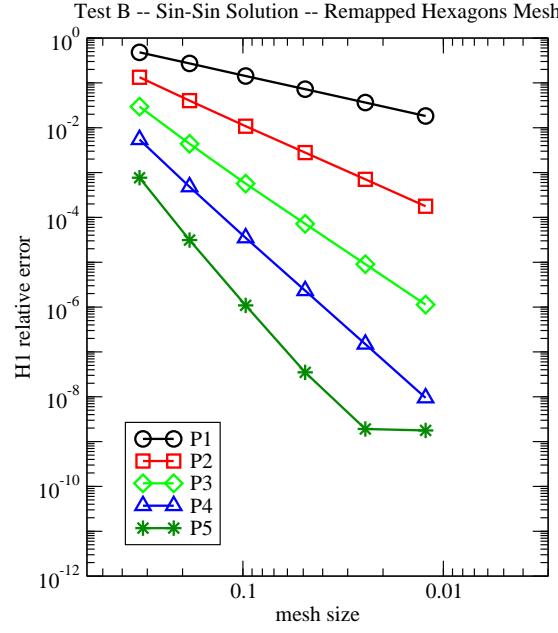
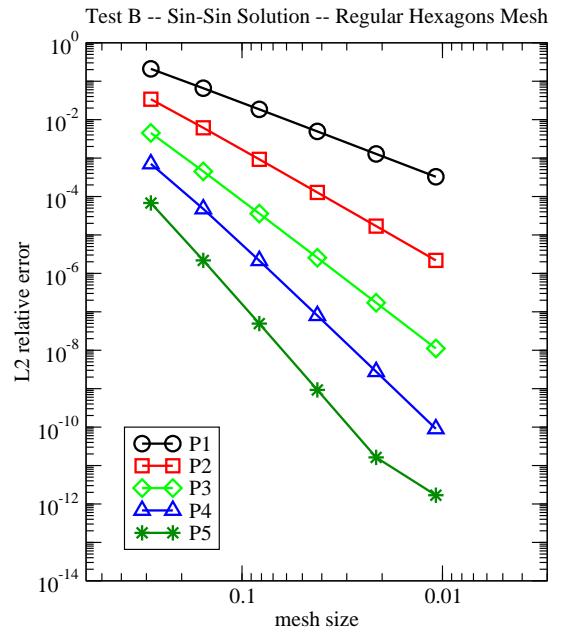


Fig. 166. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

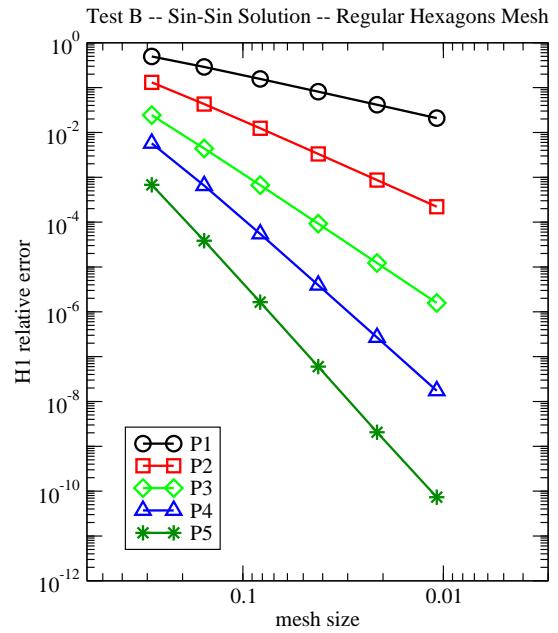
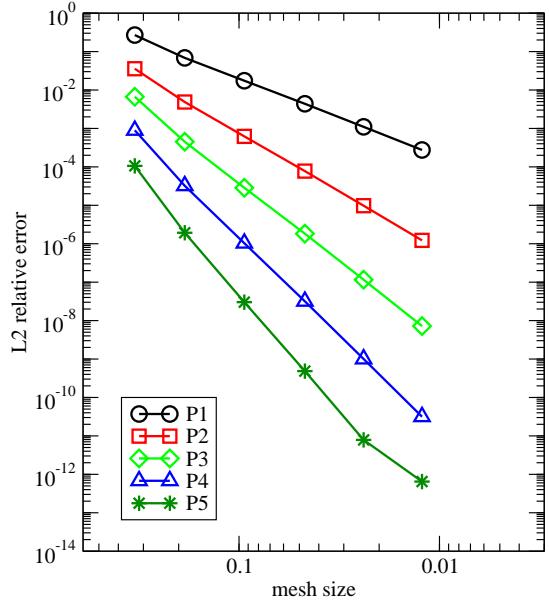


Fig. 167. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

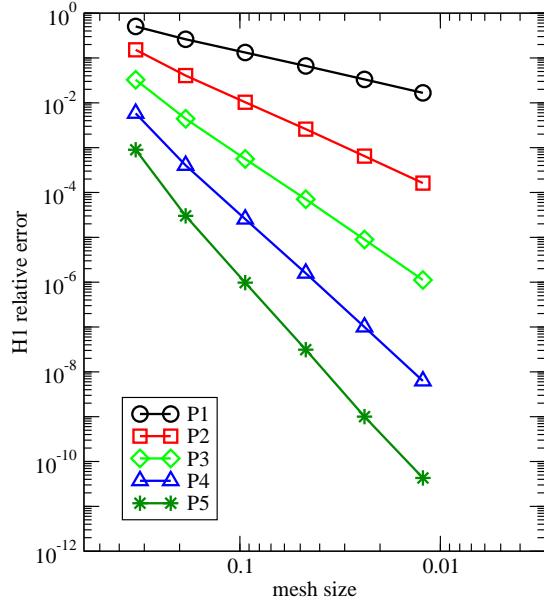
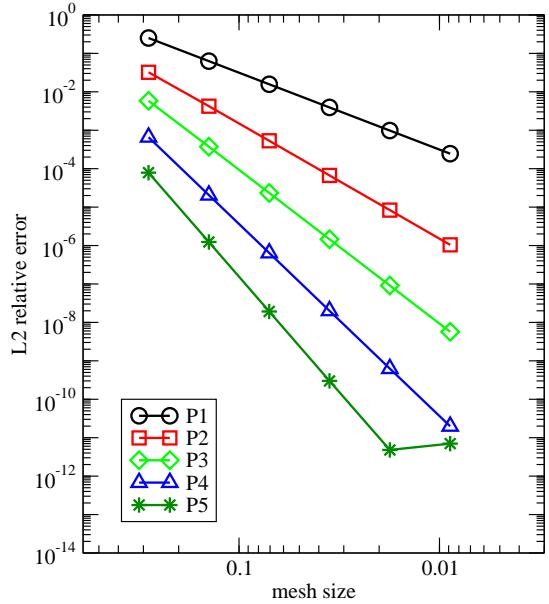


Fig. 168. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

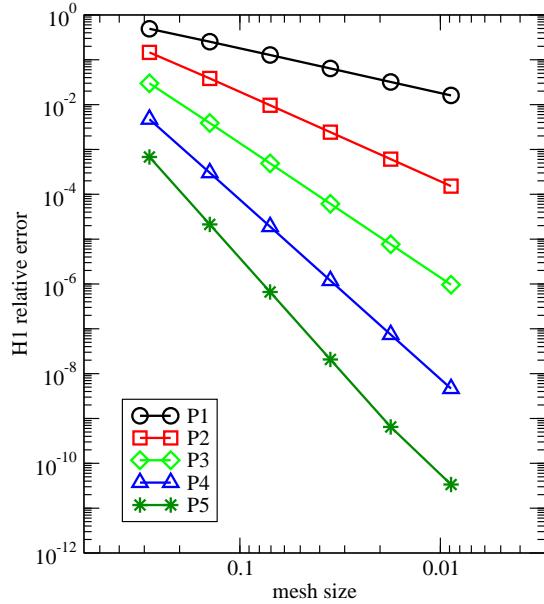
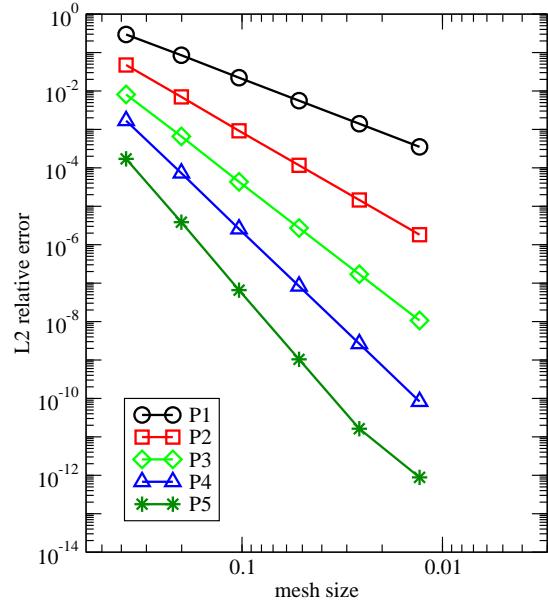


Fig. 169. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

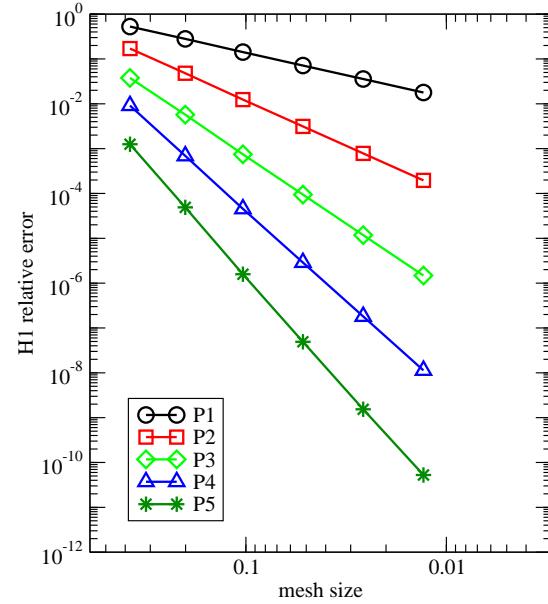
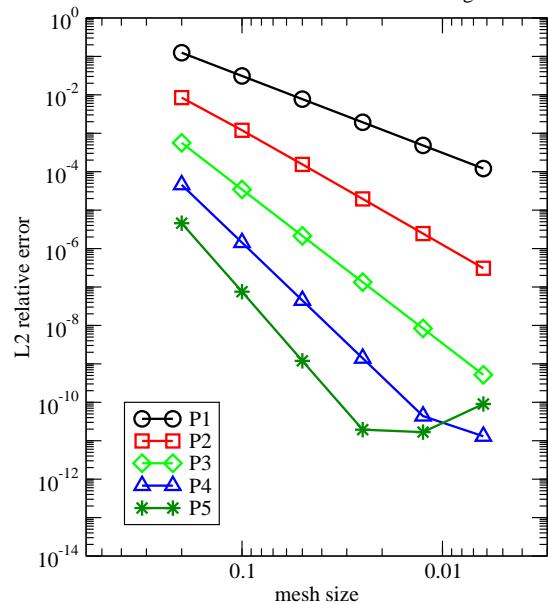


Fig. 170. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Sin Solution -- Criss-cross triangular Mesh

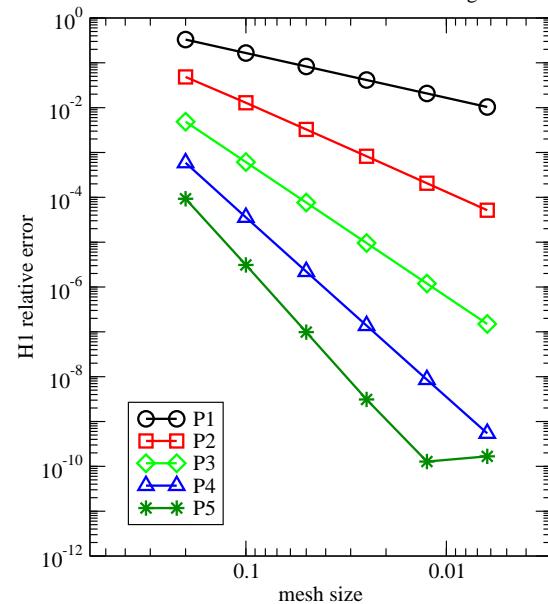
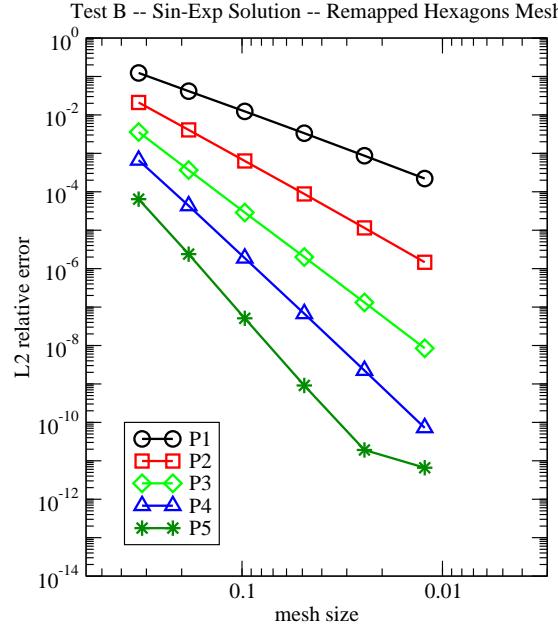


Fig. 171. Internal VEM formulation with constant coefficients; Test B; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

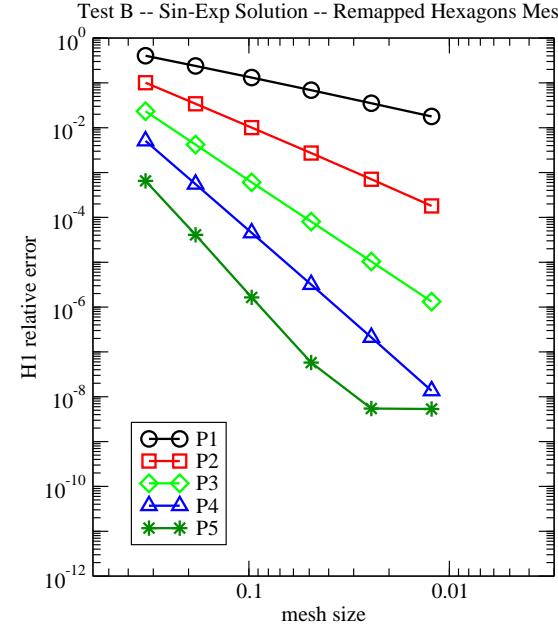
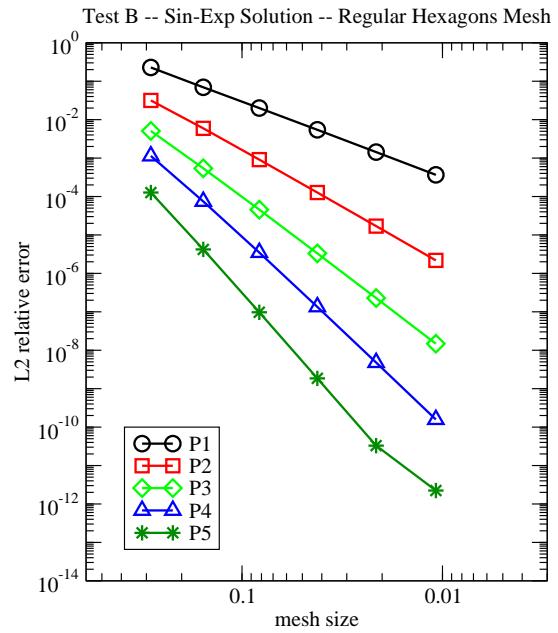


Fig. 172. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

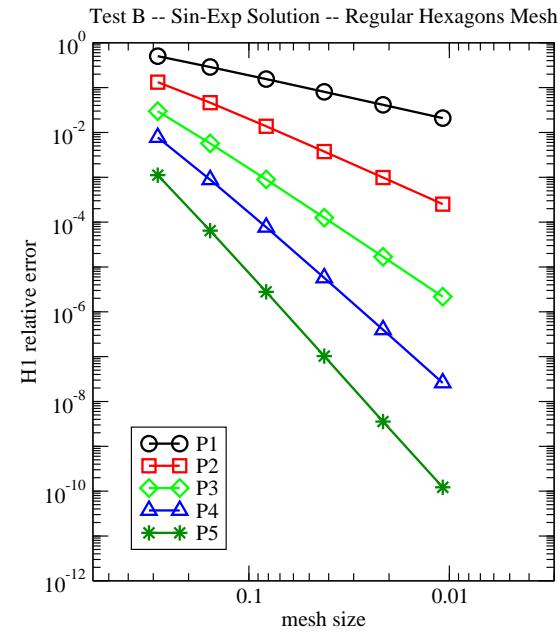
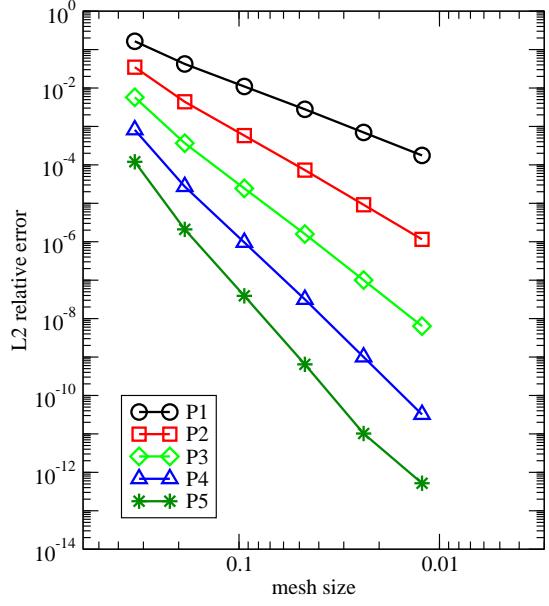


Fig. 173. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular hexagons.

**Internal/External VEM - Constant Coeffs.**  
Test B -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
Test B -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

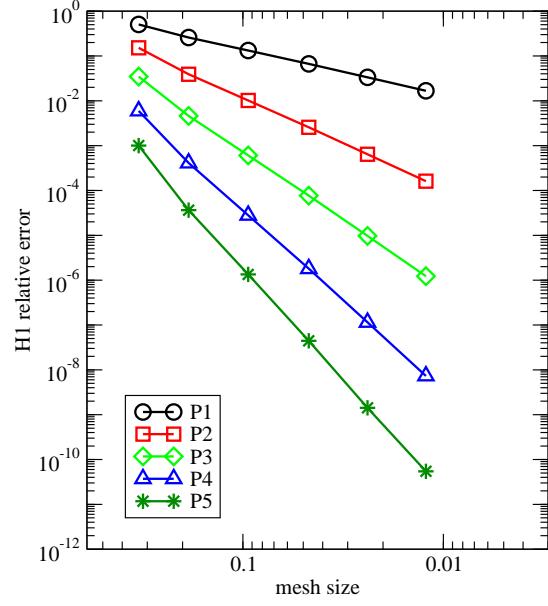
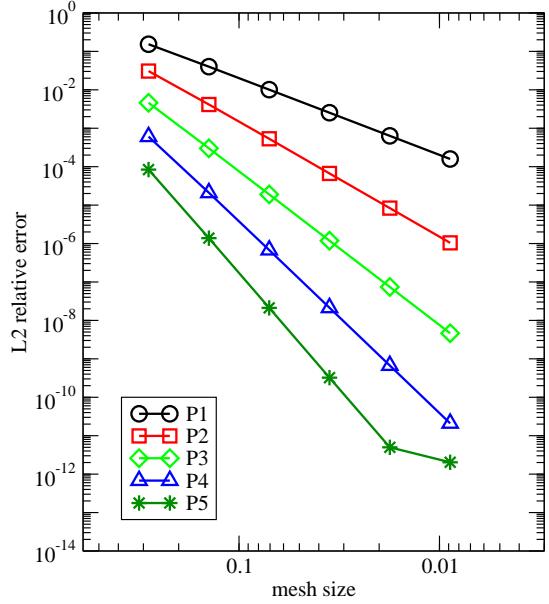


Fig. 174. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of randomized quadrilateral cells.

**Internal/External VEM - Constant Coeffs.**  
Test B -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
Test B -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

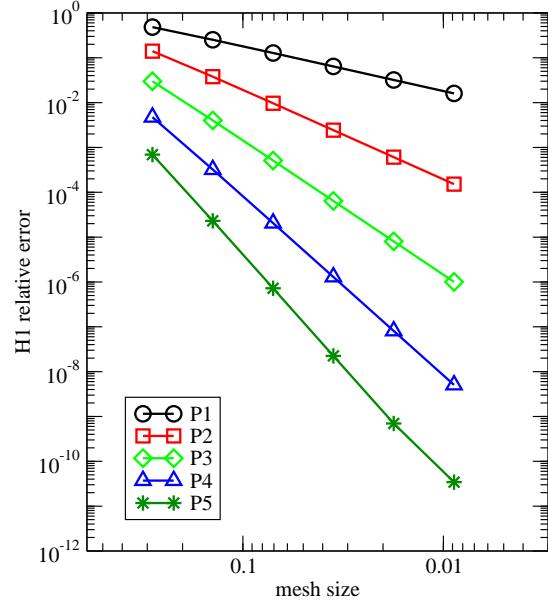
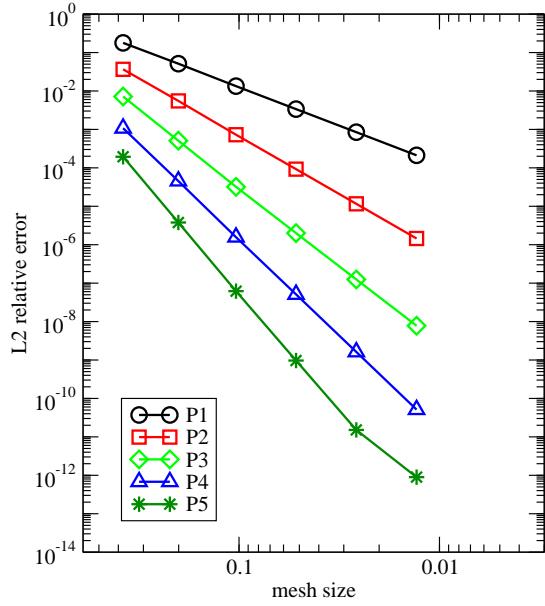


Fig. 175. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

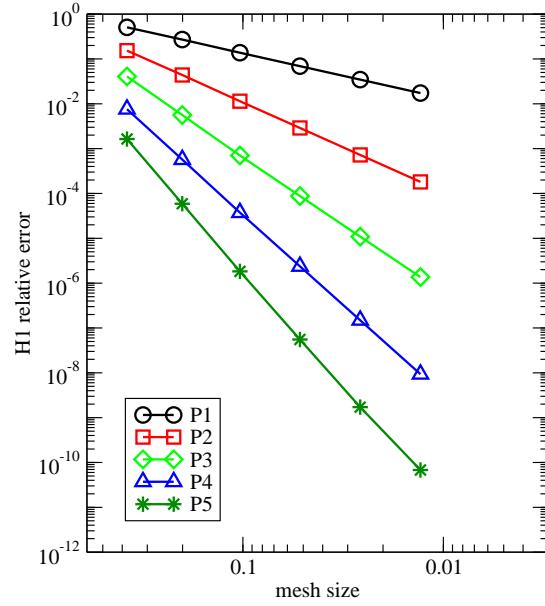
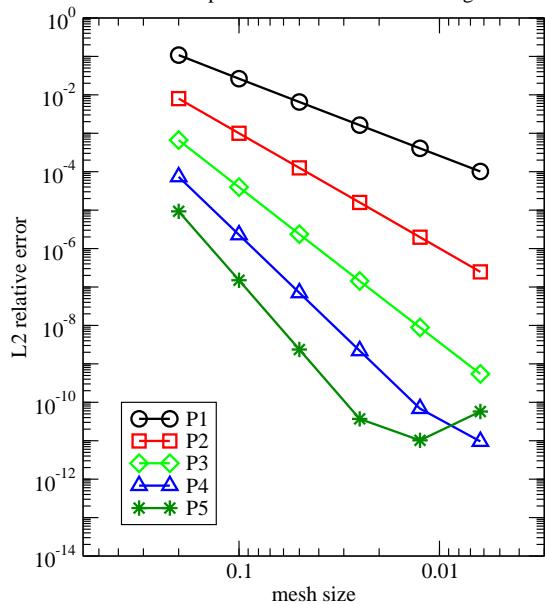


Fig. 176. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Sin-Exp Solution -- Criss-cross triangular Mesh

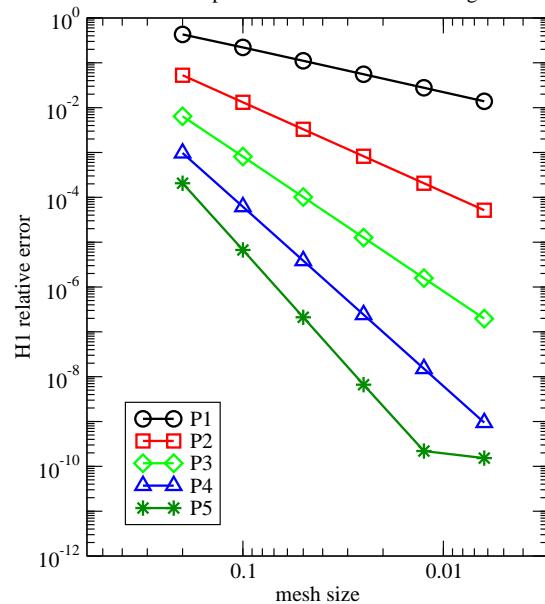
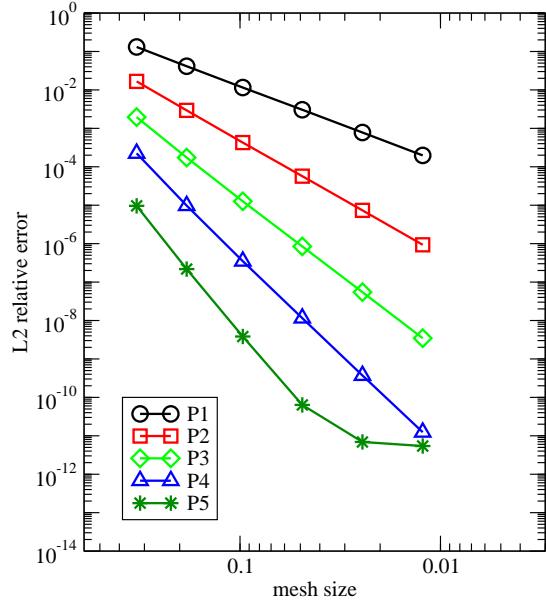


Fig. 177. Internal VEM formulation with constant coefficients; Test B; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Remapped Hexagons Mesh

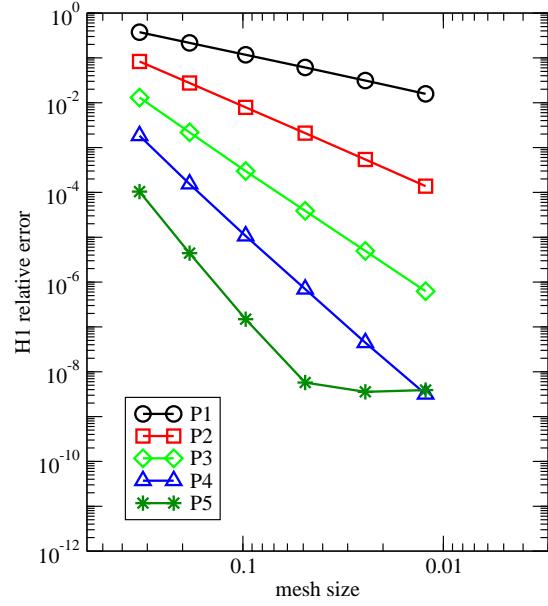
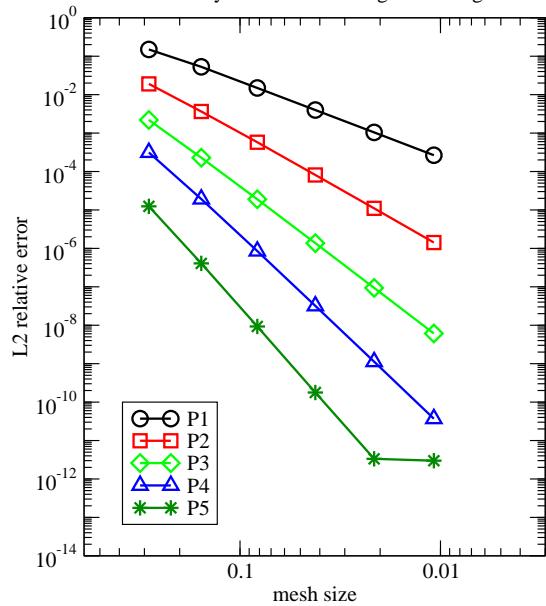


Fig. 178. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Regular Hexagons Mesh

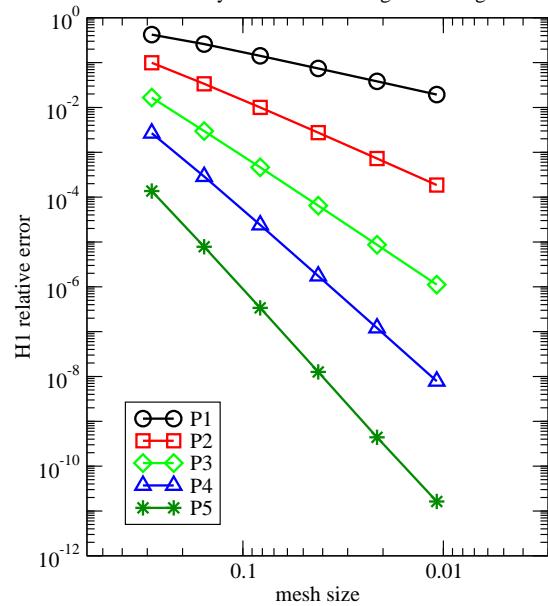
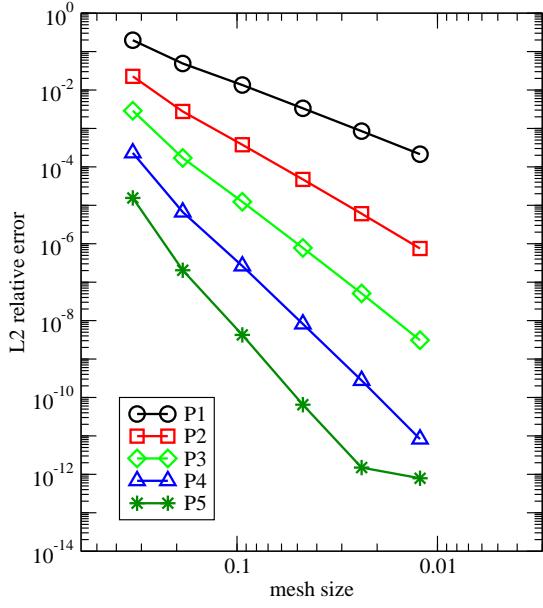


Fig. 179. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Randomized Quadrilaterals Mesh

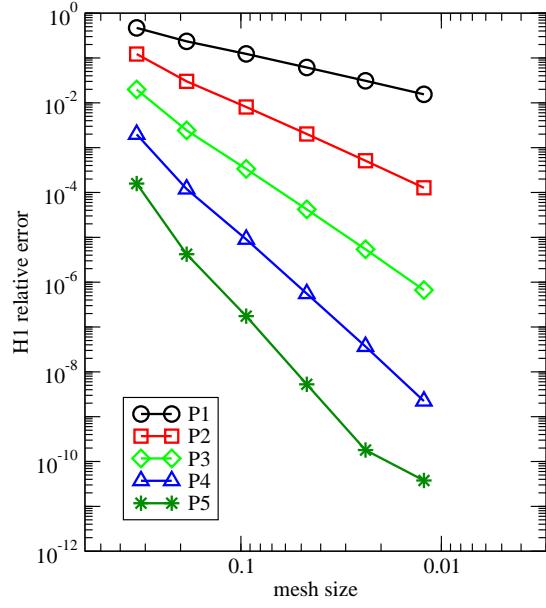
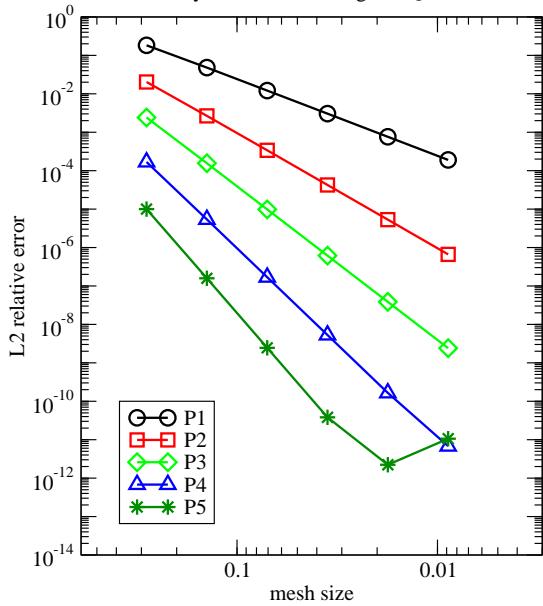


Fig. 180. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Regular Quadrilaterals Mesh

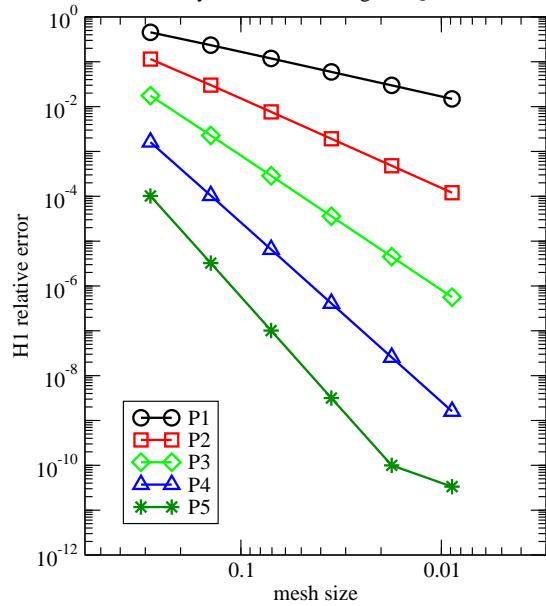
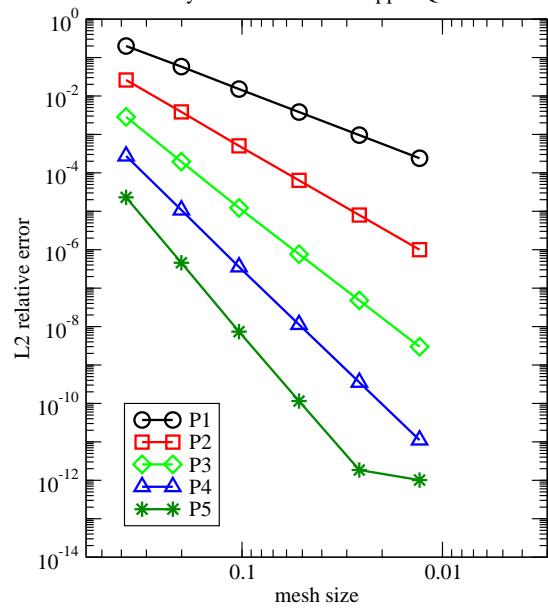


Fig. 181. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs. Test B -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs. Test B -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

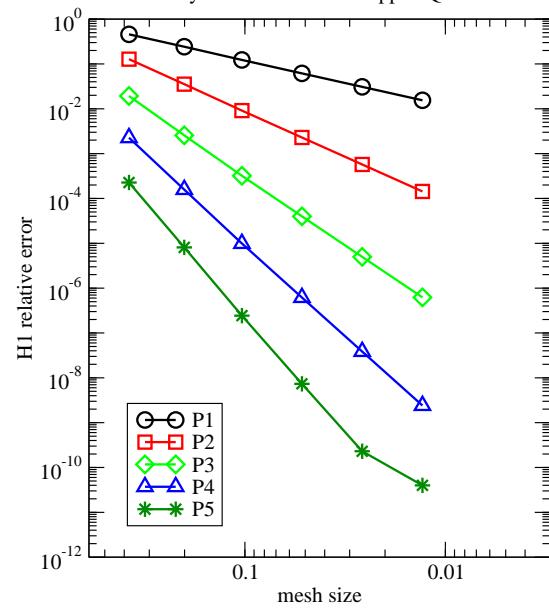
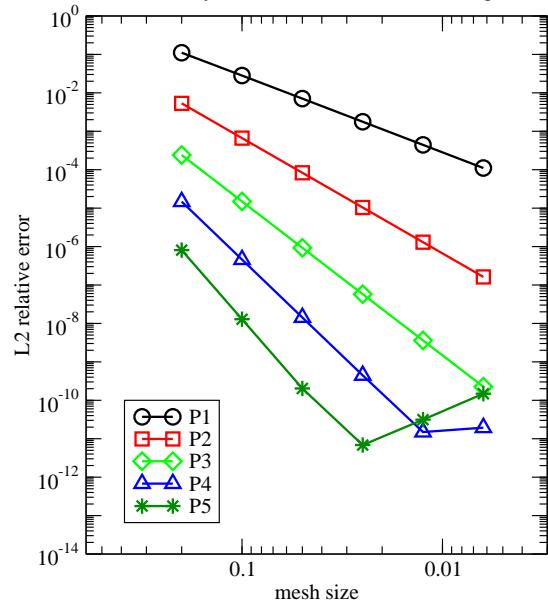


Fig. 182. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test B -- Two-Layers Solution -- Criss-cross triangular Mesh

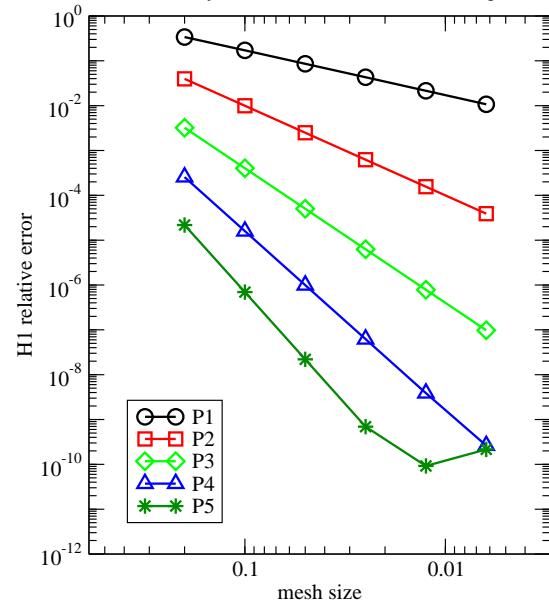
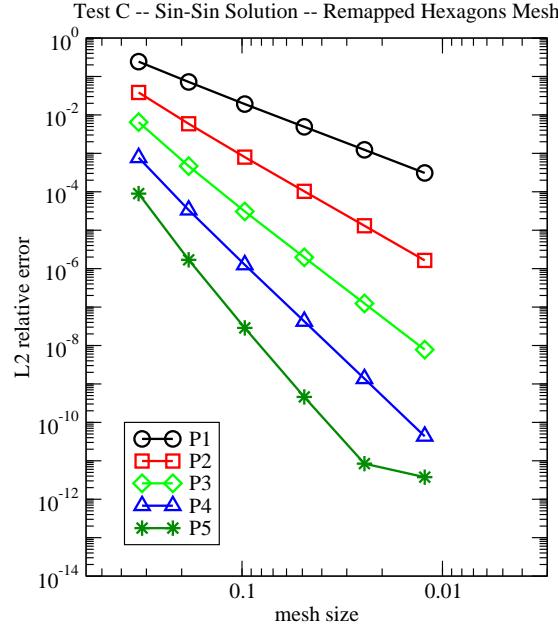


Fig. 183. Internal VEM formulation with constant coefficients; Test B; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

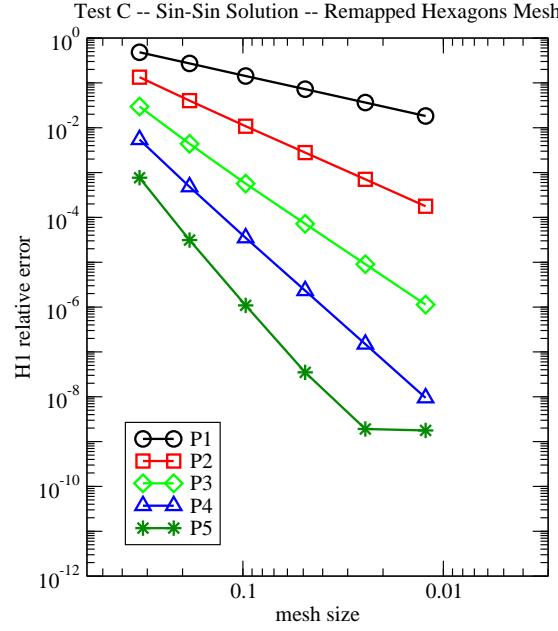
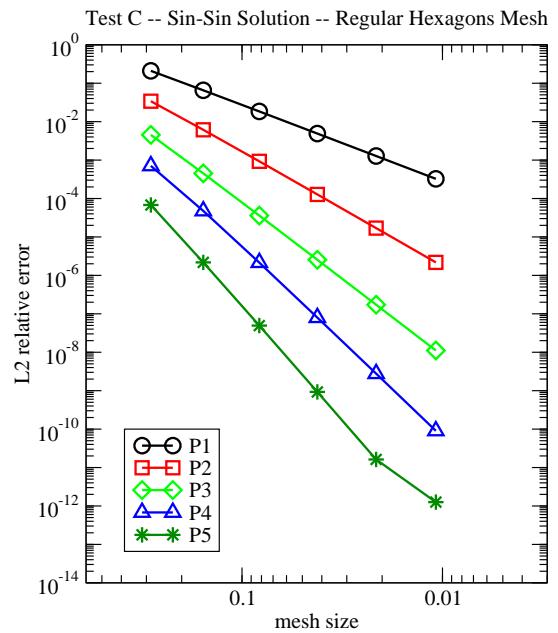


Fig. 184. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

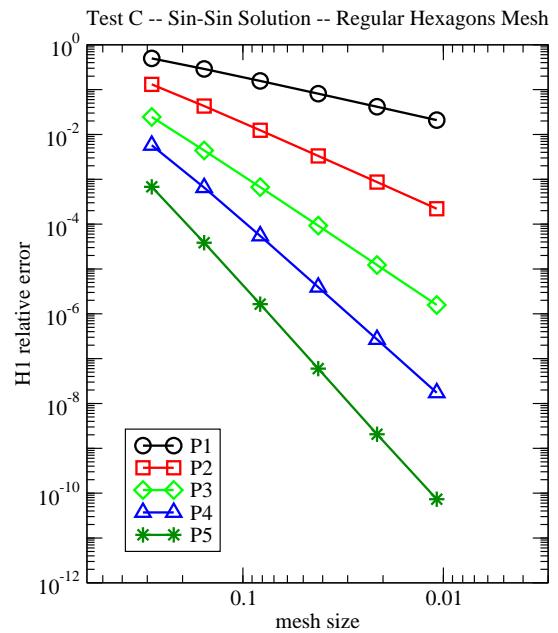
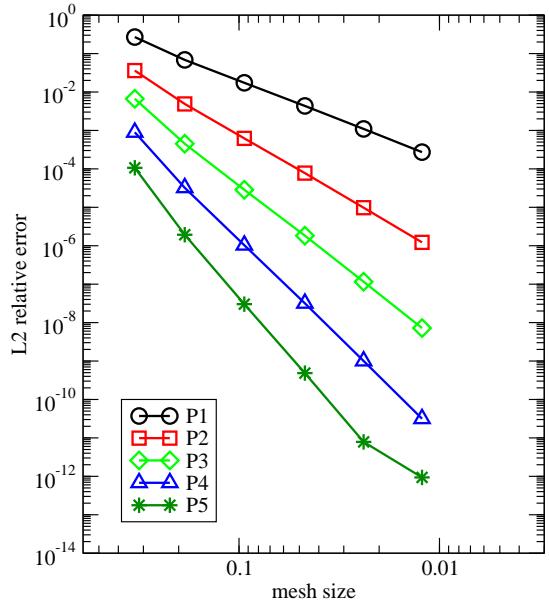


Fig. 185. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

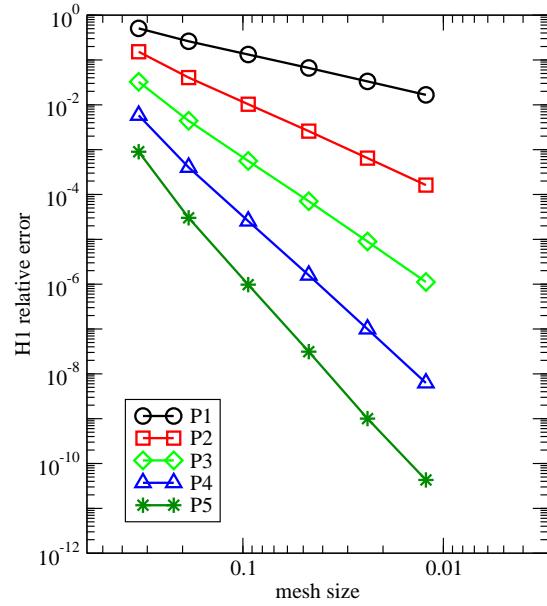
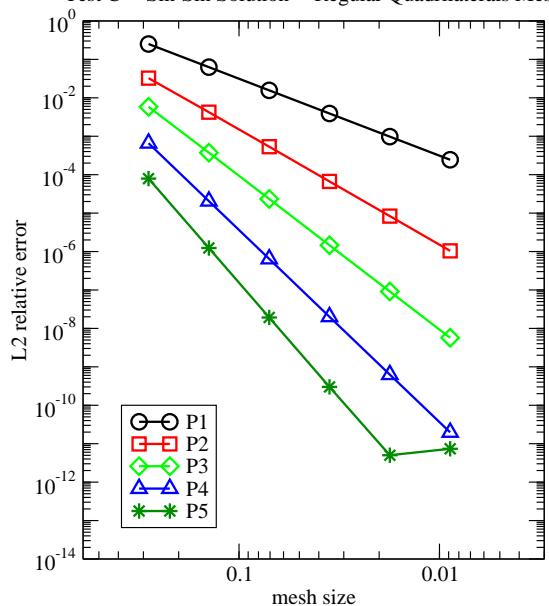


Fig. 186. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

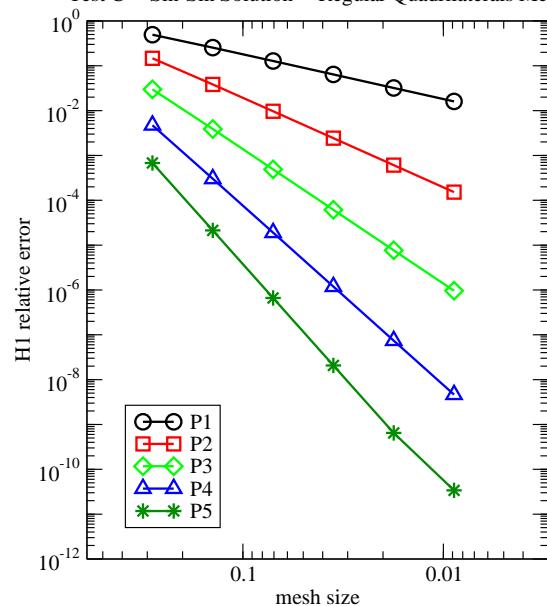
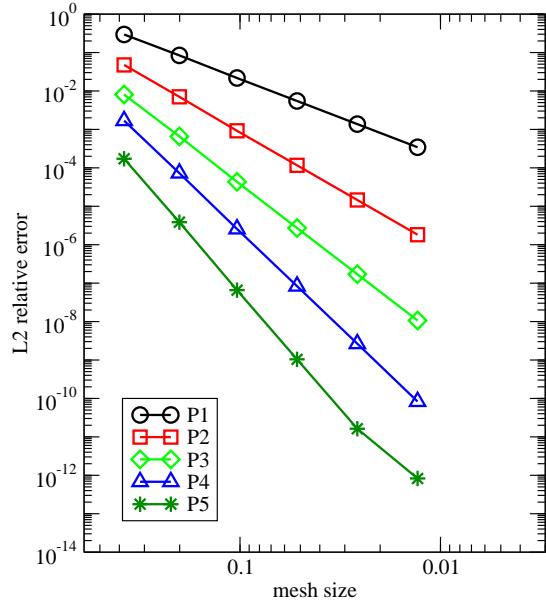


Fig. 187. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

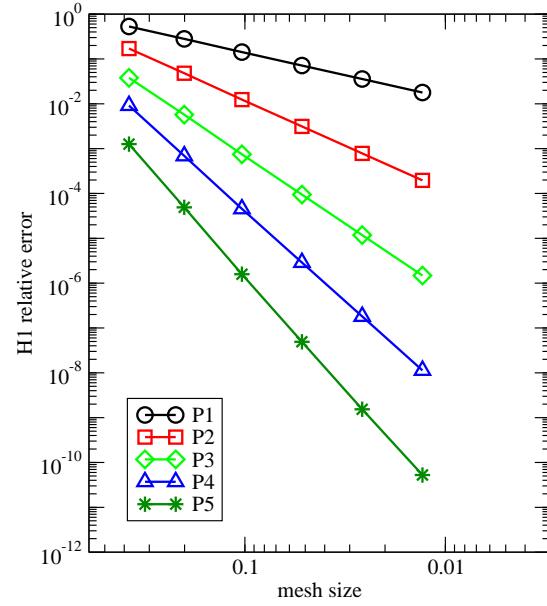
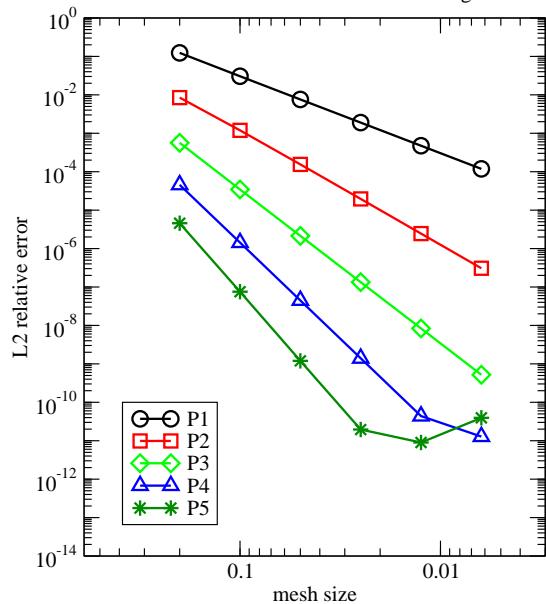


Fig. 188. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Sin Solution -- Criss-cross triangular Mesh

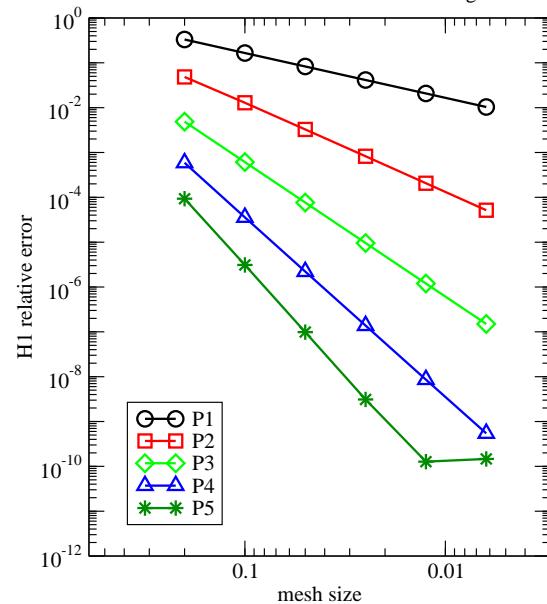
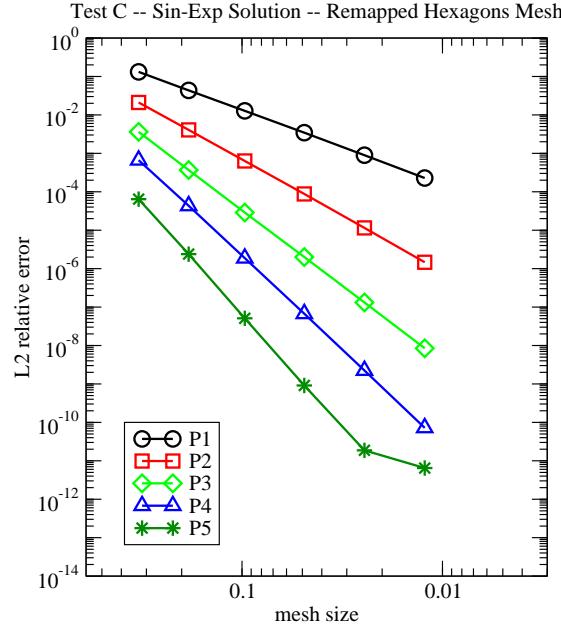


Fig. 189. Internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

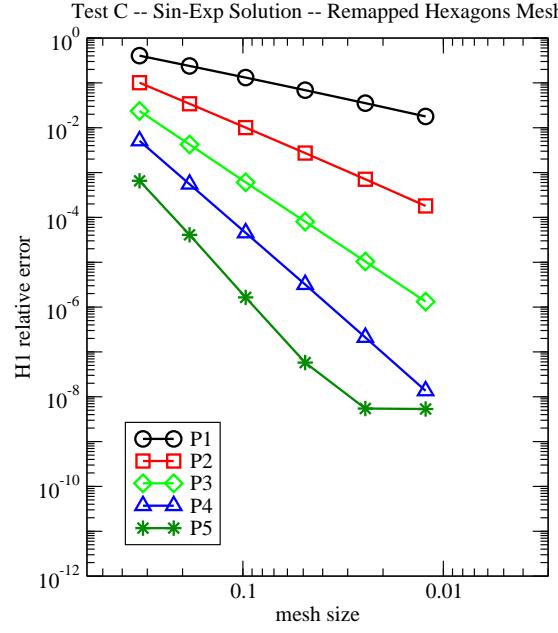
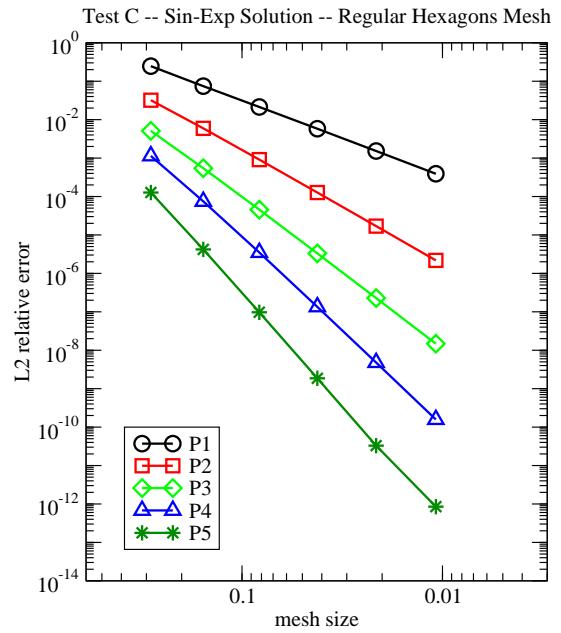


Fig. 190. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

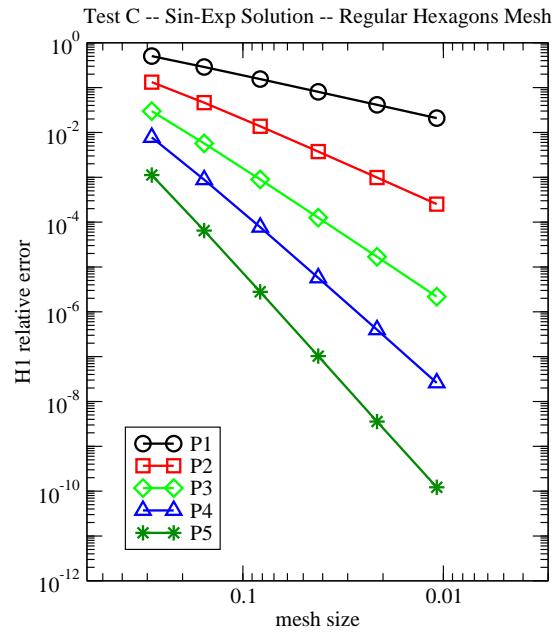
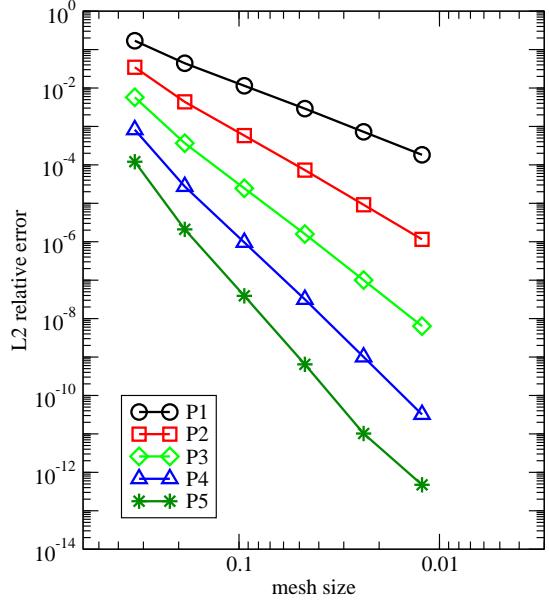


Fig. 191. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

**Internal/External VEM - Constant Coeffs.**  
Test C -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
Test C -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

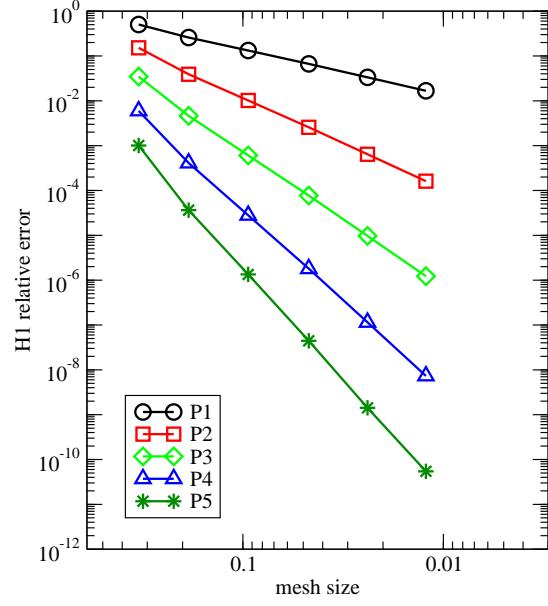
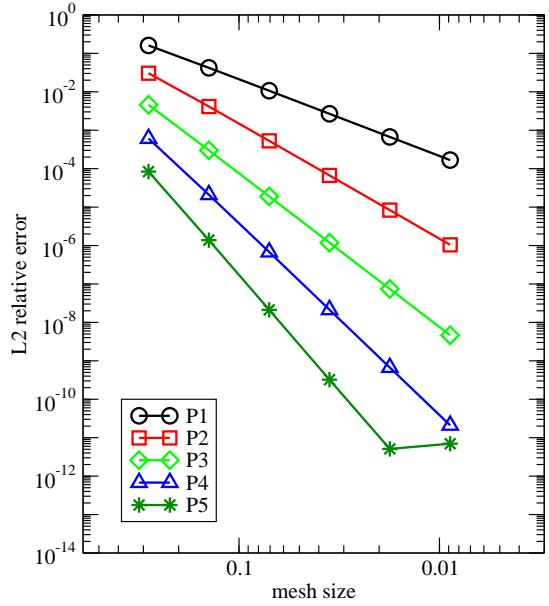


Fig. 192. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

**Internal/External VEM - Constant Coeffs.**  
Test C -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
Test C -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

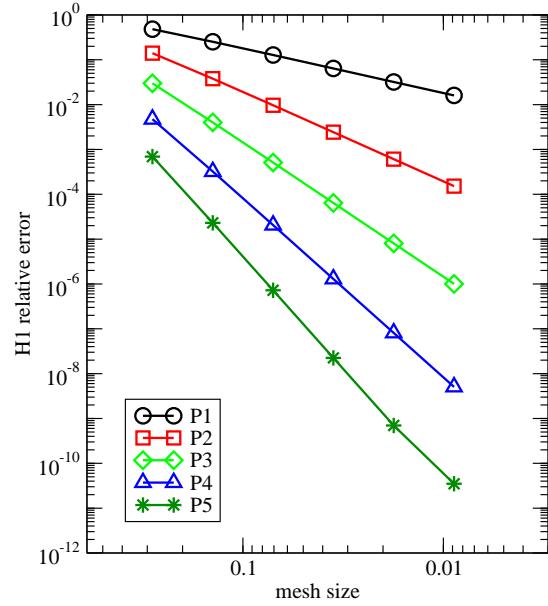
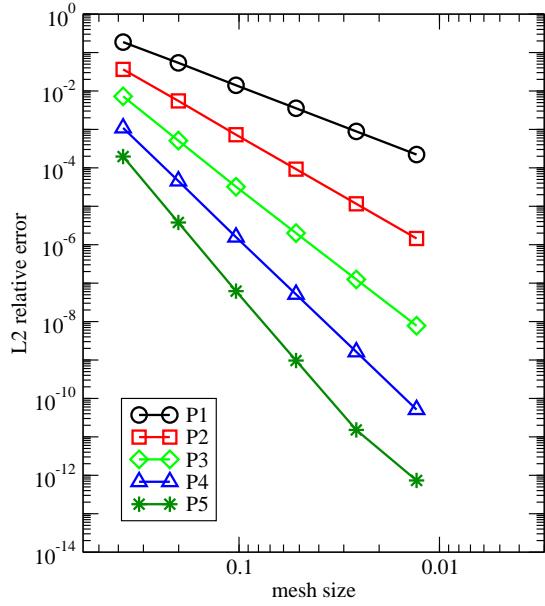


Fig. 193. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

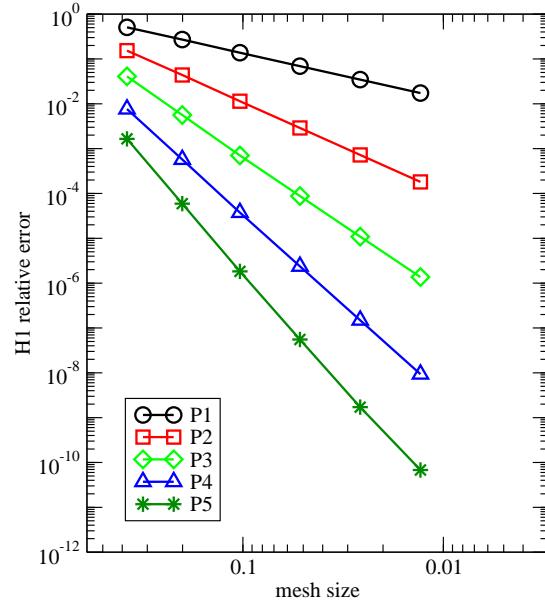
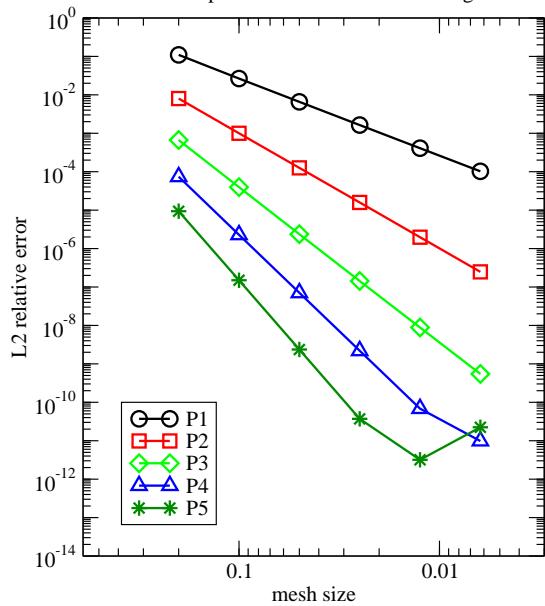


Fig. 194. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Sin-Exp Solution -- Criss-cross triangular Mesh

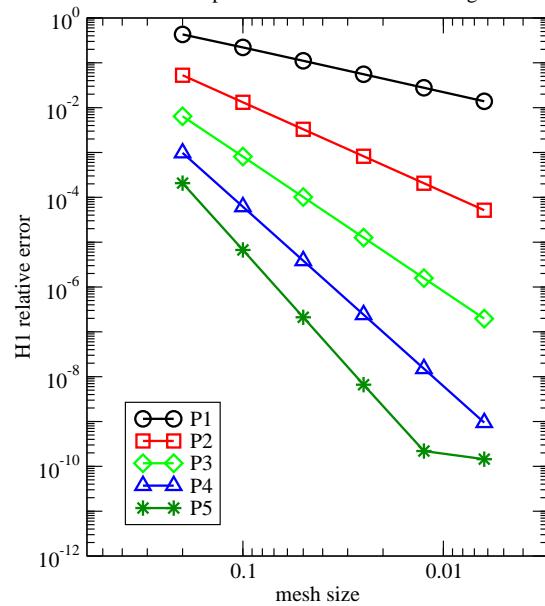
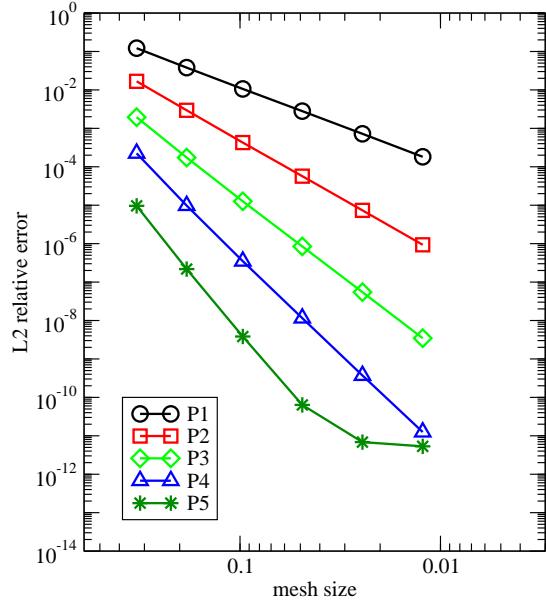


Fig. 195. Internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.

Test C -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Two-Layers Solution -- Remapped Hexagons Mesh

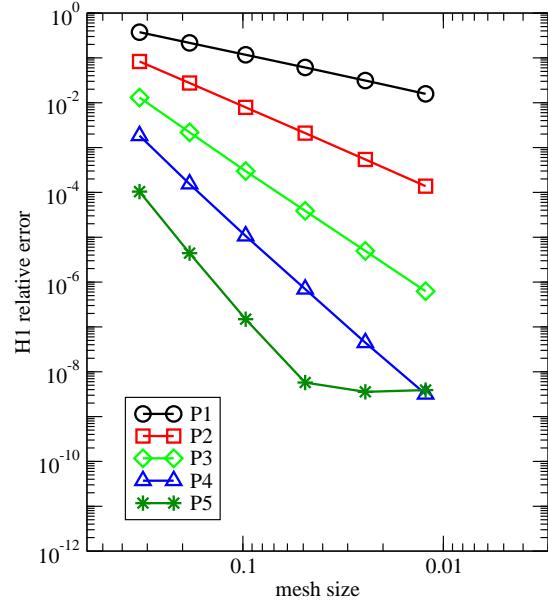
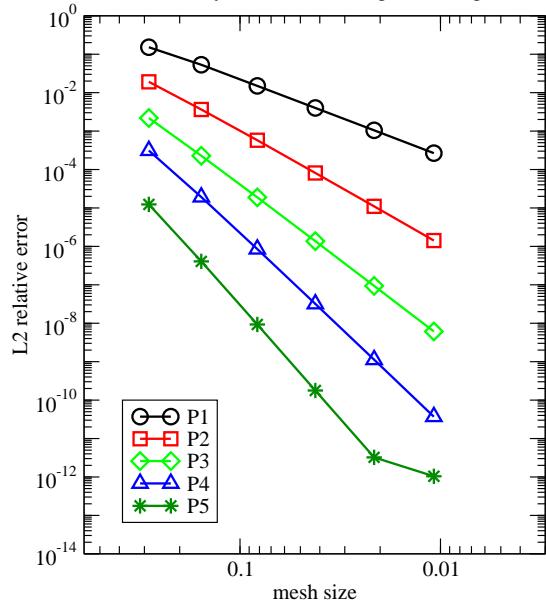


Fig. 196. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.

Test C -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test C -- Two-Layers Solution -- Regular Hexagons Mesh

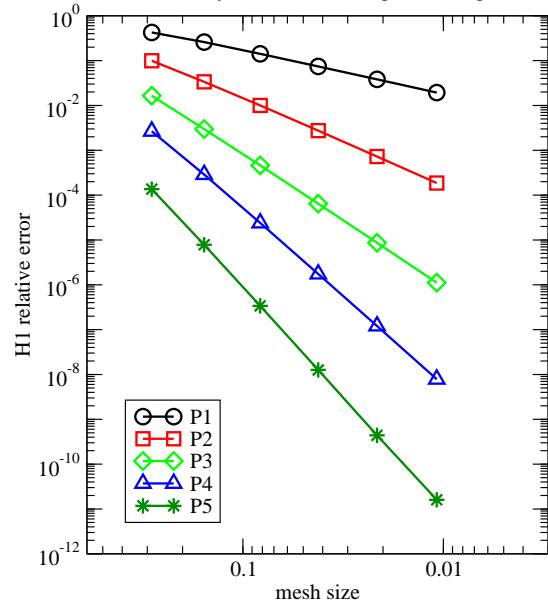


Fig. 197. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular hexagons.

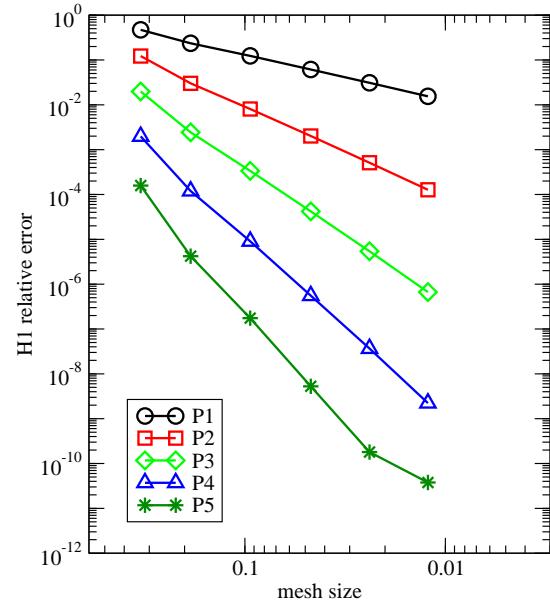
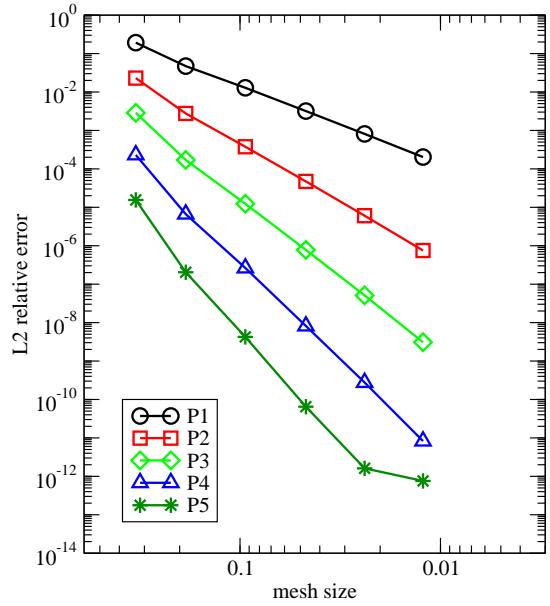


Fig. 198. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

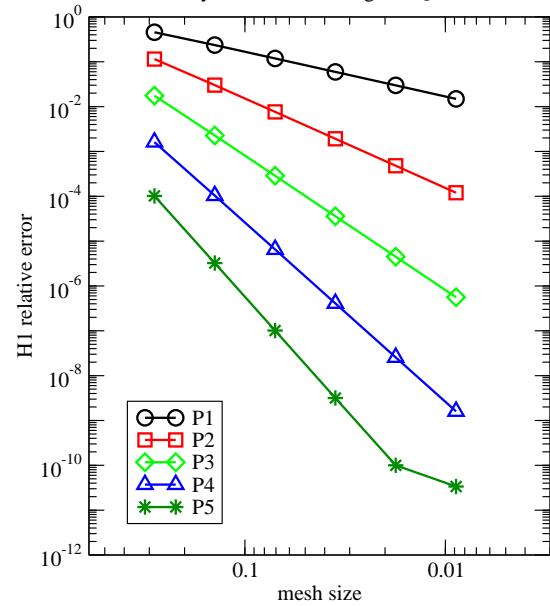
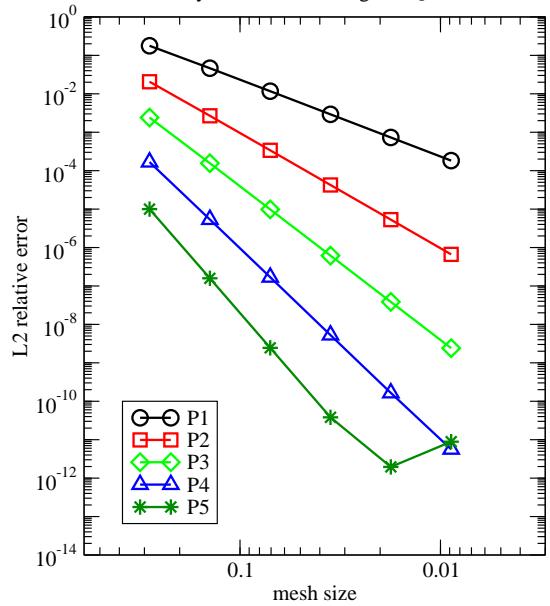
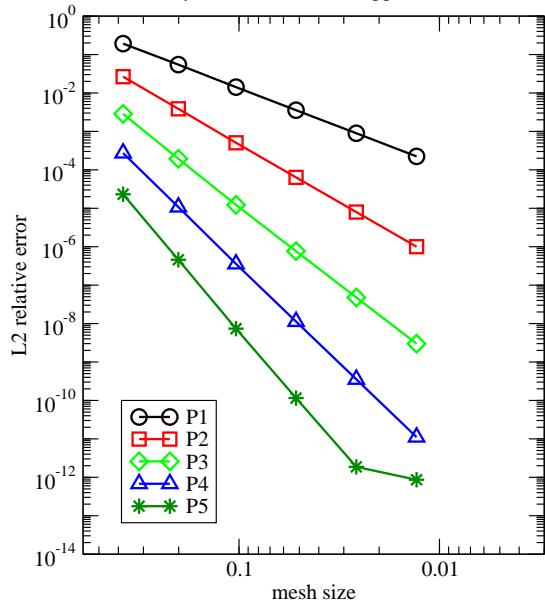


Fig. 199. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

**Internal/External VEM - Constant Coeffs.**  
 Test C -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test C -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

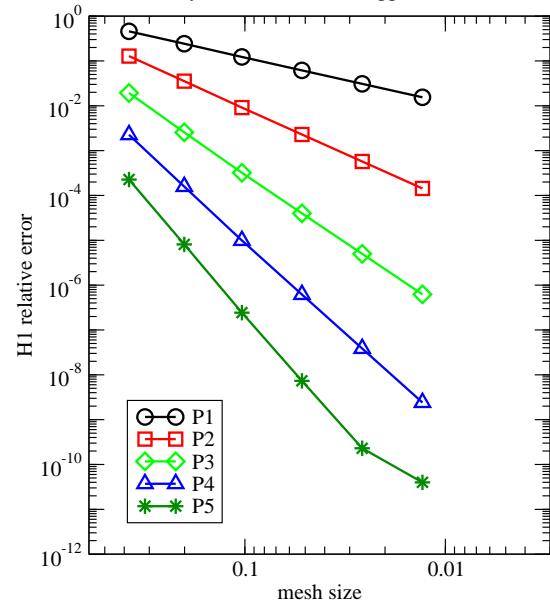
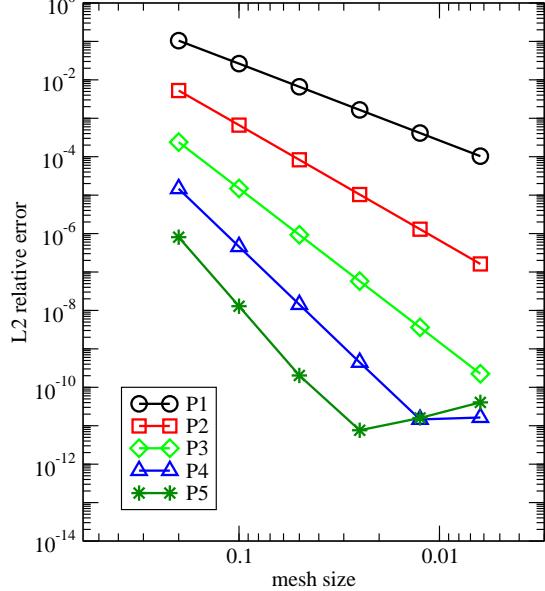


Fig. 200. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

**Internal/External VEM - Constant Coeffs.**  
 Test C -- Two-Layers Solution -- Criss-cross triangular Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test C -- Two-Layers Solution -- Criss-cross triangular Mesh

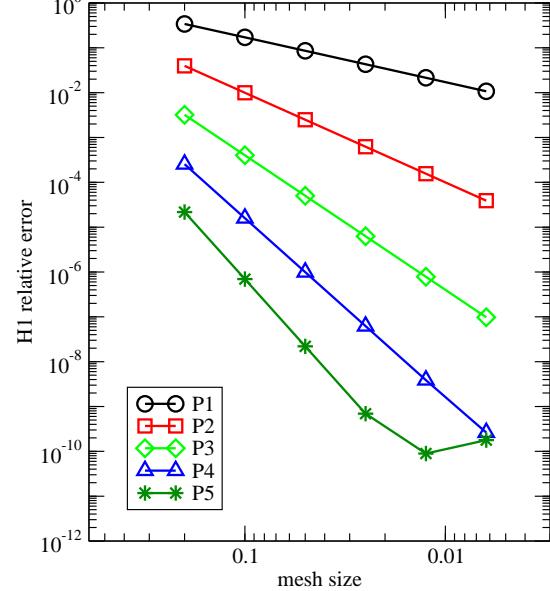
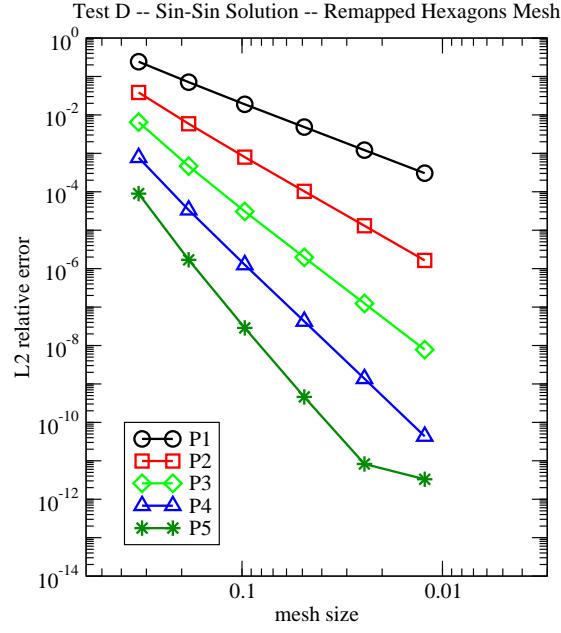


Fig. 201. Internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

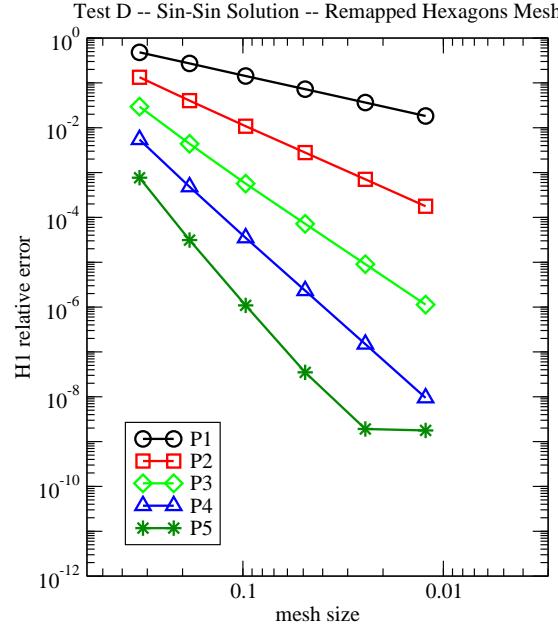
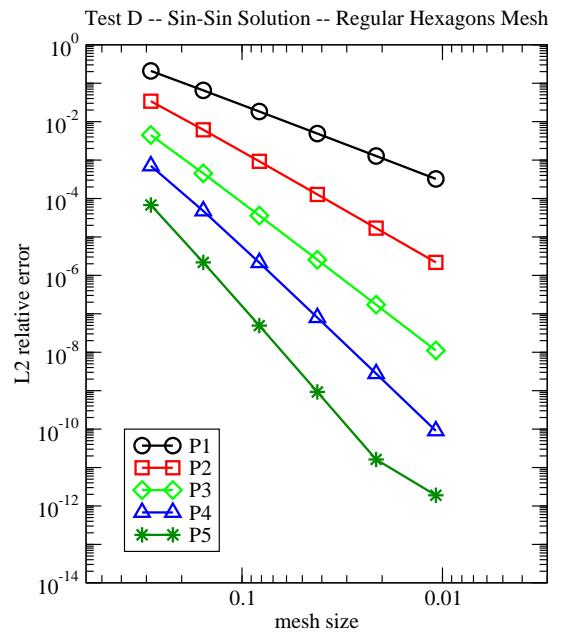


Fig. 202. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

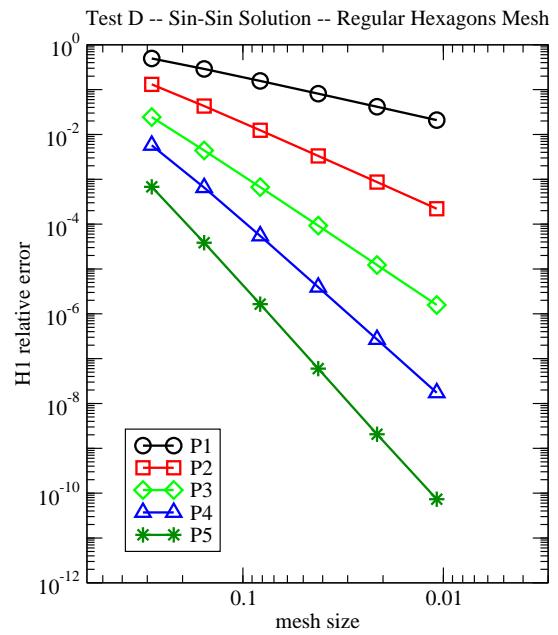
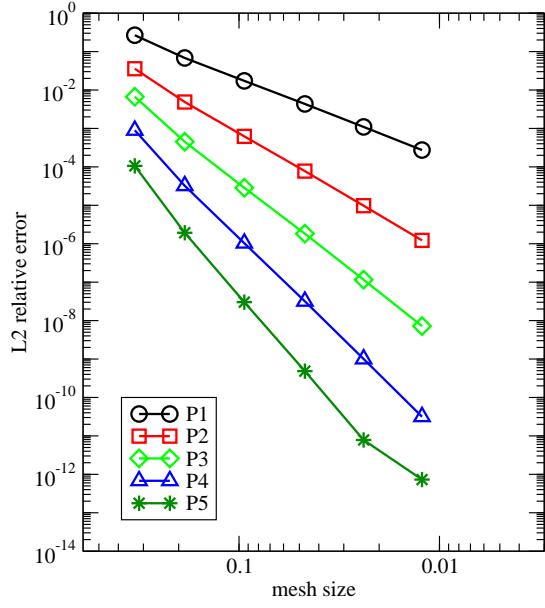


Fig. 203. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

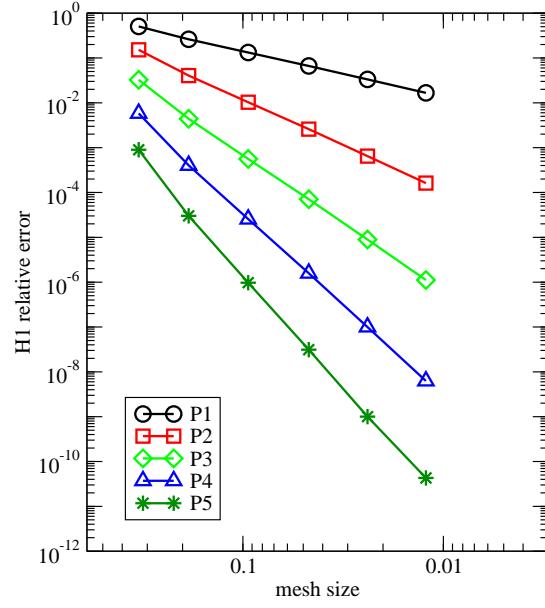
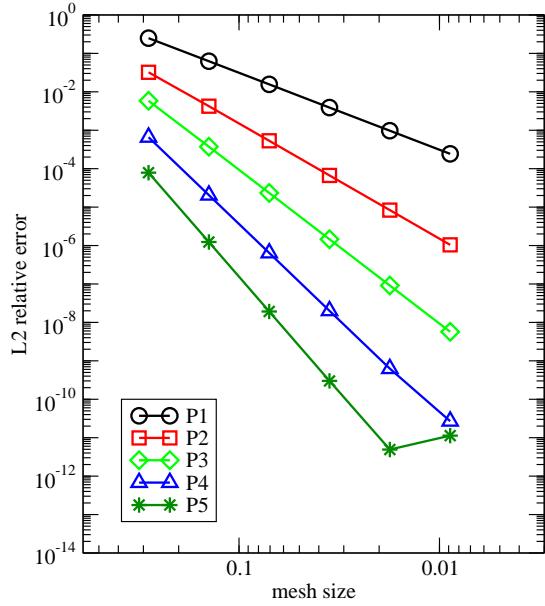


Fig. 204. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

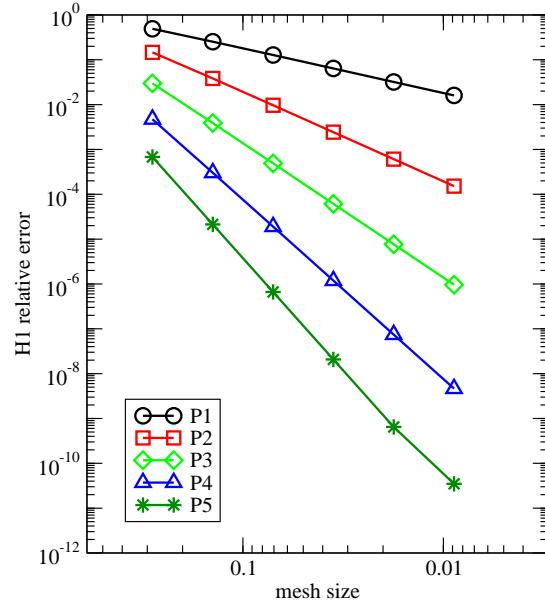
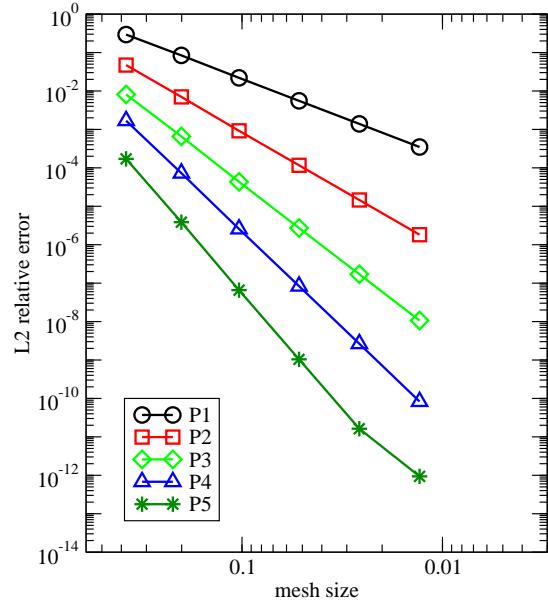


Fig. 205. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

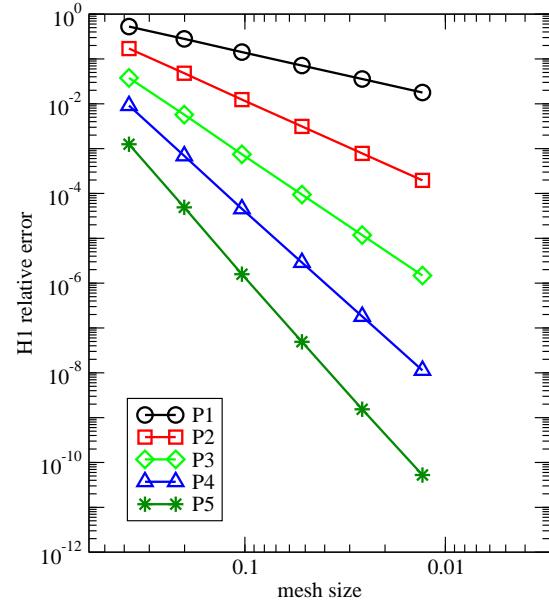
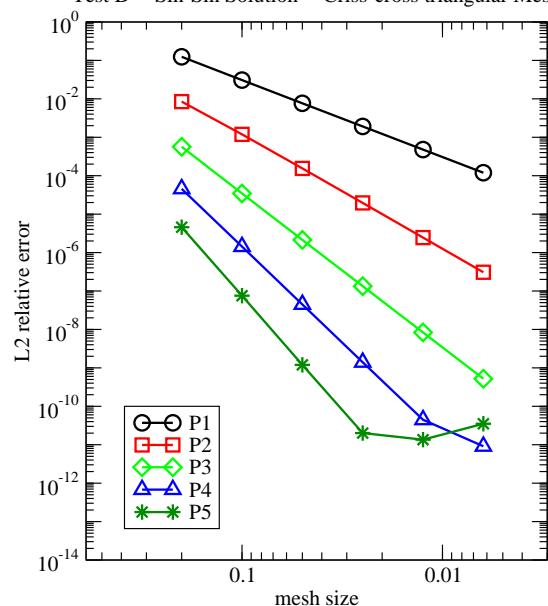


Fig. 206. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Sin Solution -- Criss-cross triangular Mesh

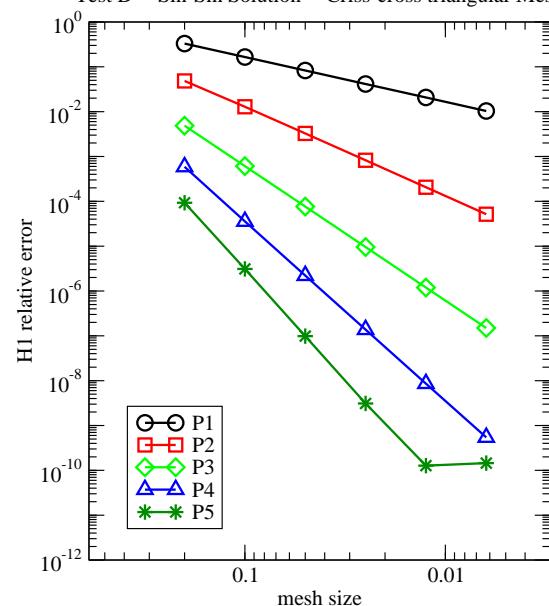
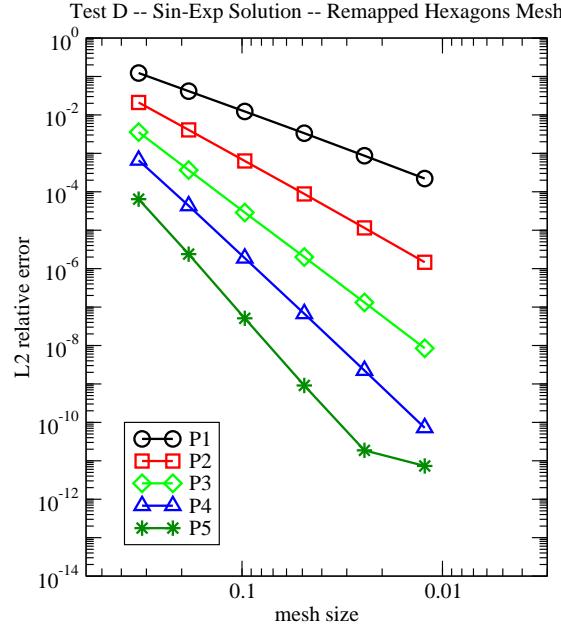


Fig. 207. Internal VEM formulation with constant coefficients; Test D; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

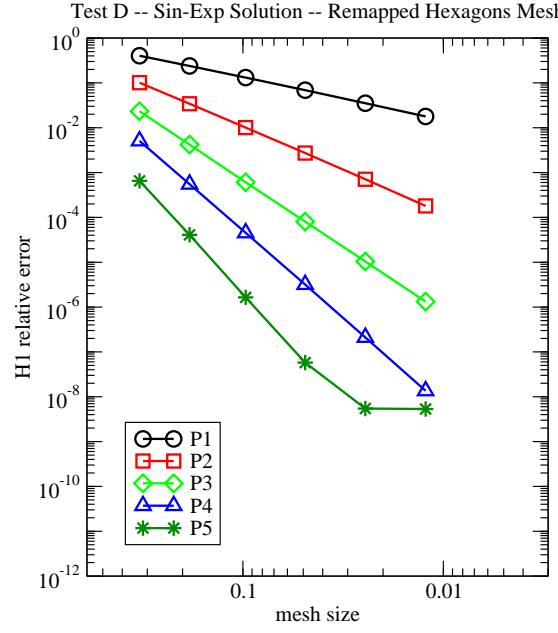
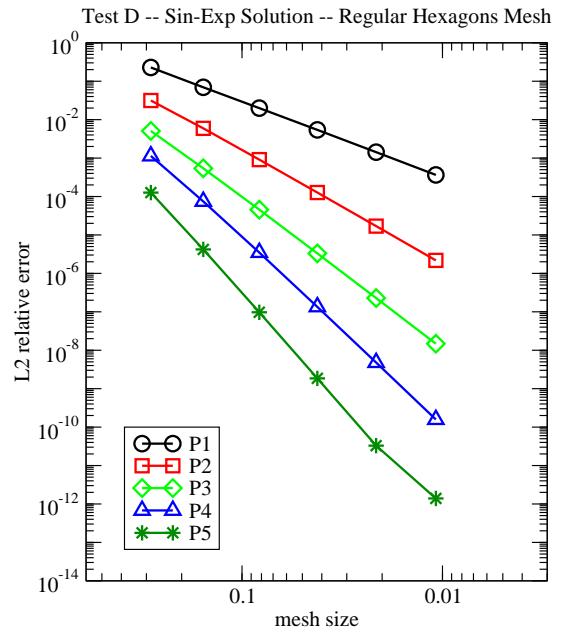


Fig. 208. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.



### Internal/External VEM - Constant Coeffs.

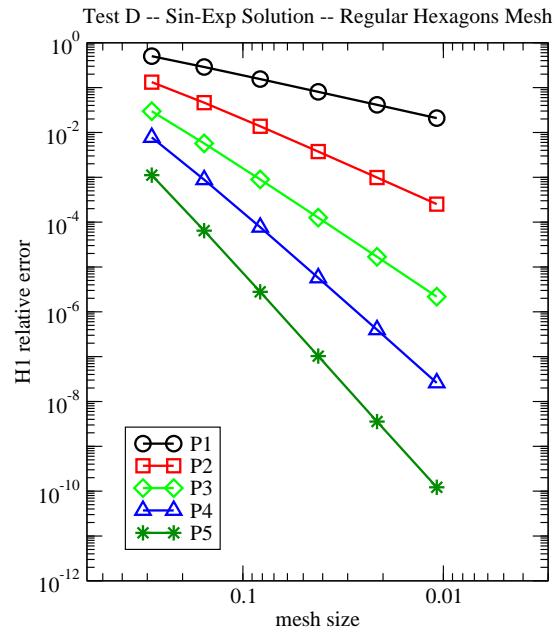
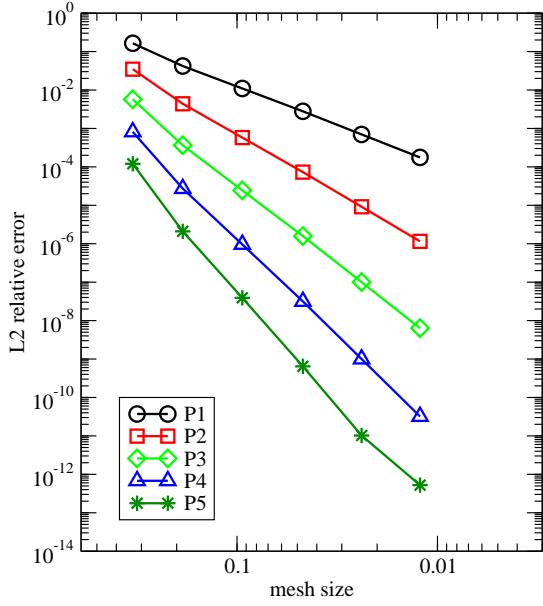


Fig. 209. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular hexagons.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

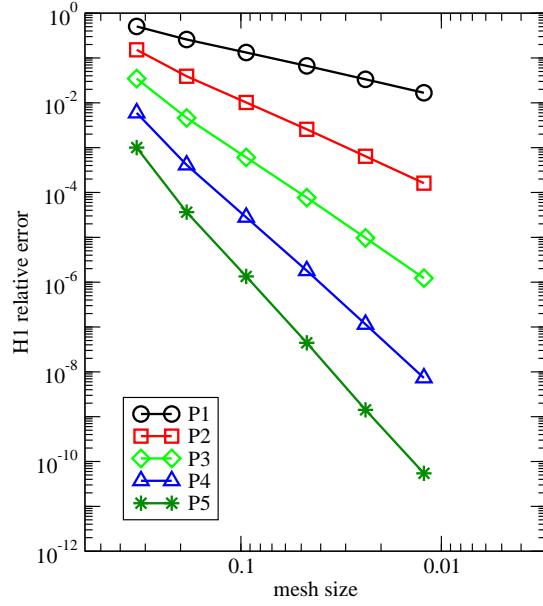
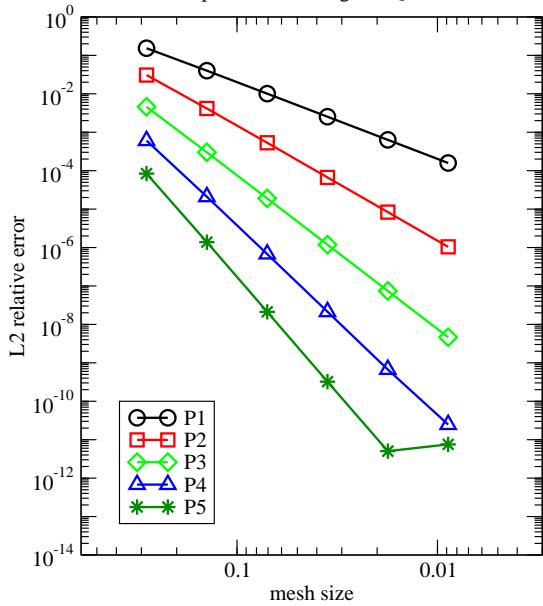


Fig. 210. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

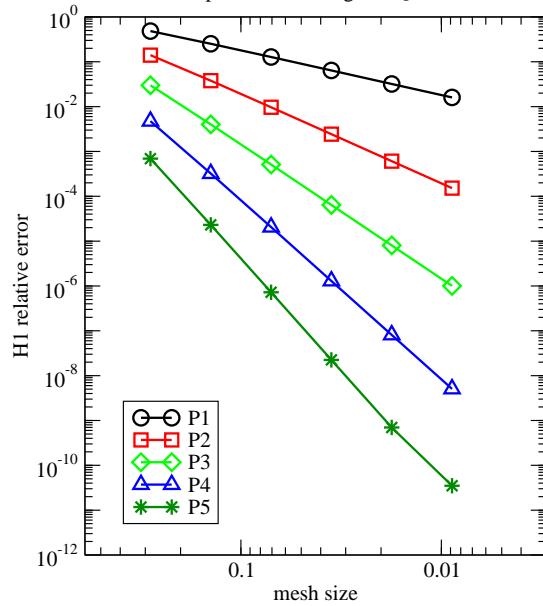
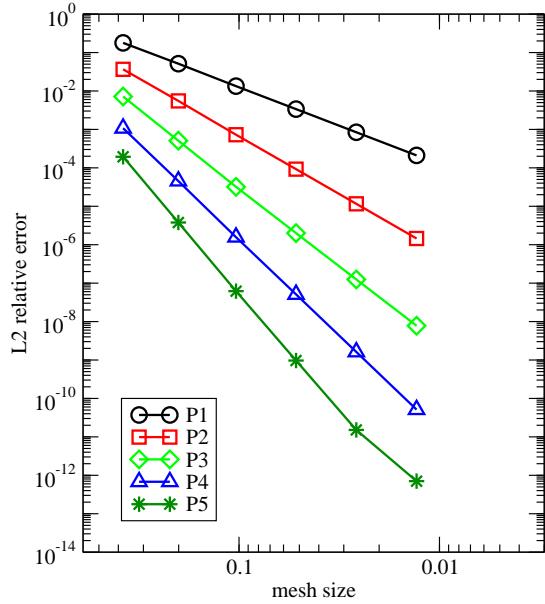


Fig. 211. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

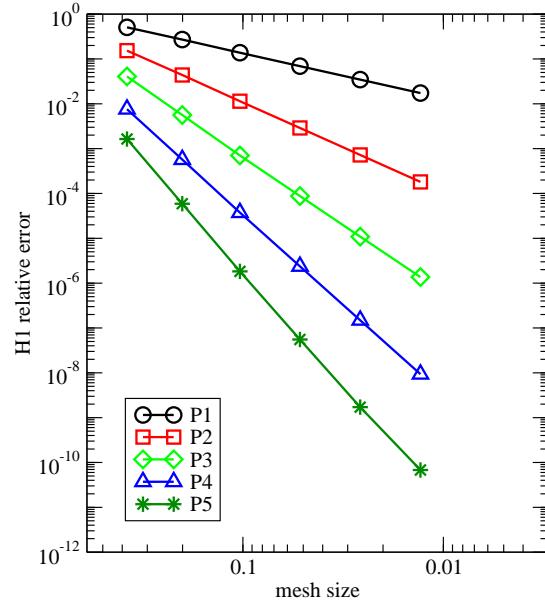
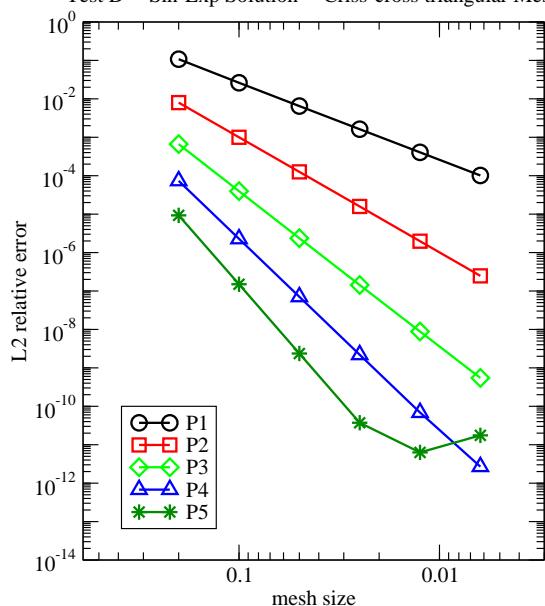


Fig. 212. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Sin-Exp Solution -- Criss-cross triangular Mesh

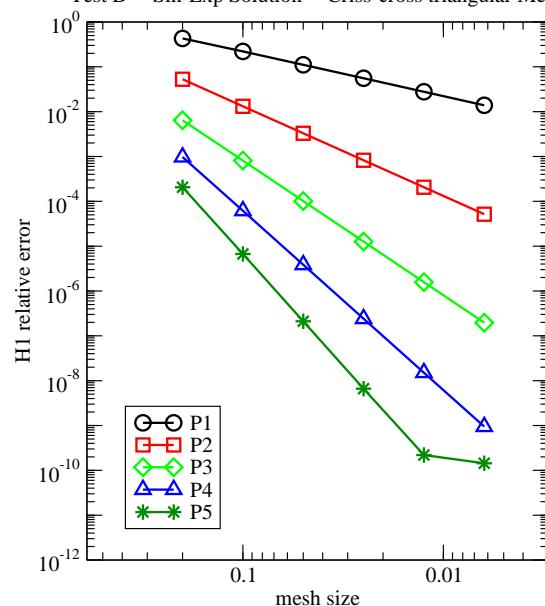
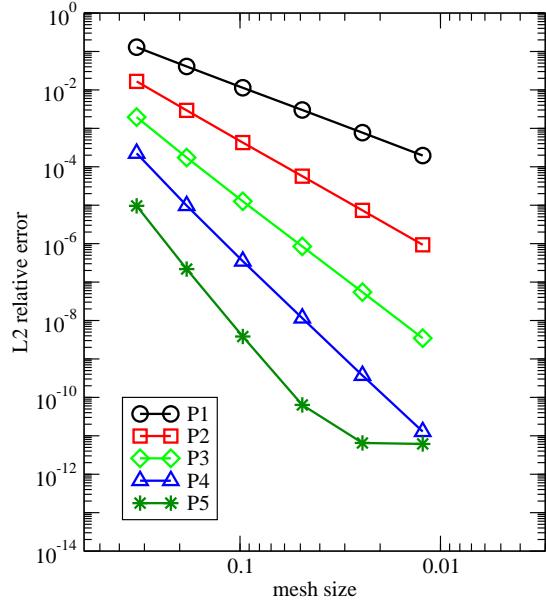


Fig. 213. Internal VEM formulation with constant coefficients; Test D; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Constant Coeffs.

Test D -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Two-Layers Solution -- Remapped Hexagons Mesh

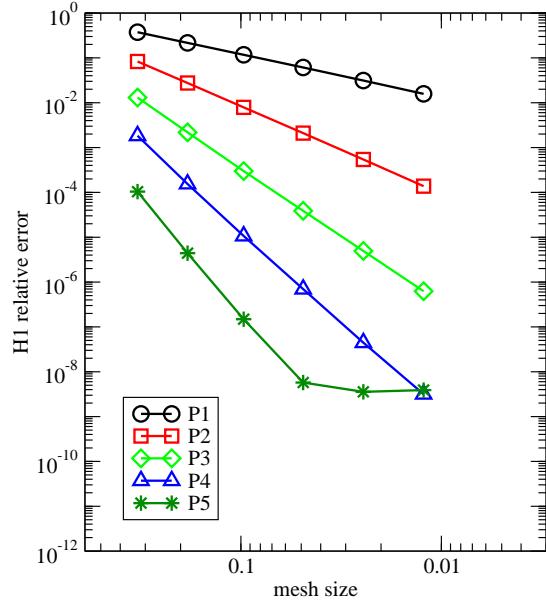
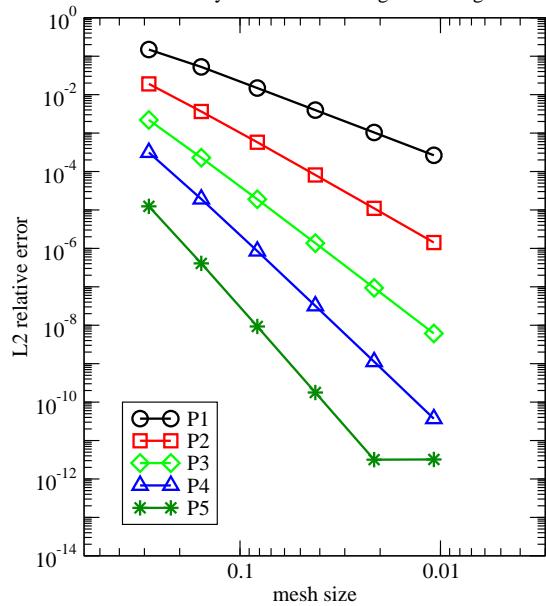


Fig. 214. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Constant Coeffs.

Test D -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Constant Coeffs.

Test D -- Two-Layers Solution -- Regular Hexagons Mesh

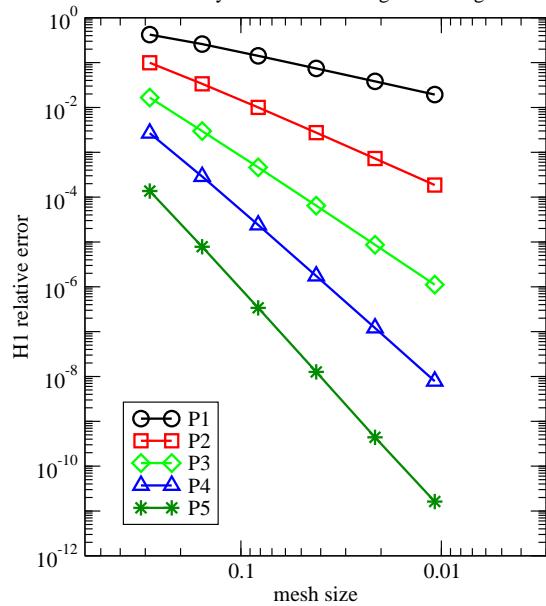


Fig. 215. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular hexagons.

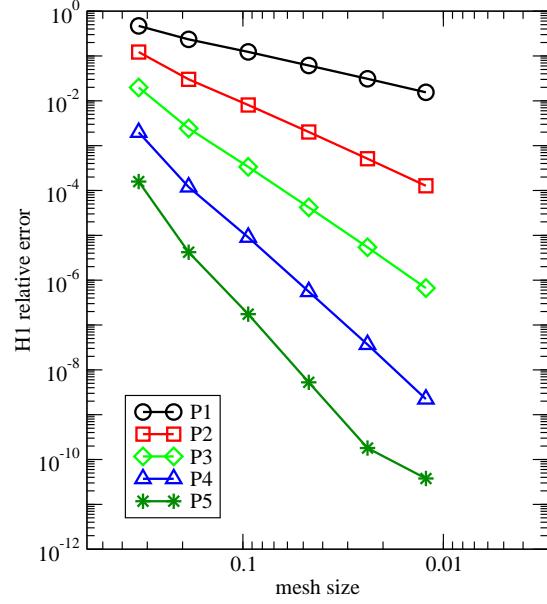
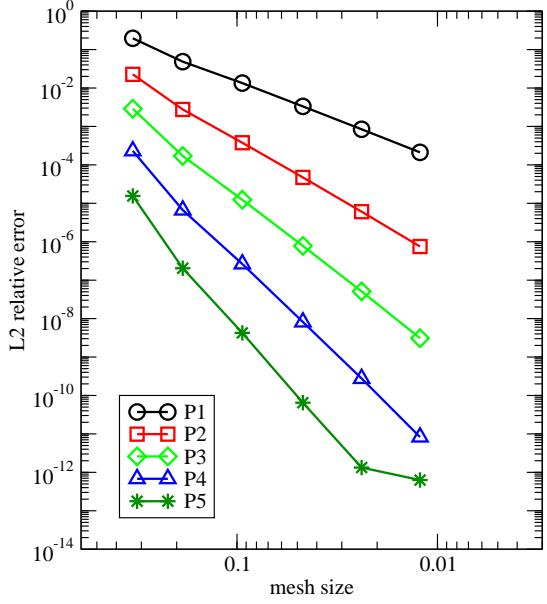


Fig. 216. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of randomized quadrilateral cells.

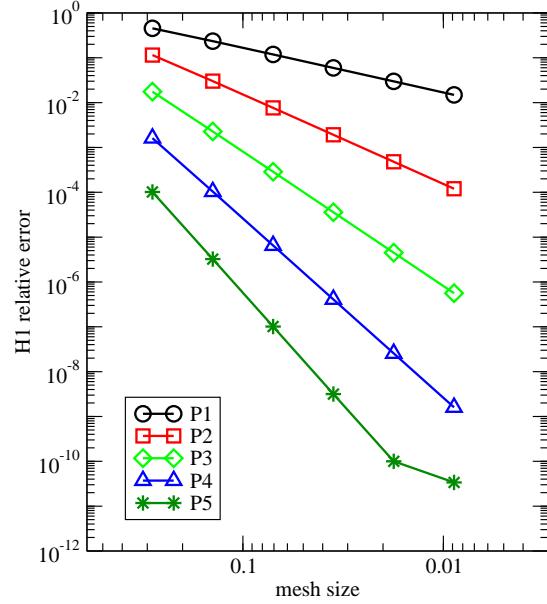
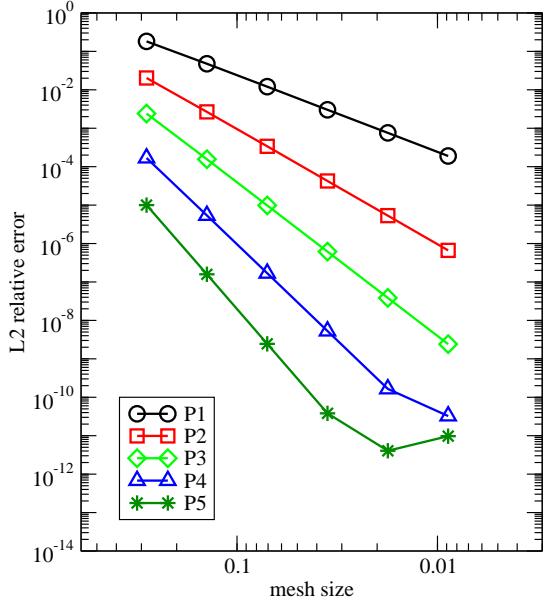
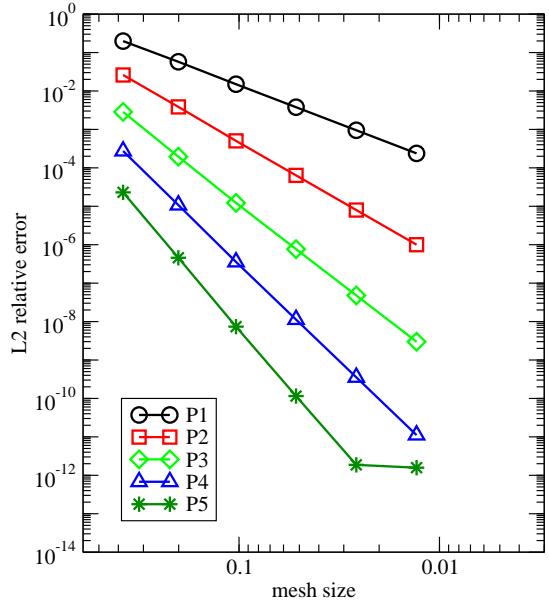


Fig. 217. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular quadrilateral cells (squares).

**Internal/External VEM - Constant Coeffs.**  
 Test D -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test D -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

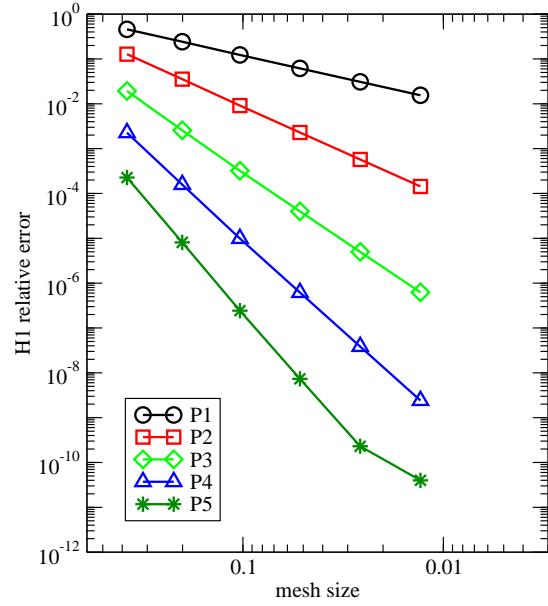
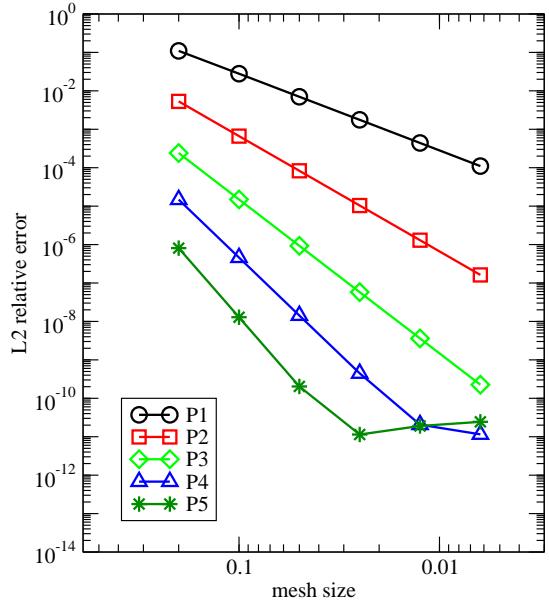


Fig. 218. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

**Internal/External VEM - Constant Coeffs.**  
 Test D -- Two-Layers Solution -- Criss-cross triangular Mesh



**Internal/External VEM - Constant Coeffs.**  
 Test D -- Two-Layers Solution -- Criss-cross triangular Mesh

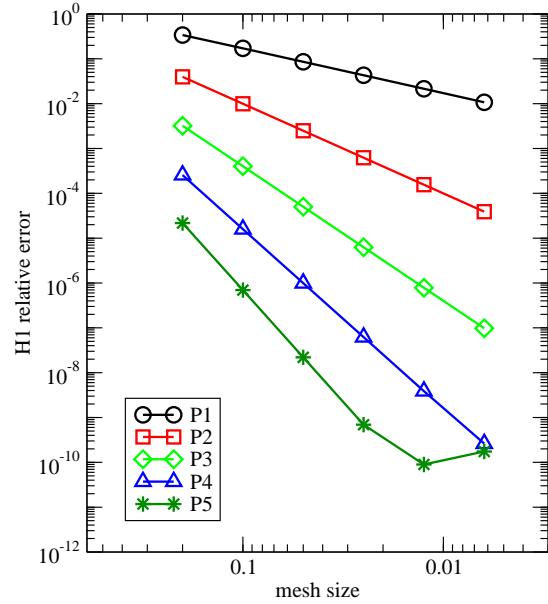
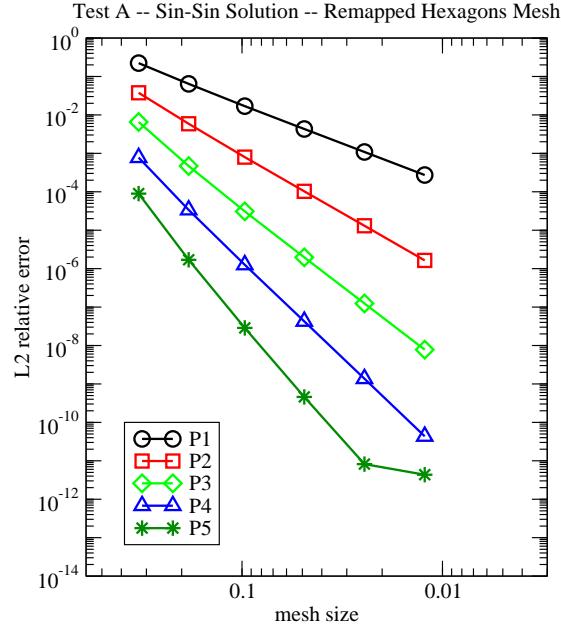


Fig. 219. Internal VEM formulation with constant coefficients; Test D; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

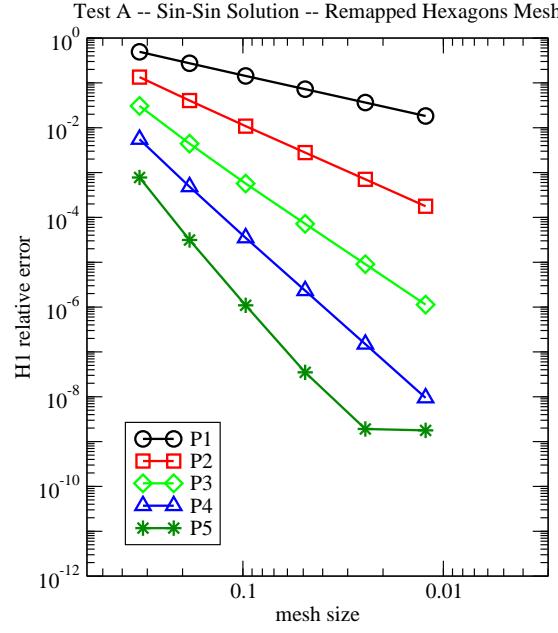
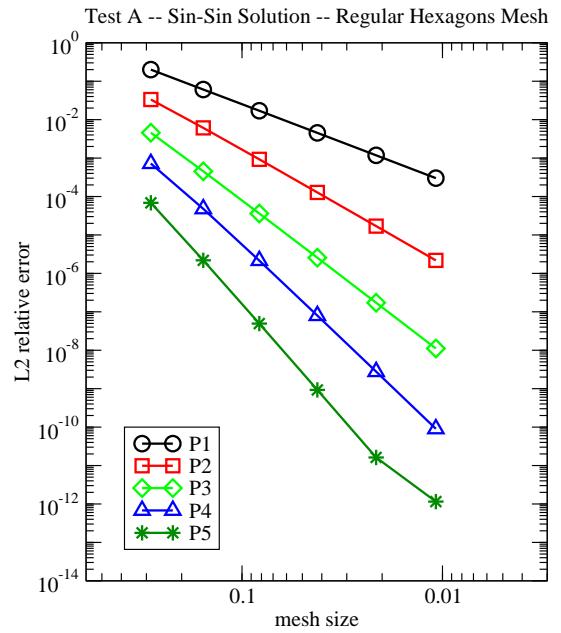


Fig. 220. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

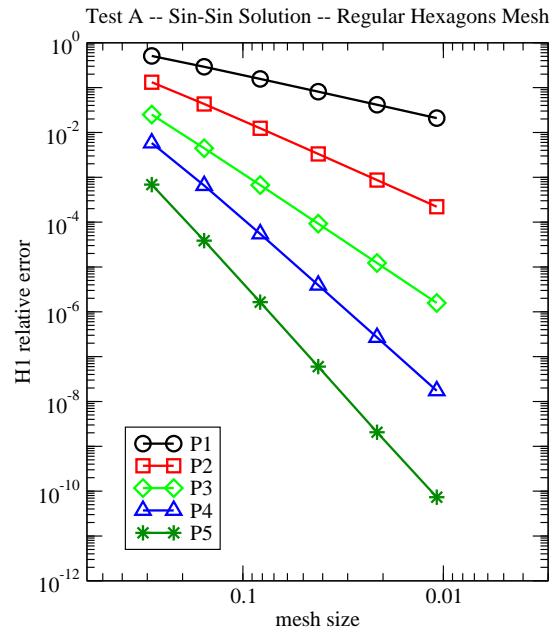
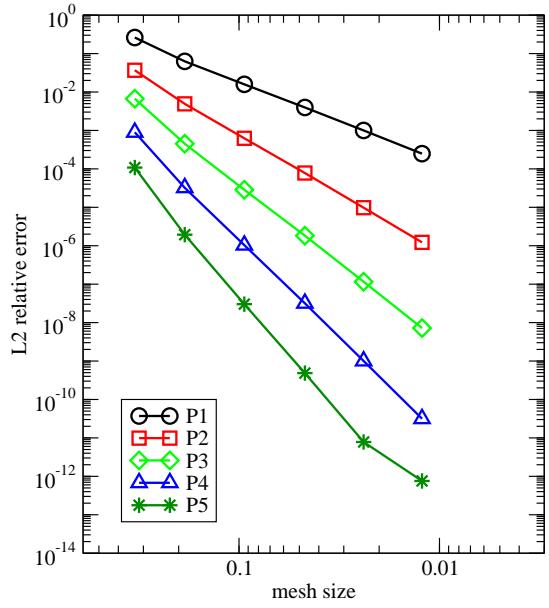


Fig. 221. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

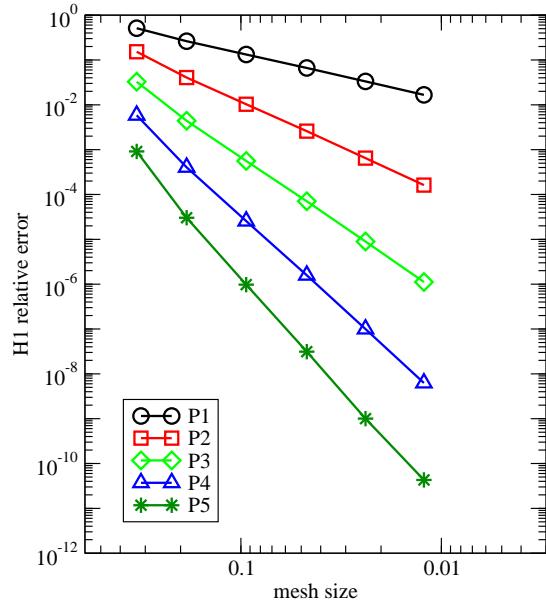
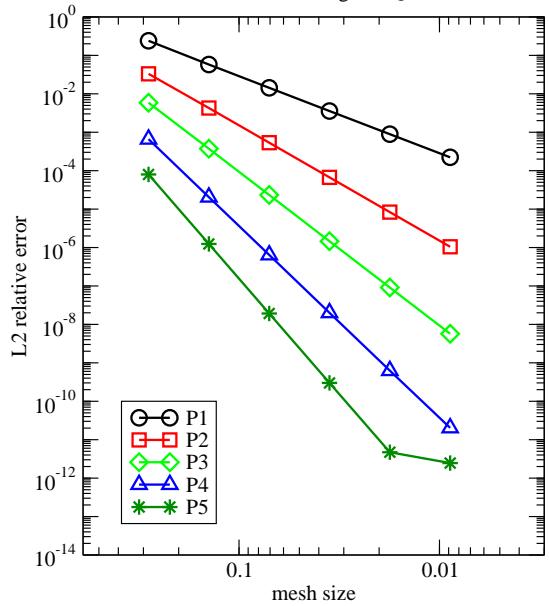


Fig. 222. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

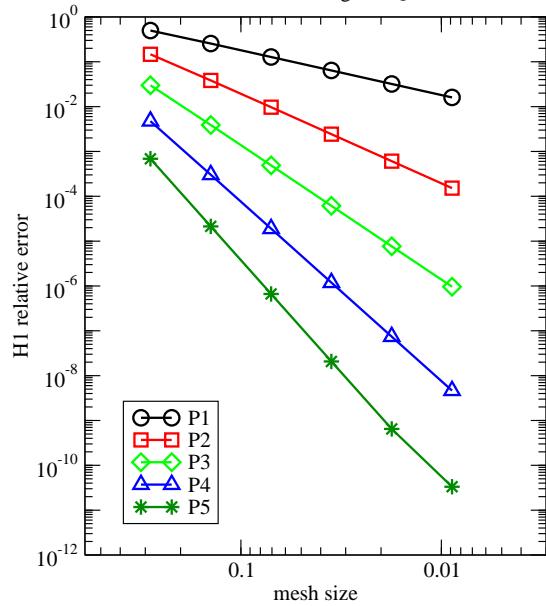
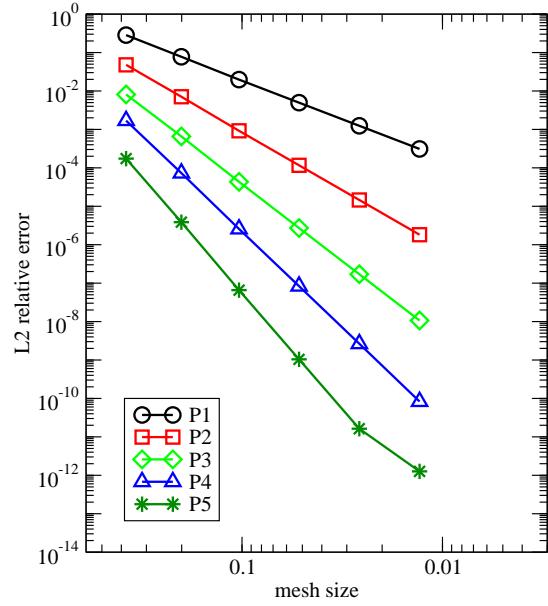


Fig. 223. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

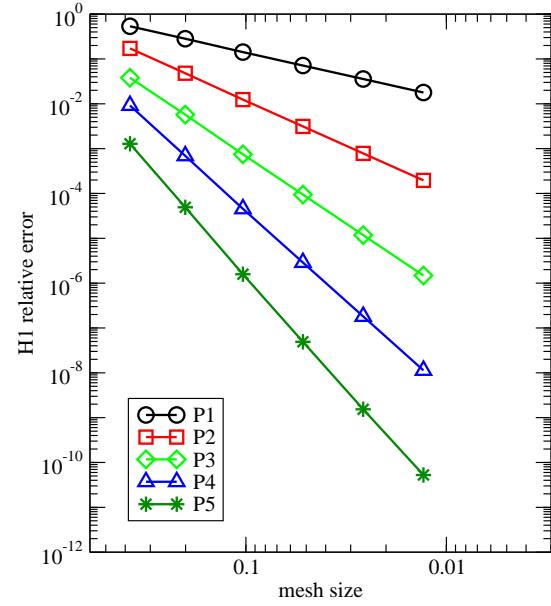
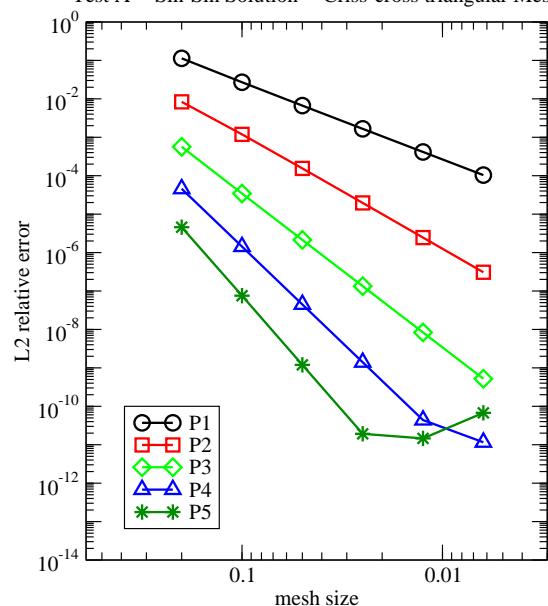


Fig. 224. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Sin Solution -- Criss-cross triangular Mesh

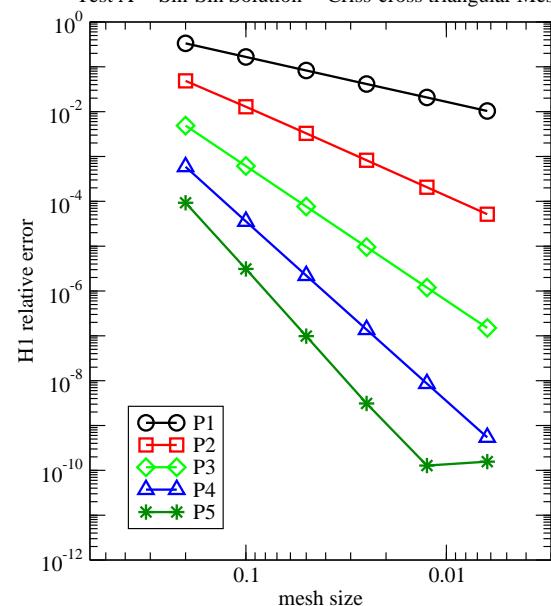
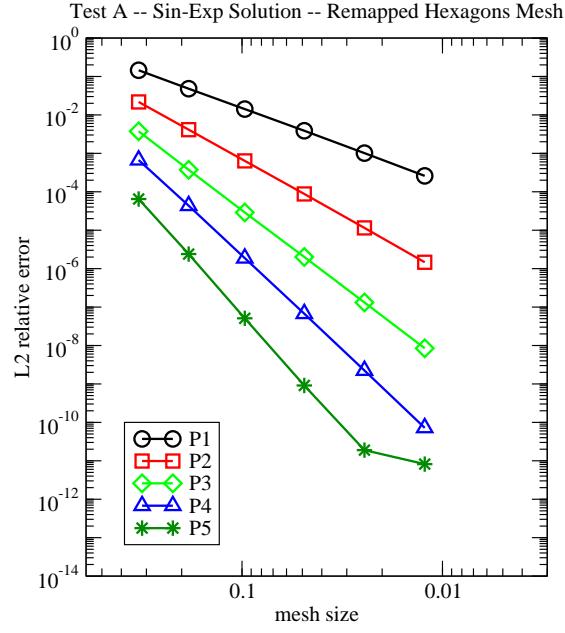


Fig. 225. Internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

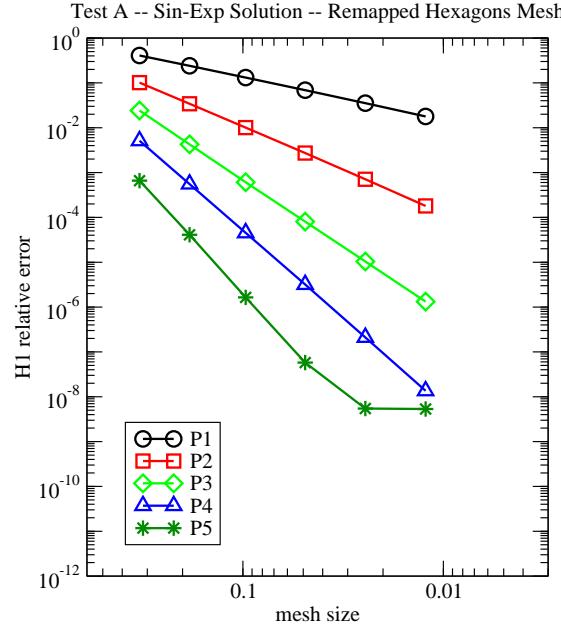
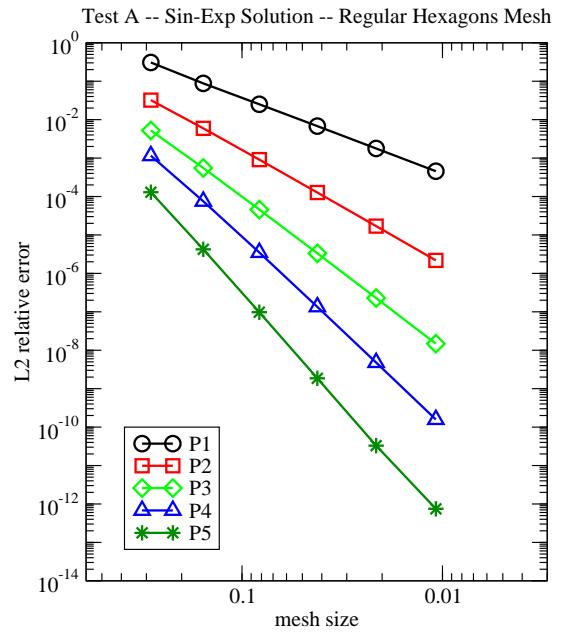


Fig. 226. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

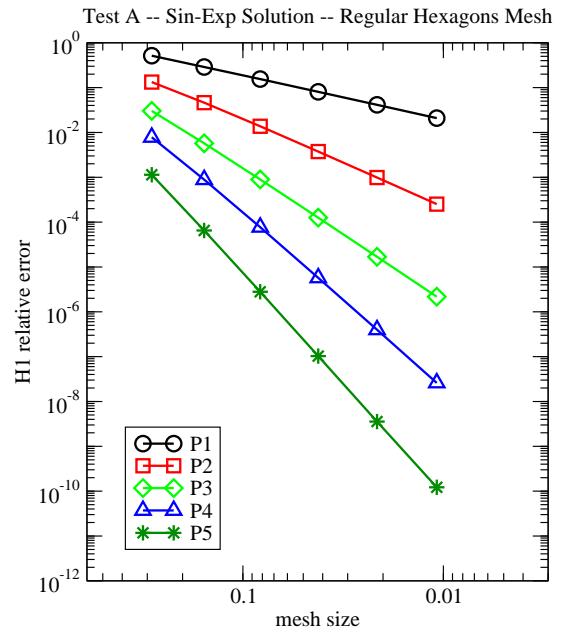
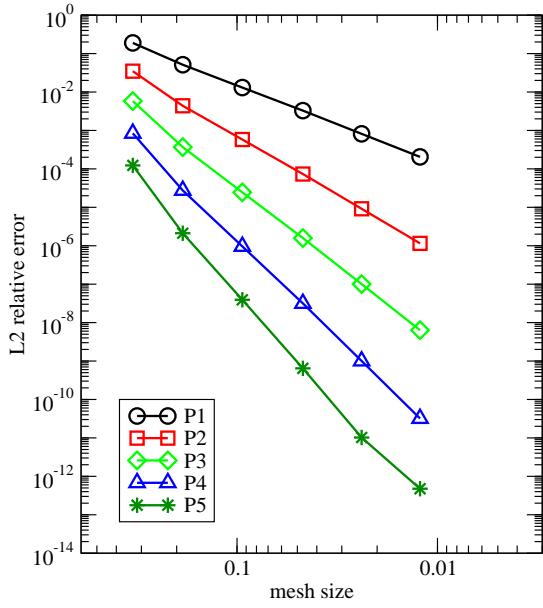


Fig. 227. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

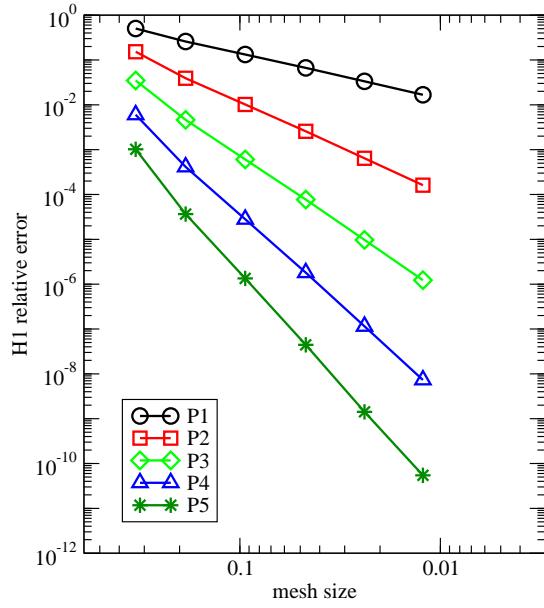
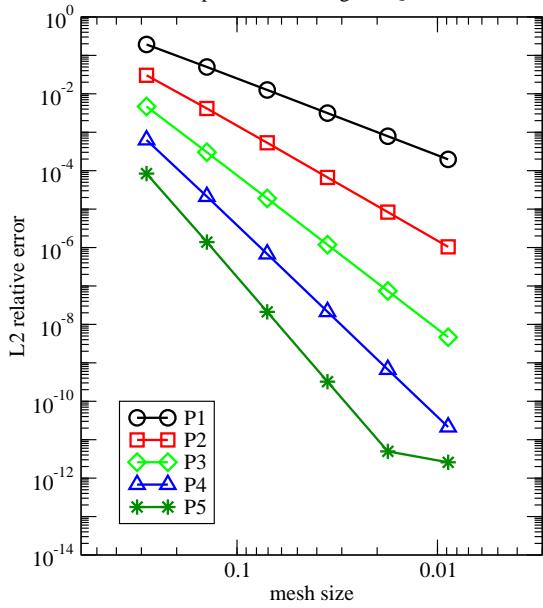


Fig. 228. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

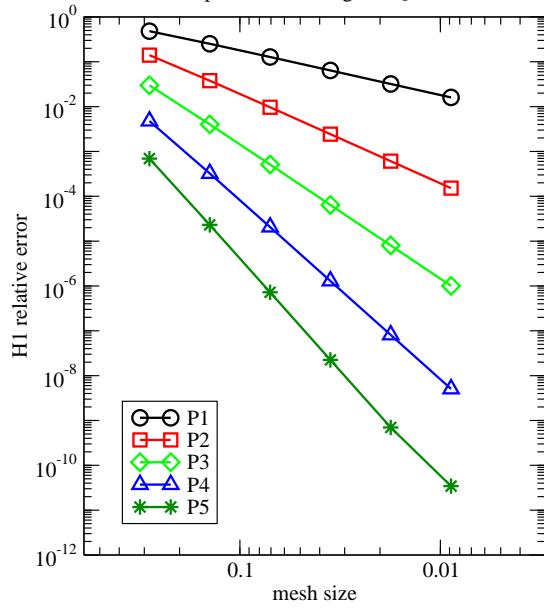
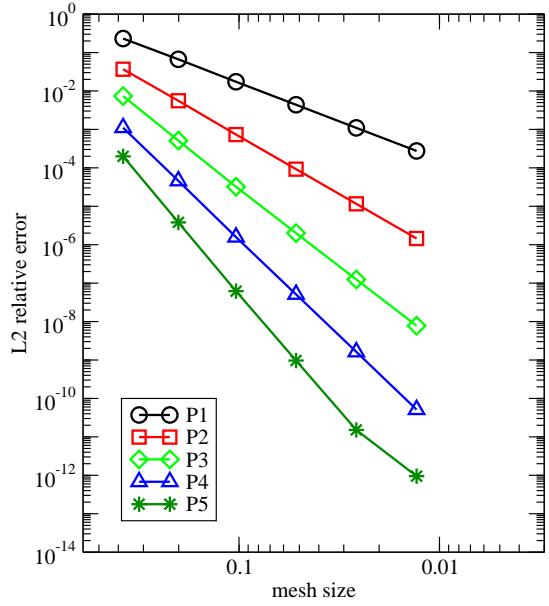


Fig. 229. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

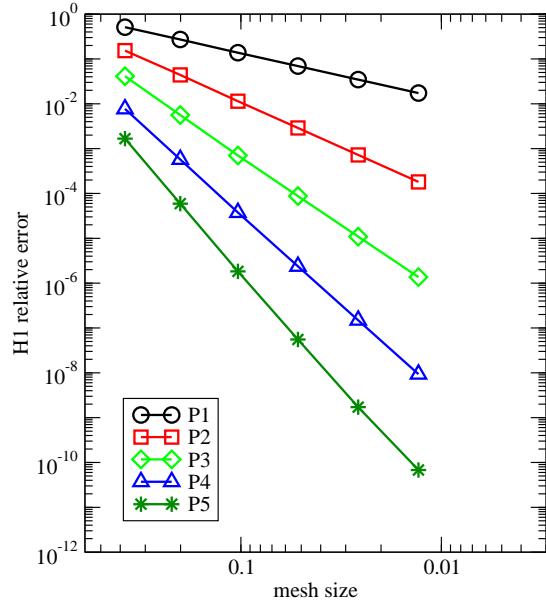
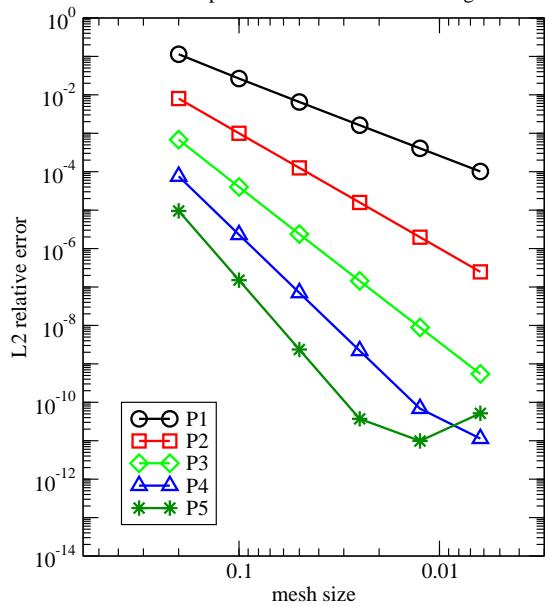


Fig. 230. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Sin-Exp Solution -- Criss-cross triangular Mesh

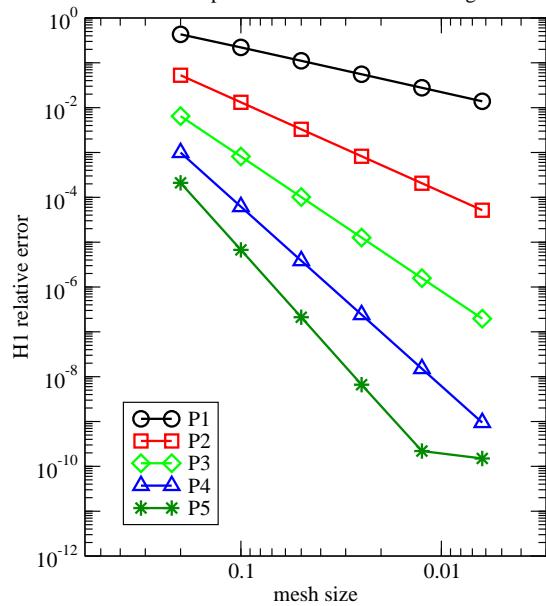
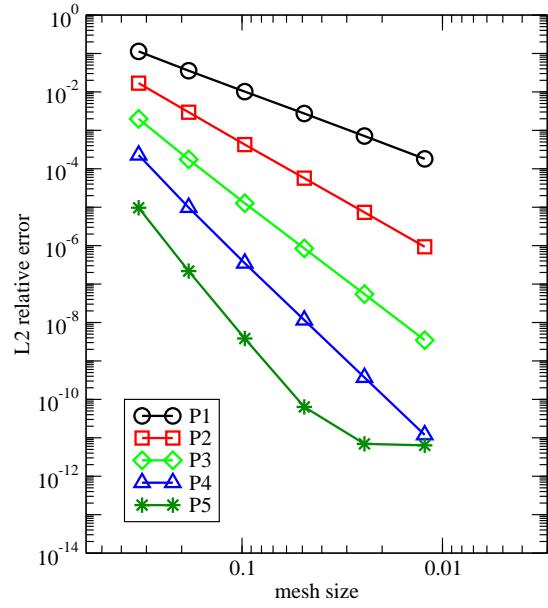


Fig. 231. Internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Remapped Hexagons Mesh

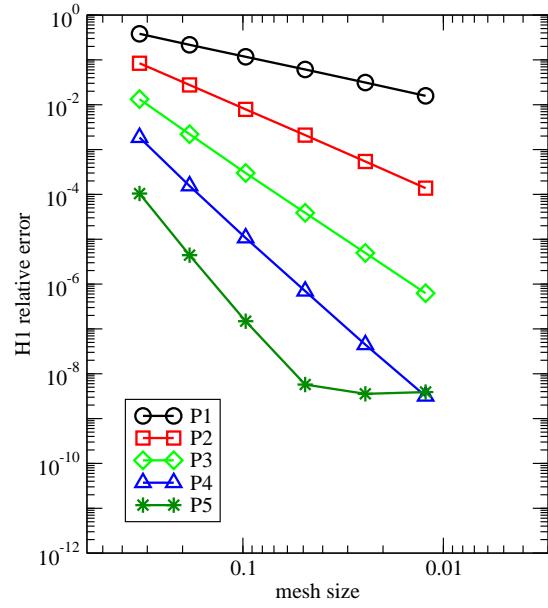
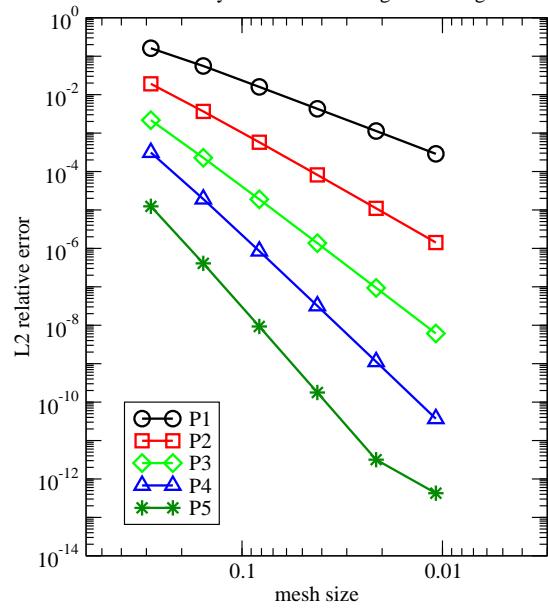


Fig. 232. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Regular Hexagons Mesh

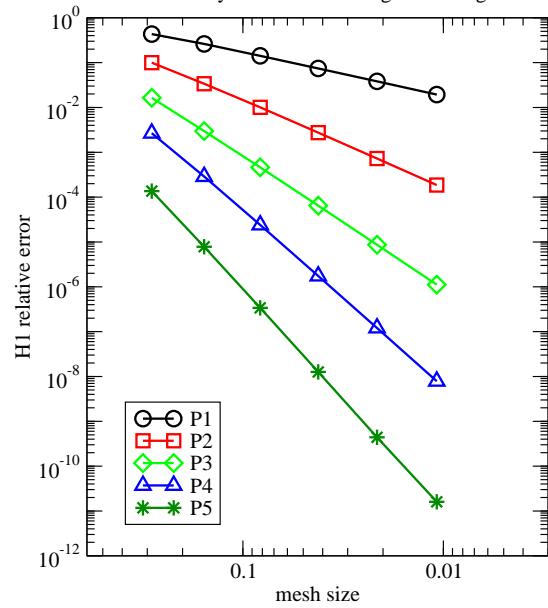


Fig. 233. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular hexagons.

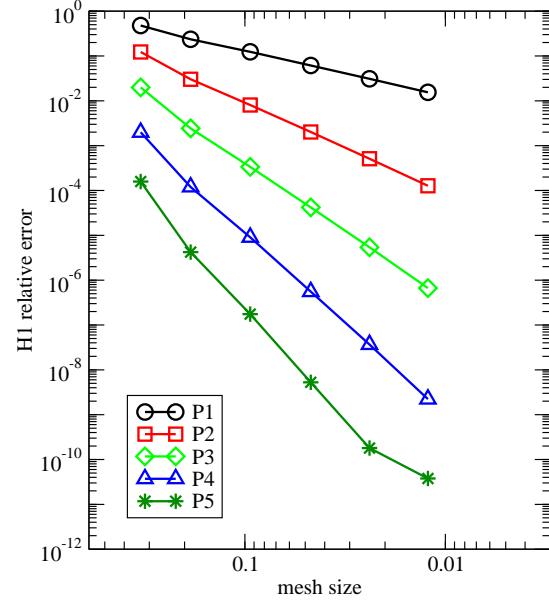
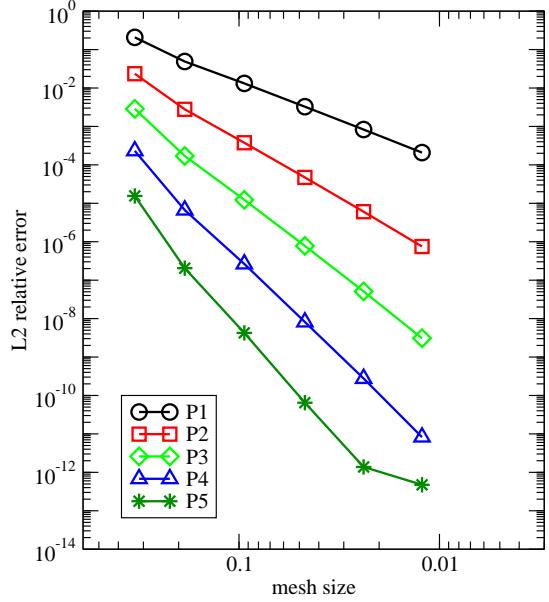


Fig. 234. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

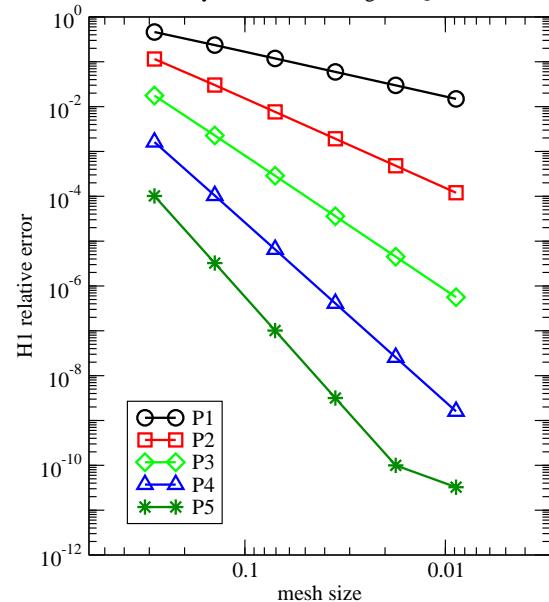
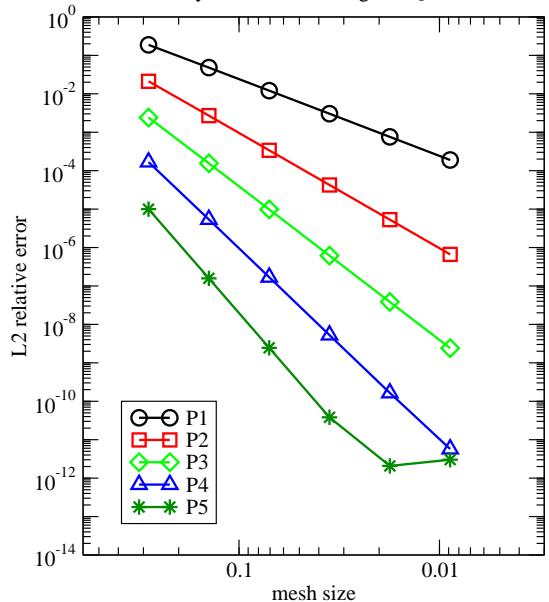
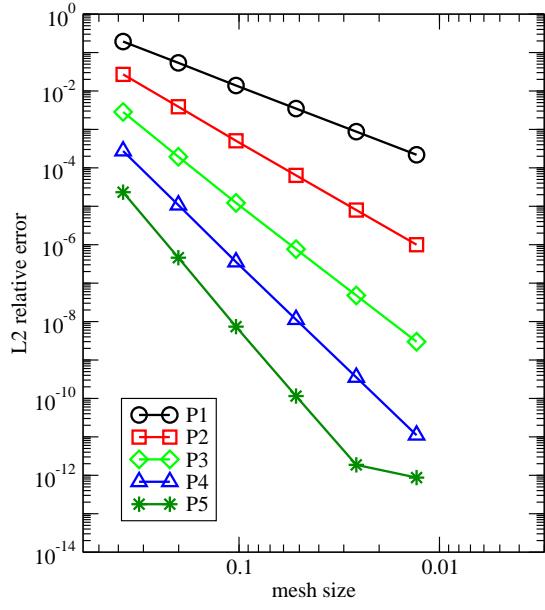


Fig. 235. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

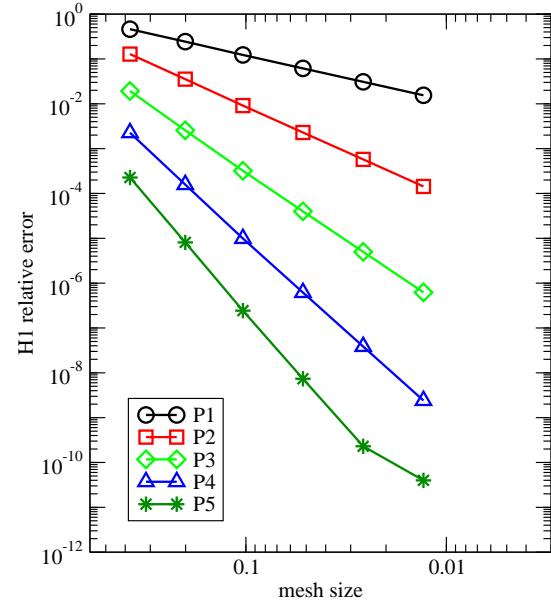
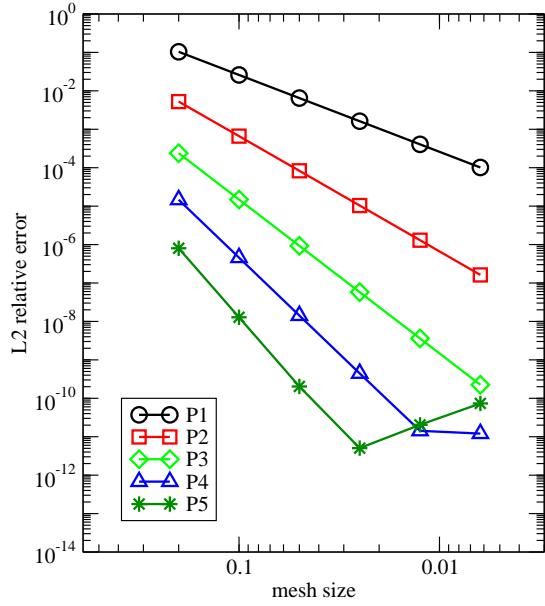


Fig. 236. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test A -- Two-Layers Solution -- Criss-cross triangular Mesh

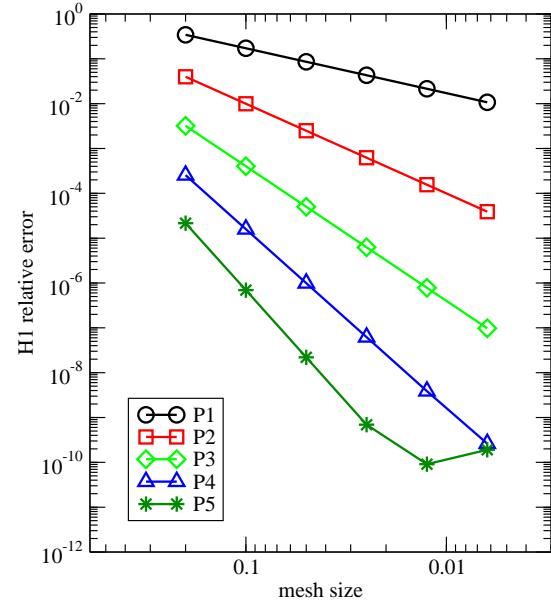
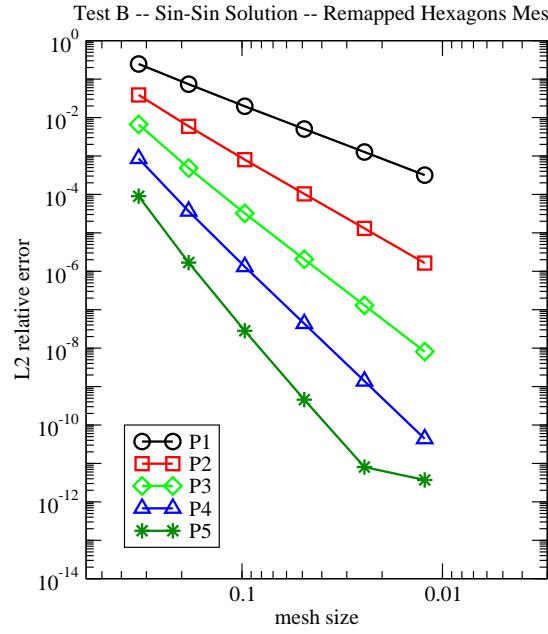


Fig. 237. Internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

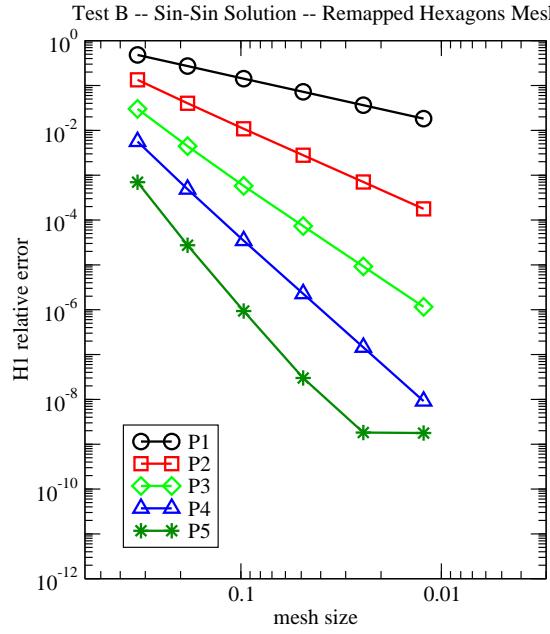
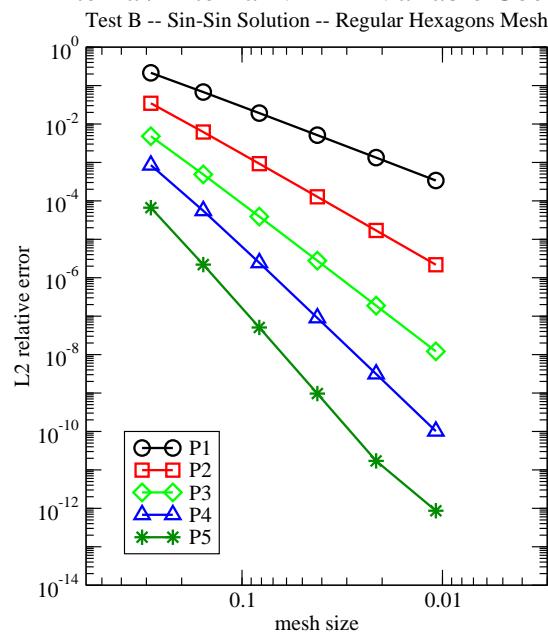


Fig. 238. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

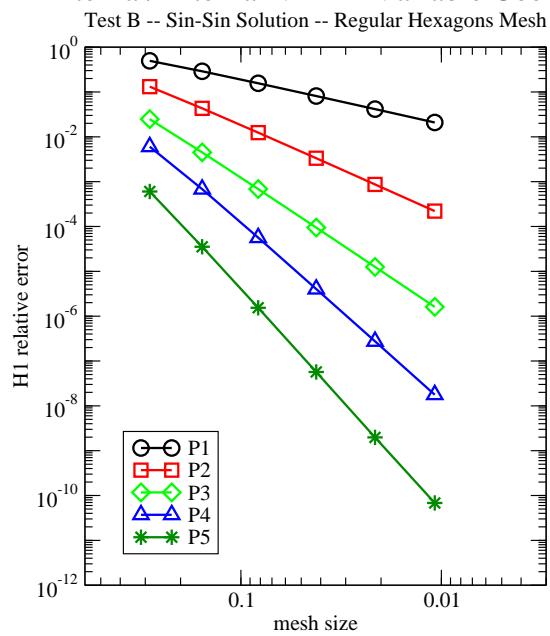
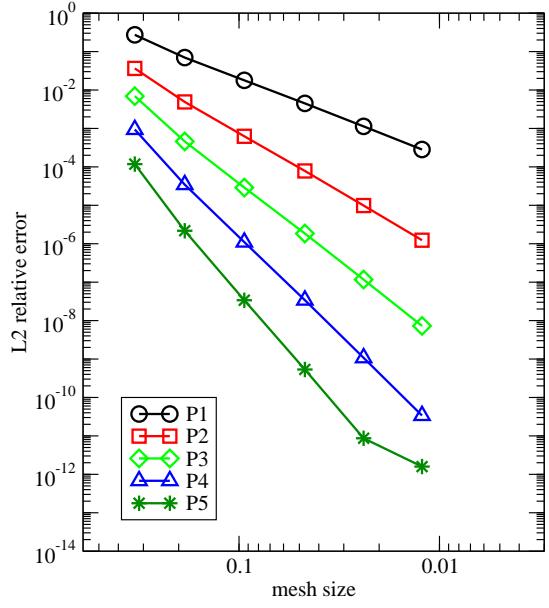


Fig. 239. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

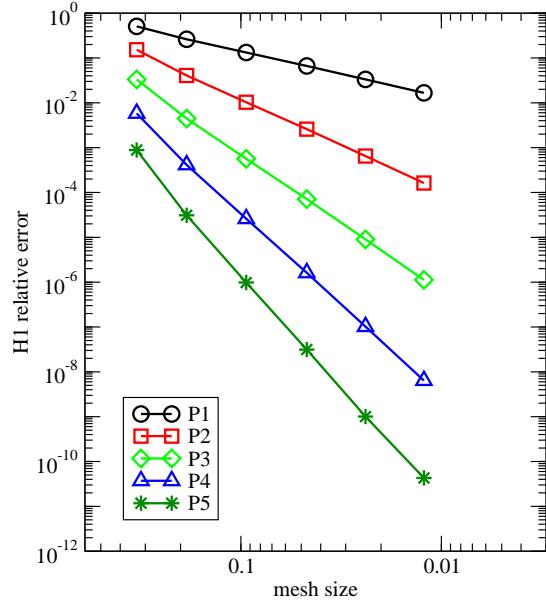
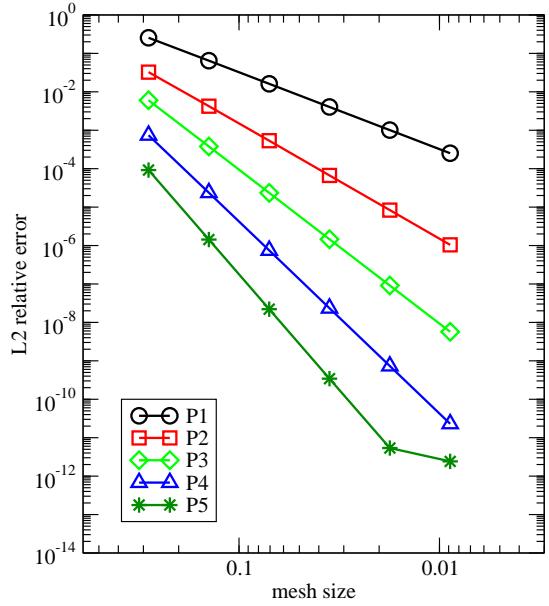


Fig. 240. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

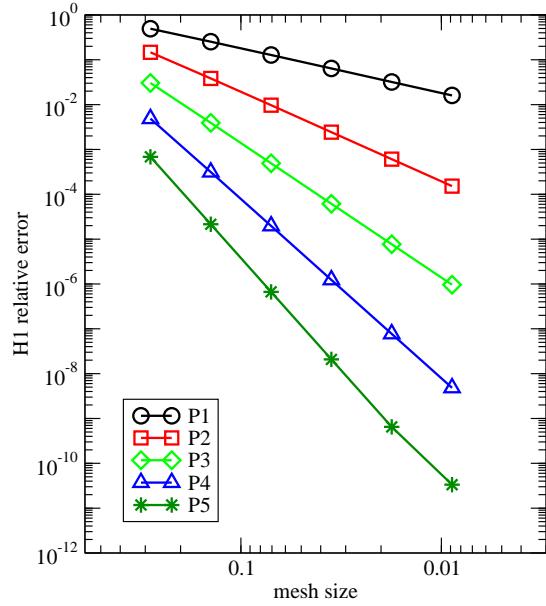
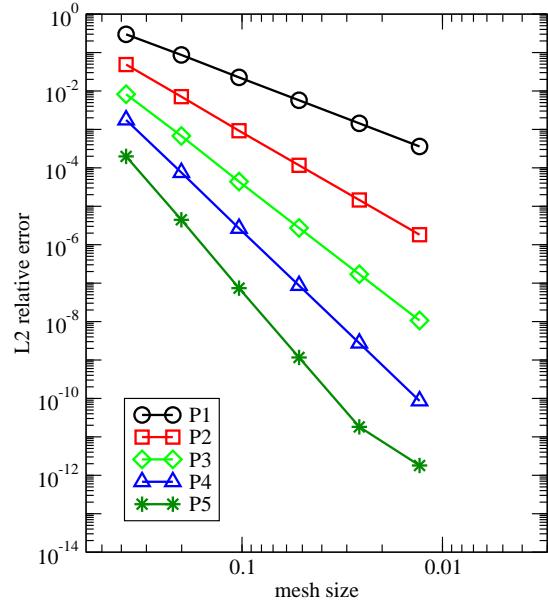


Fig. 241. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

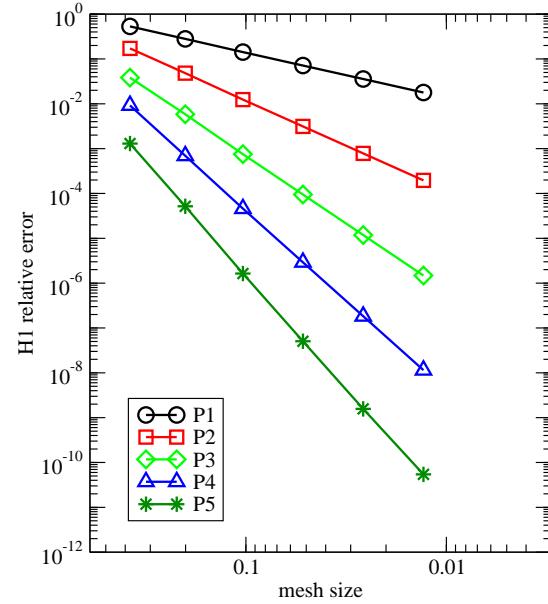
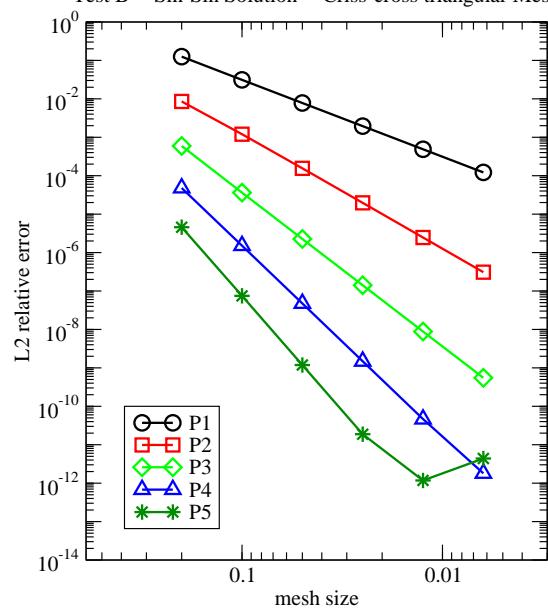


Fig. 242. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Sin Solution -- Criss-cross triangular Mesh

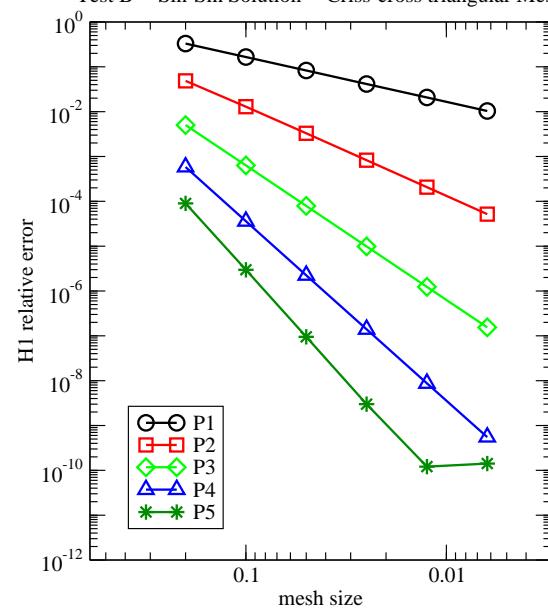
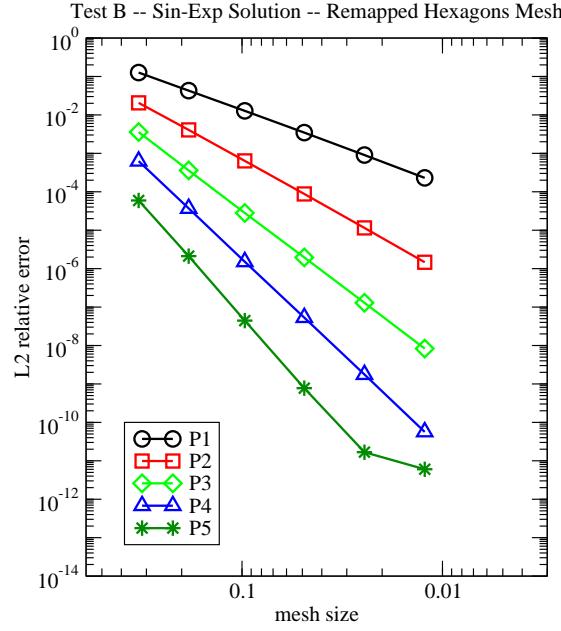


Fig. 243. Internal VEM formulation with variable coefficients; Test B; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

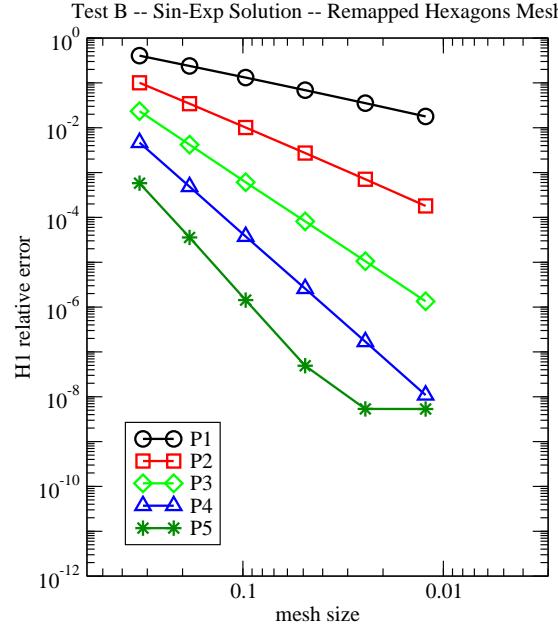
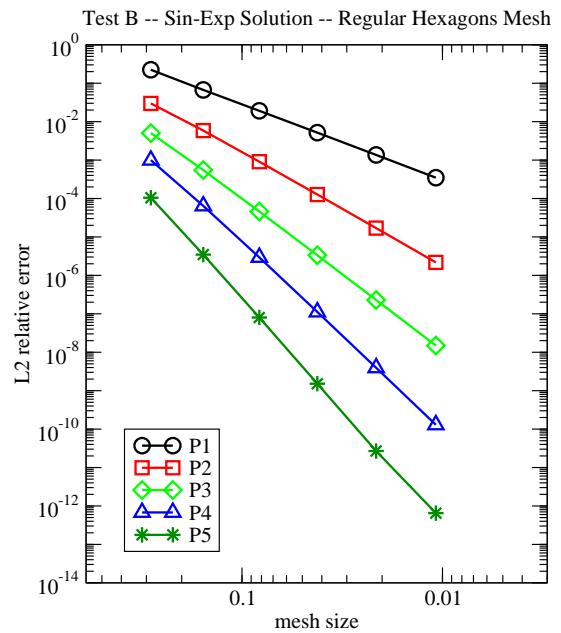


Fig. 244. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

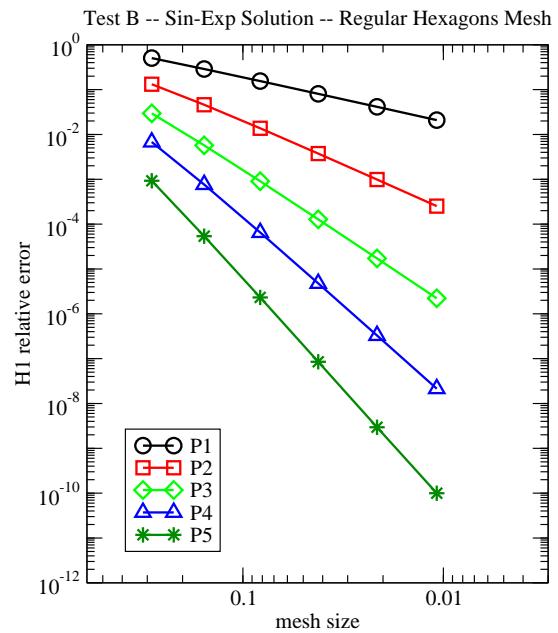
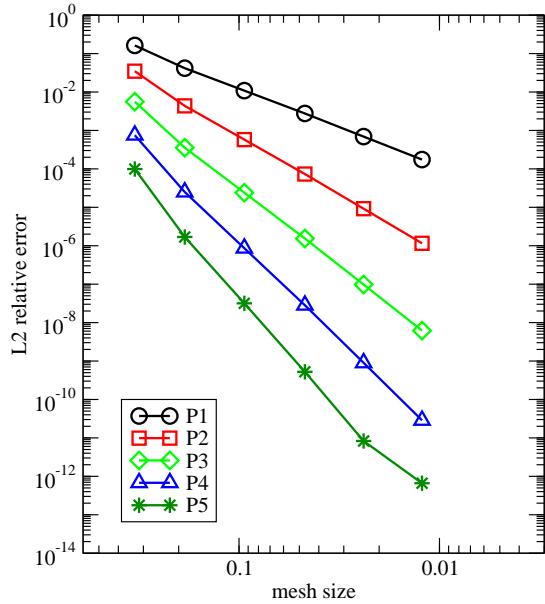


Fig. 245. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

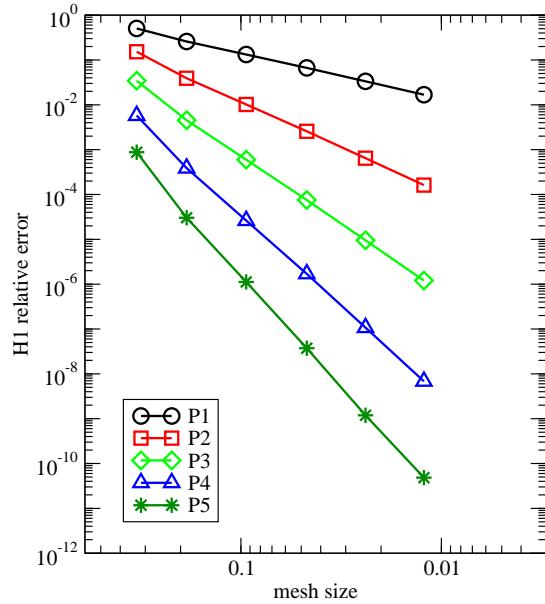
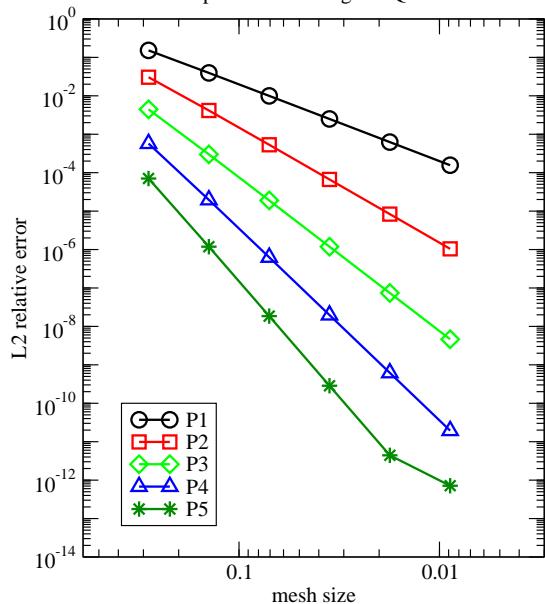


Fig. 246. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

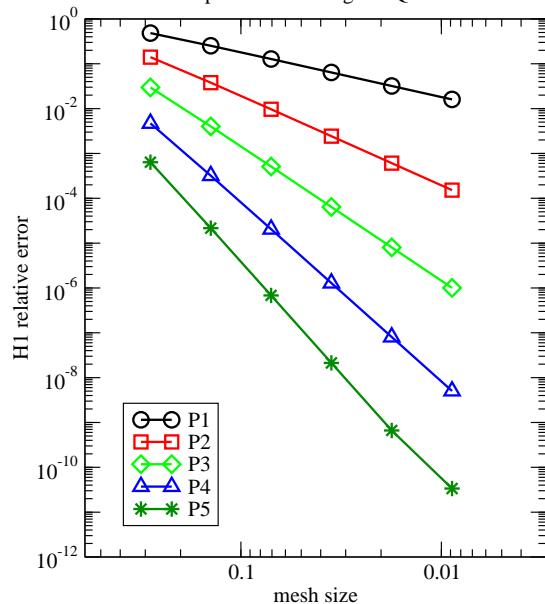
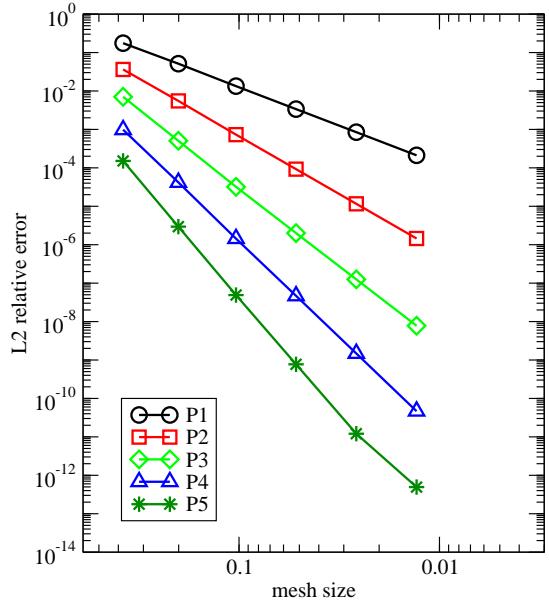


Fig. 247. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

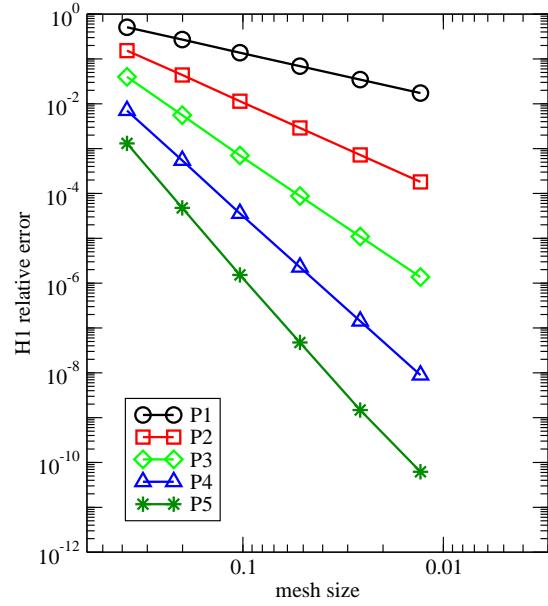
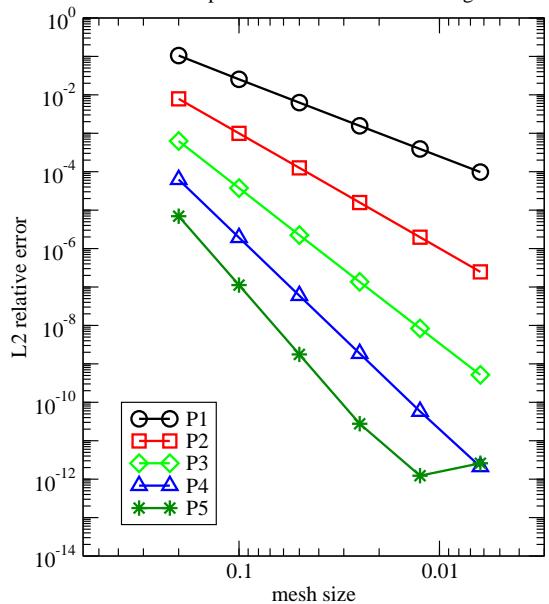


Fig. 248. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Sin-Exp Solution -- Criss-cross triangular Mesh

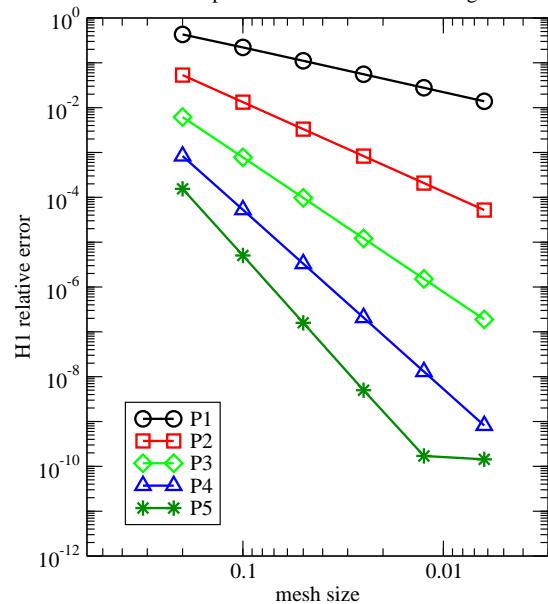
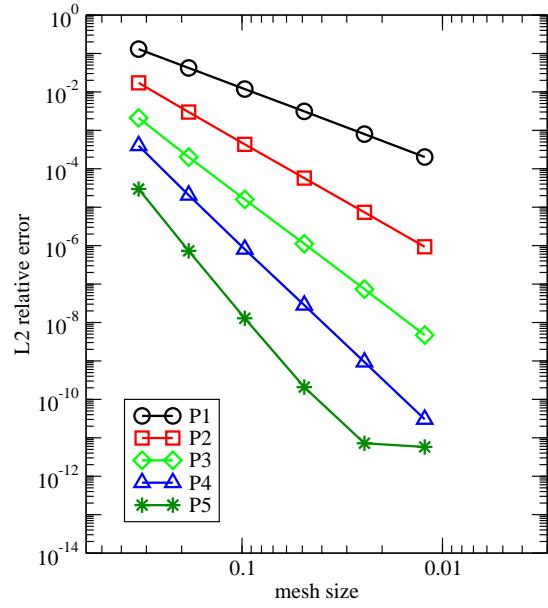


Fig. 249. Internal VEM formulation with variable coefficients; Test B; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Remapped Hexagons Mesh

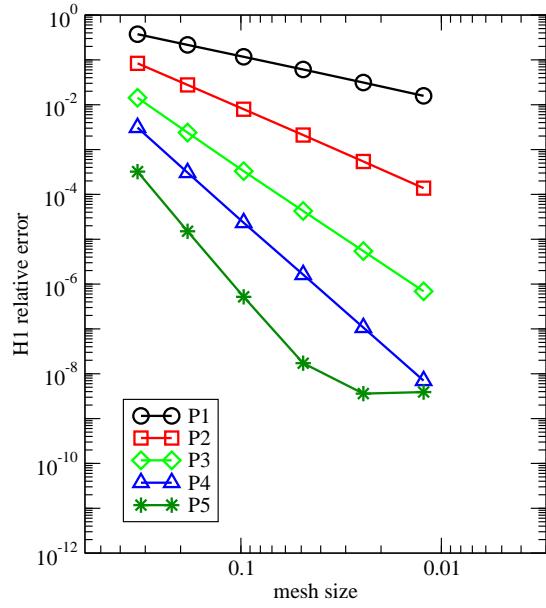
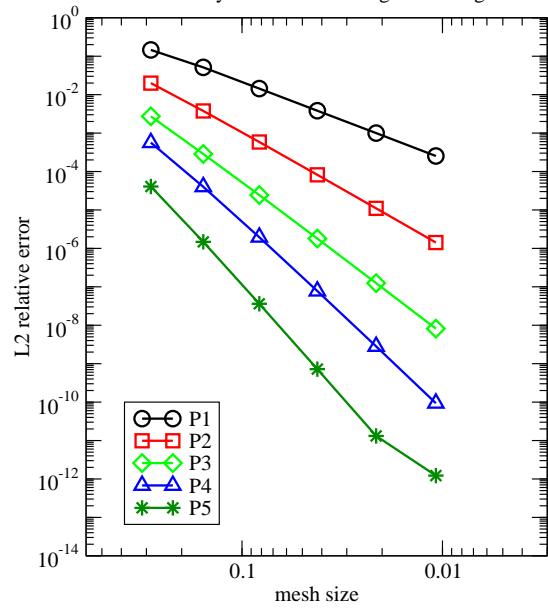


Fig. 250. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Regular Hexagons Mesh

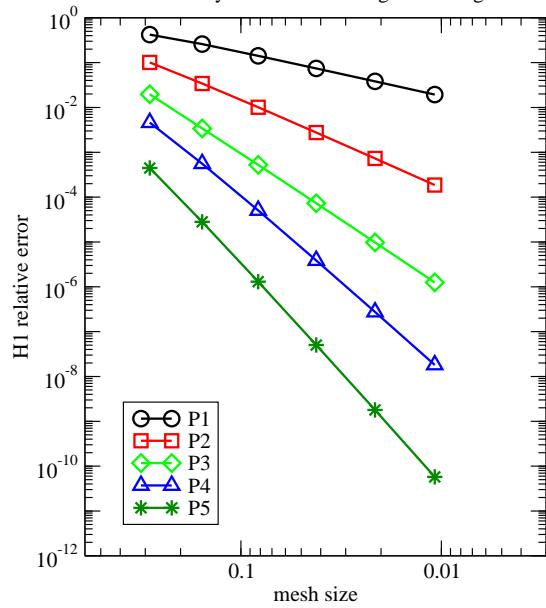


Fig. 251. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular hexagons.

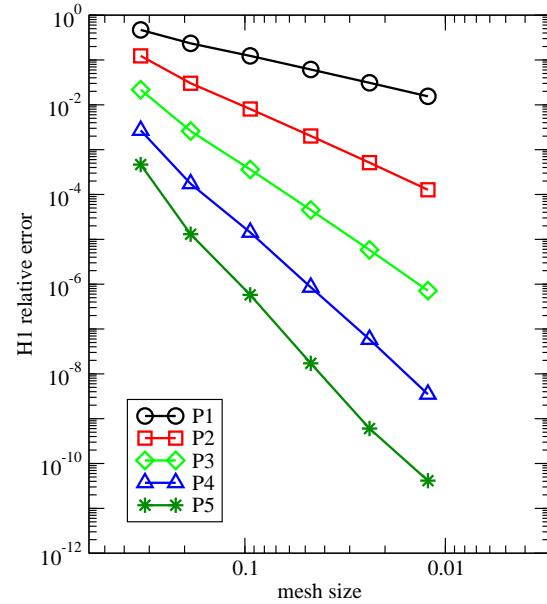
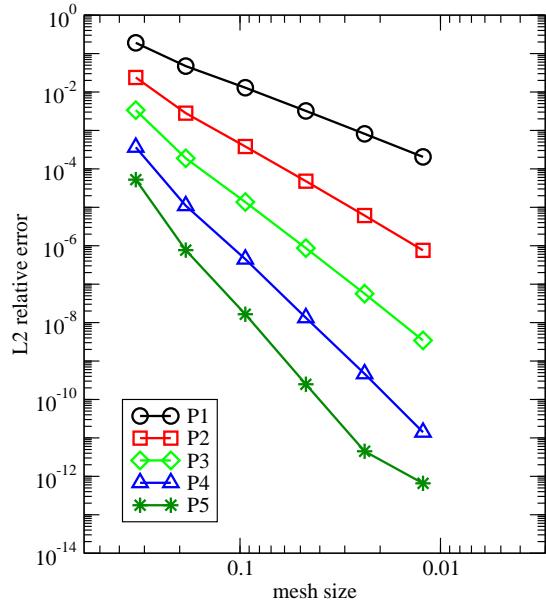


Fig. 252. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of randomized quadrilateral cells.

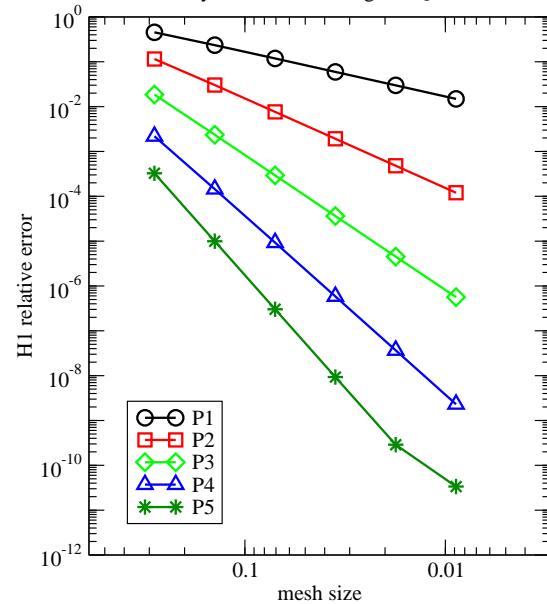
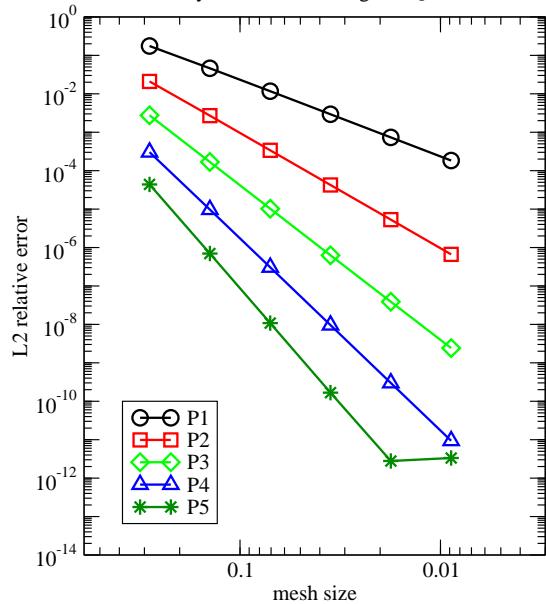
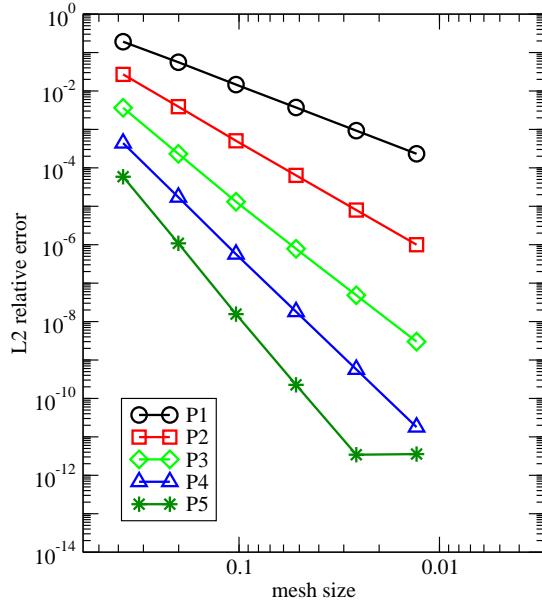


Fig. 253. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

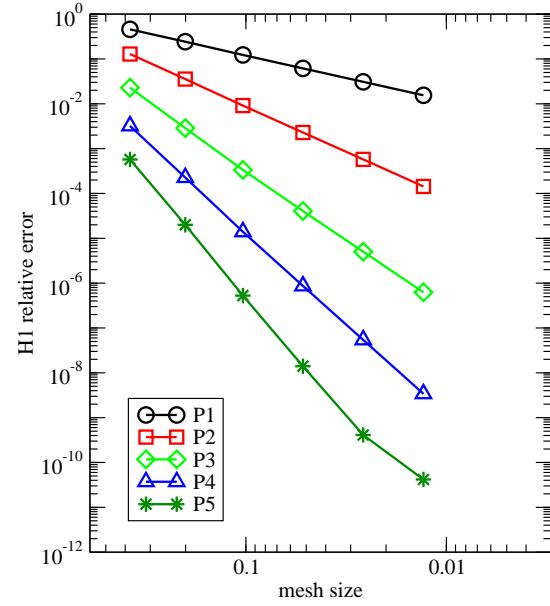
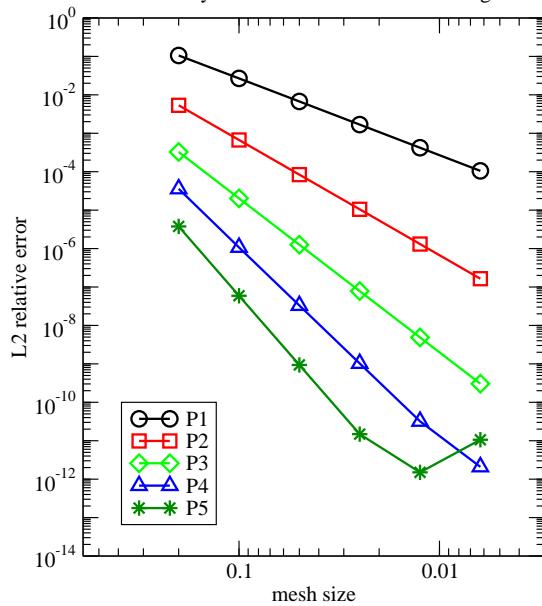


Fig. 254. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test B -- Two-Layers Solution -- Criss-cross triangular Mesh

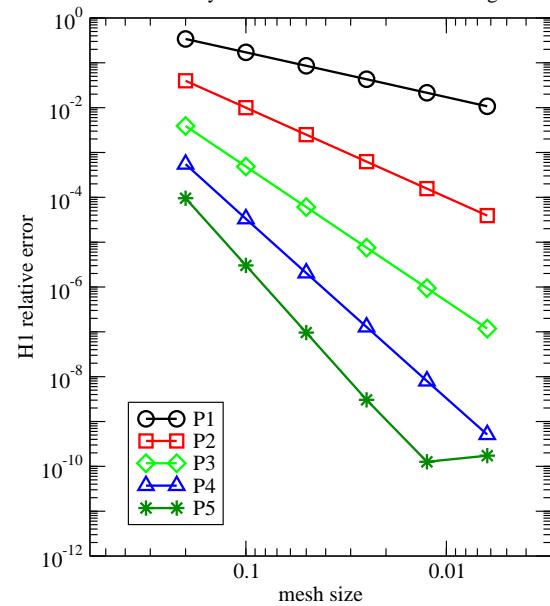
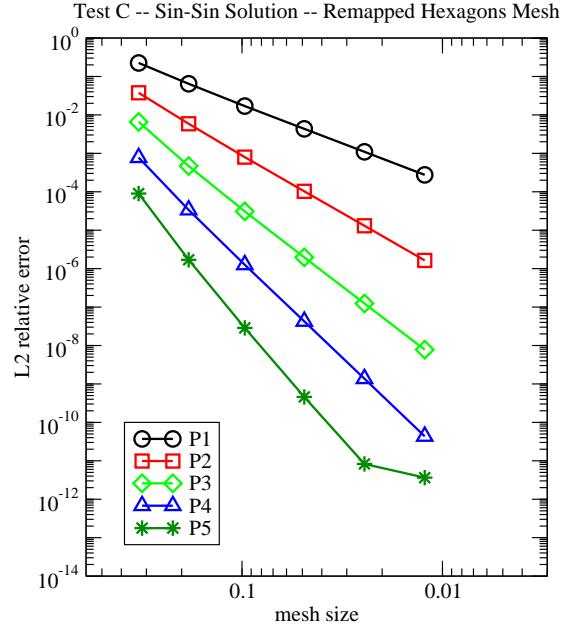


Fig. 255. Internal VEM formulation with variable coefficients; Test B; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

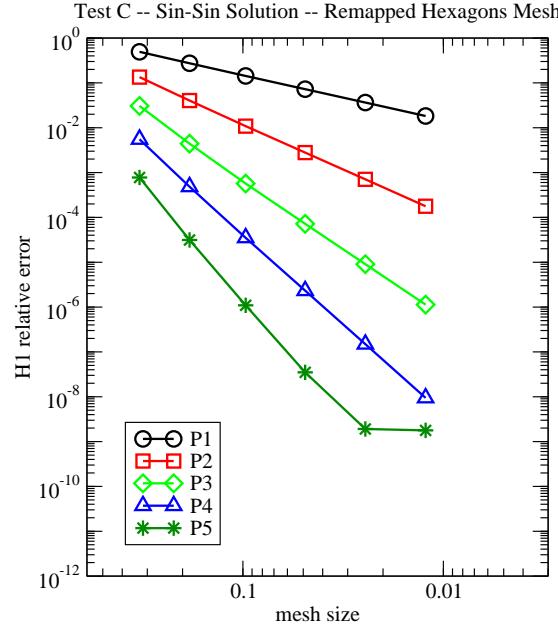
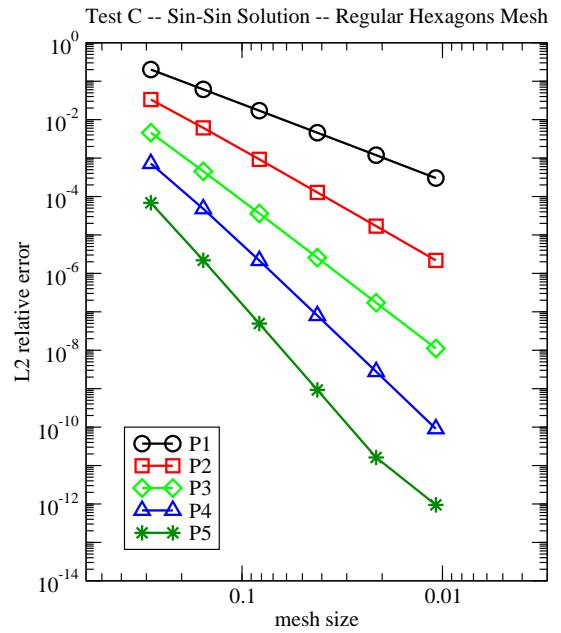


Fig. 256. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

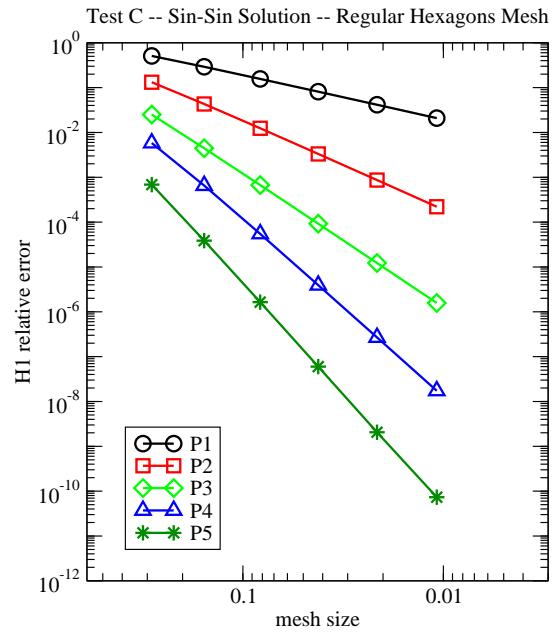
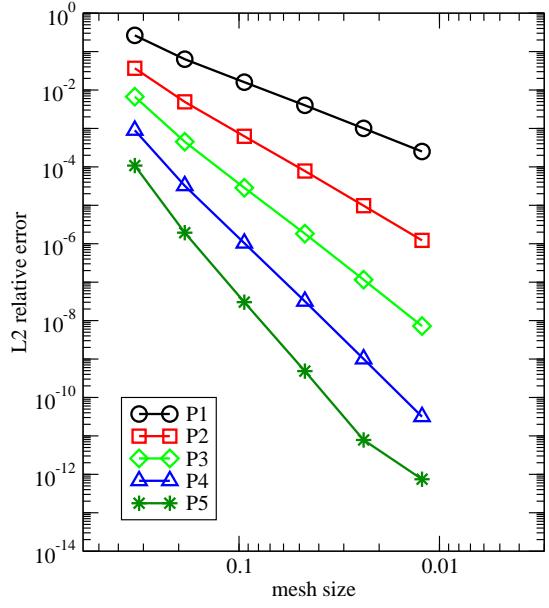


Fig. 257. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

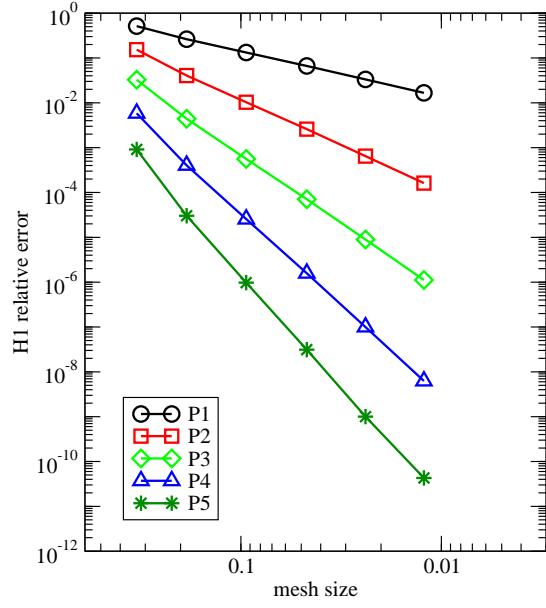
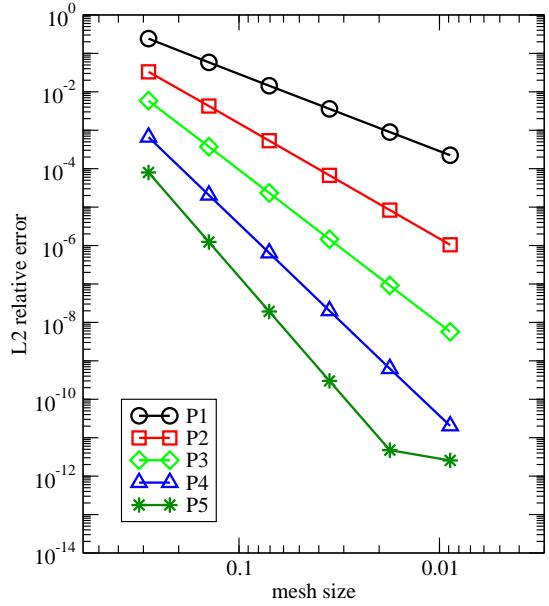


Fig. 258. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

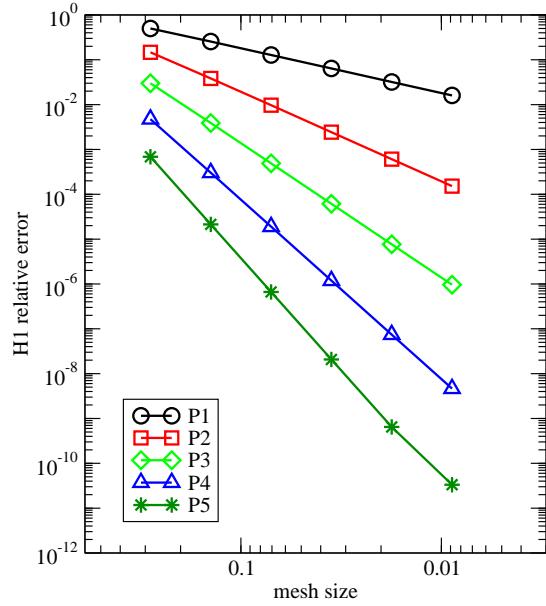
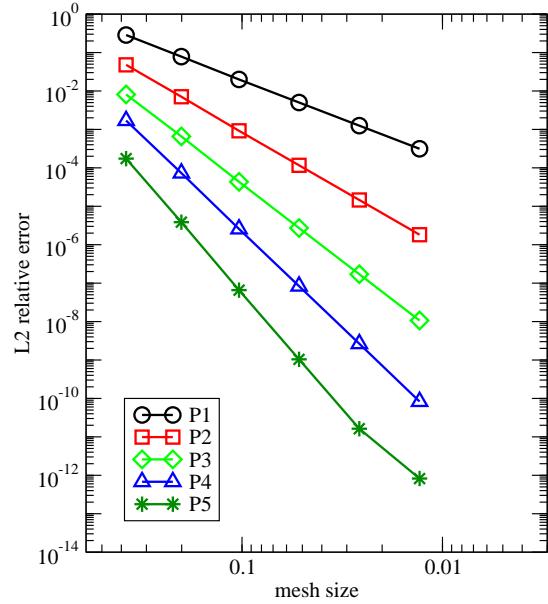


Fig. 259. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

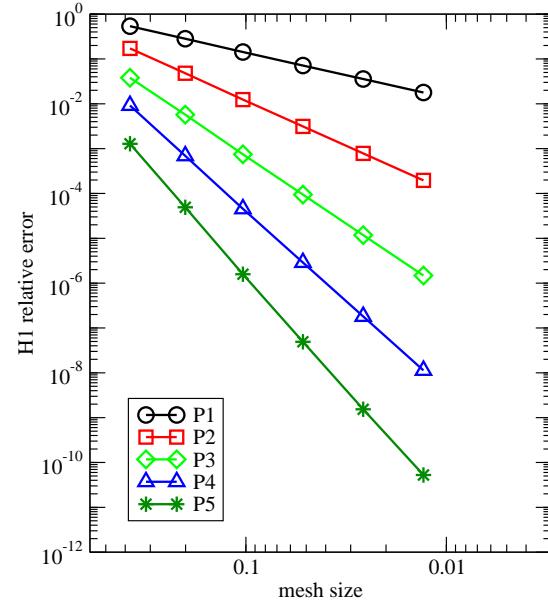
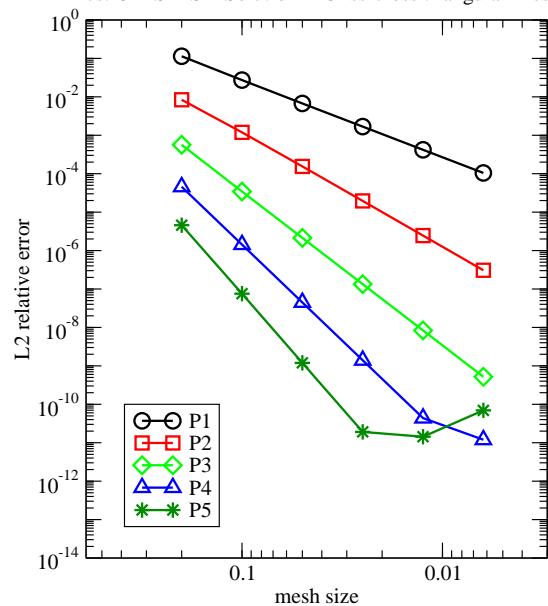


Fig. 260. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Sin Solution -- Criss-cross triangular Mesh

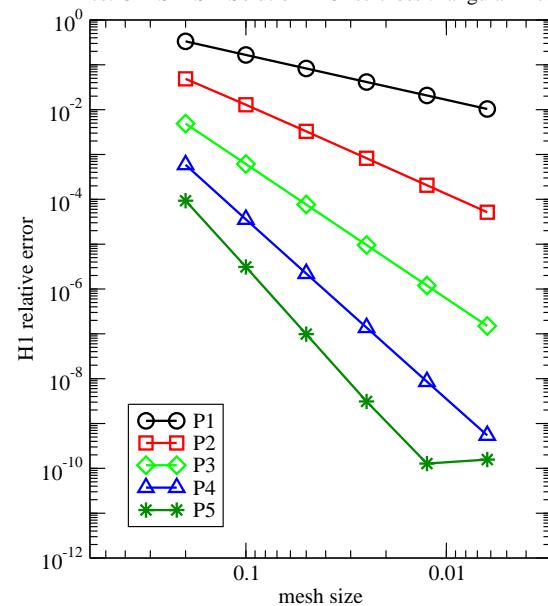
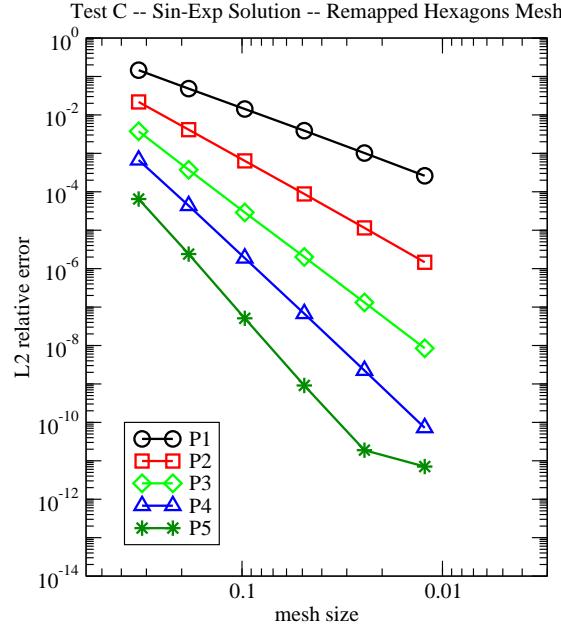


Fig. 261. Internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

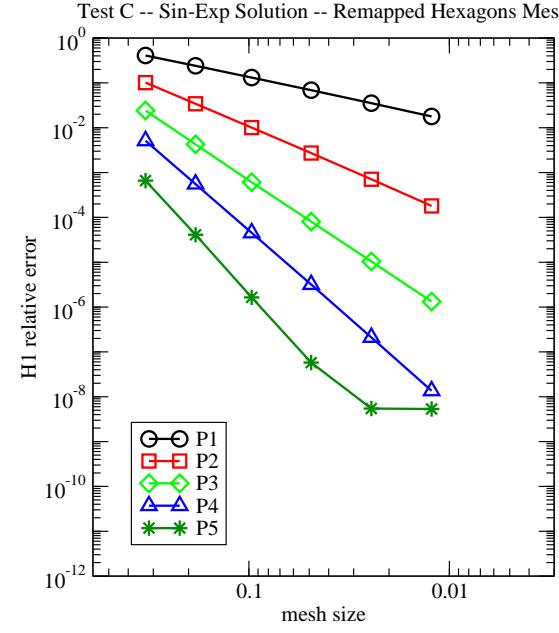
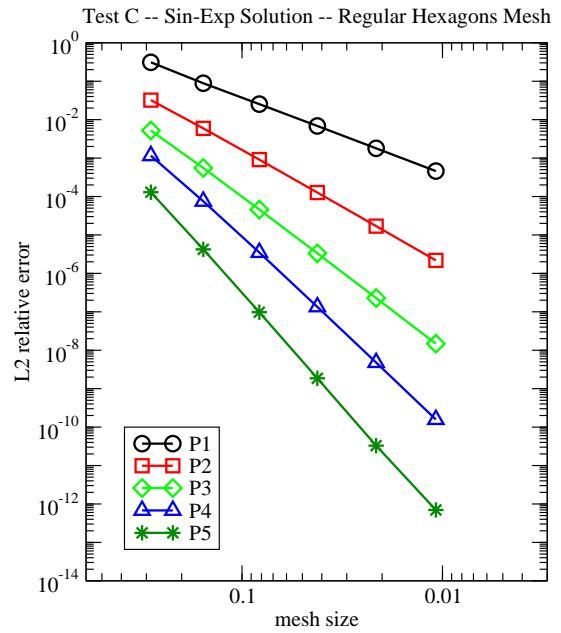


Fig. 262. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

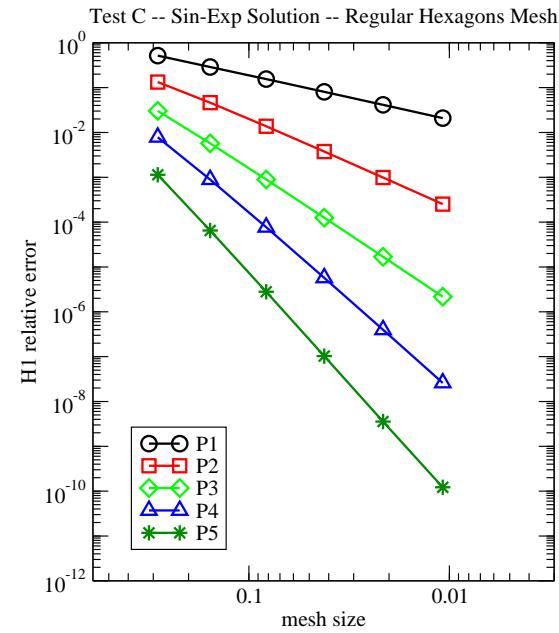
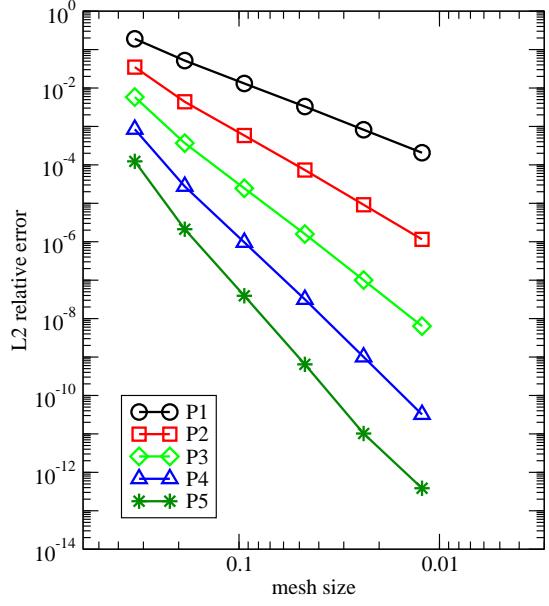


Fig. 263. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

**Internal/External VEM - Variable Coeffs.**  
 Test C -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



**Internal/External VEM - Variable Coeffs.**  
 Test C -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

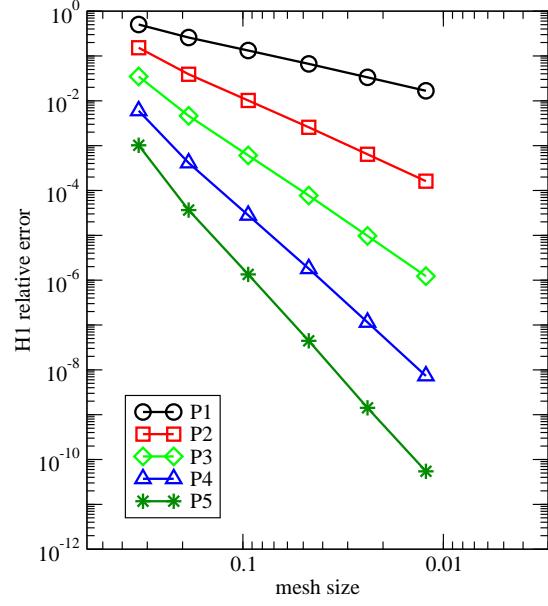
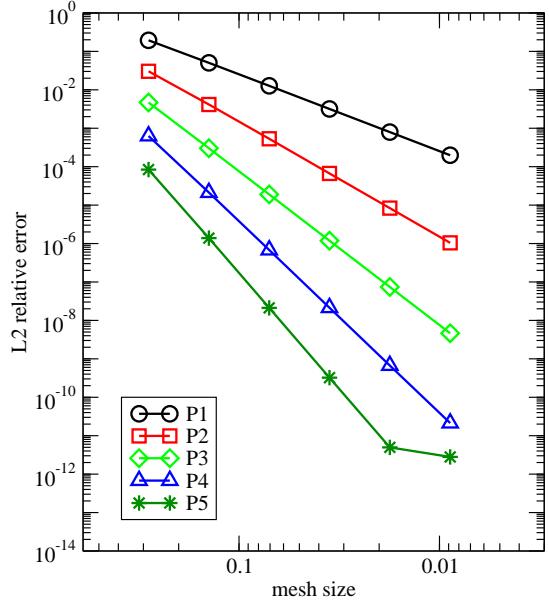


Fig. 264. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

**Internal/External VEM - Variable Coeffs.**  
 Test C -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



**Internal/External VEM - Variable Coeffs.**  
 Test C -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

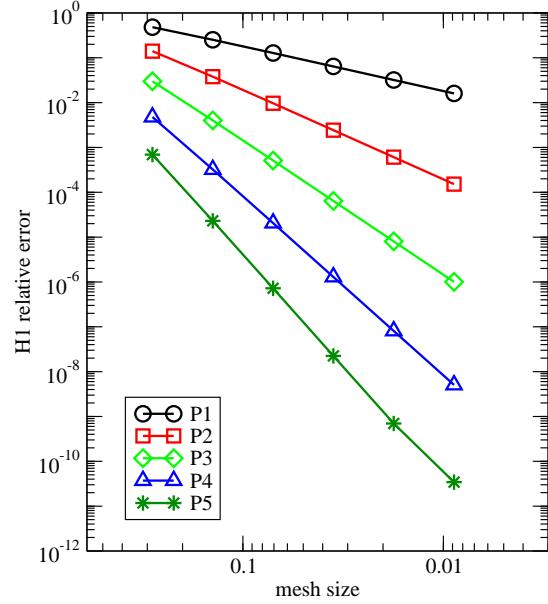
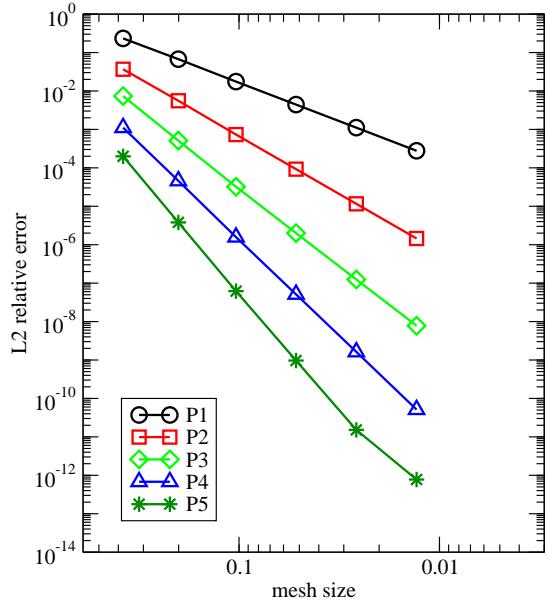


Fig. 265. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

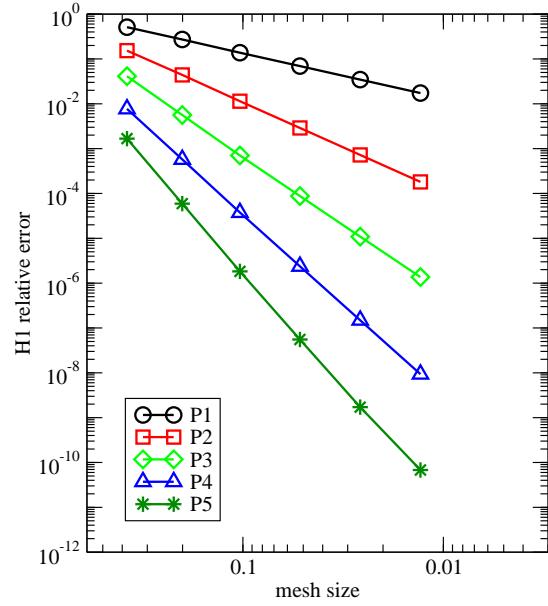
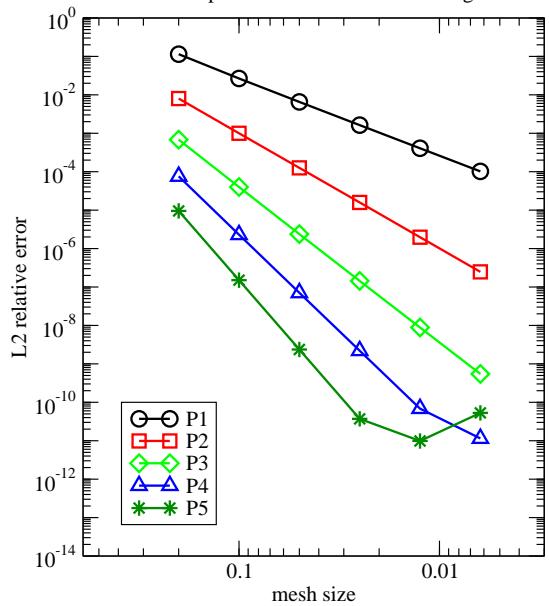


Fig. 266. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Sin-Exp Solution -- Criss-cross triangular Mesh

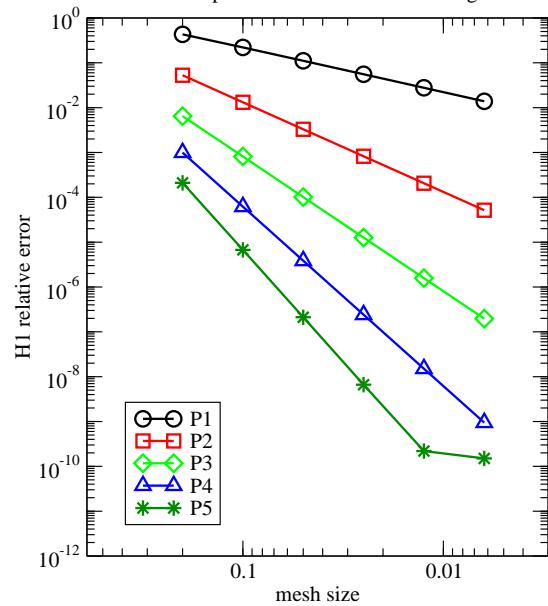
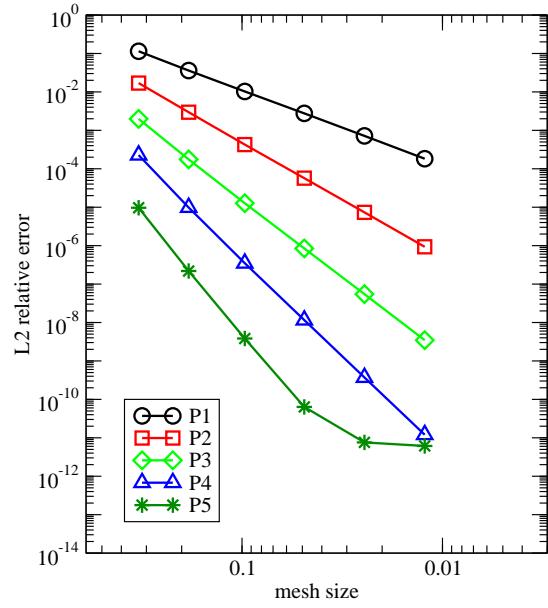


Fig. 267. Internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Remapped Hexagons Mesh

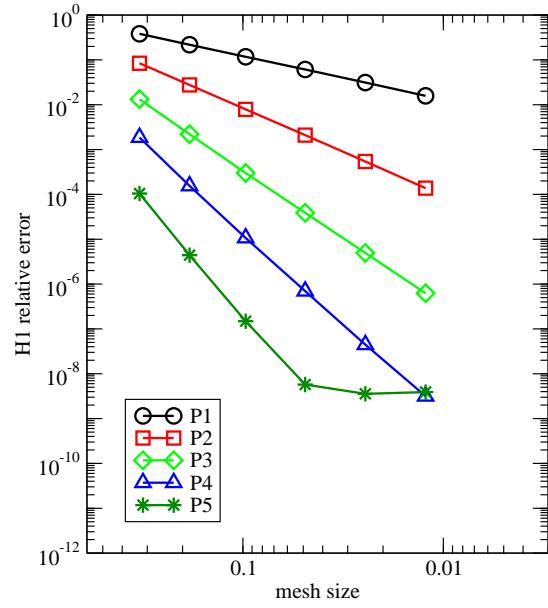
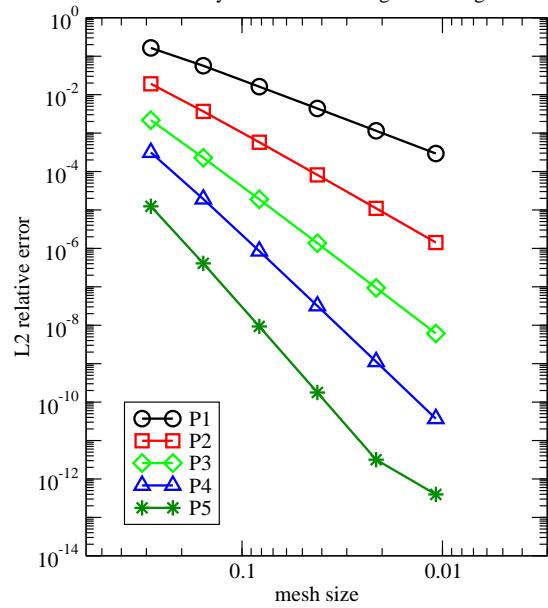


Fig. 268. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Regular Hexagons Mesh

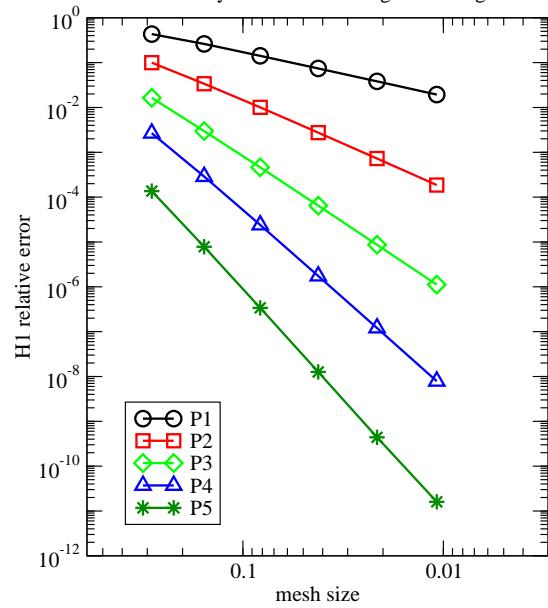
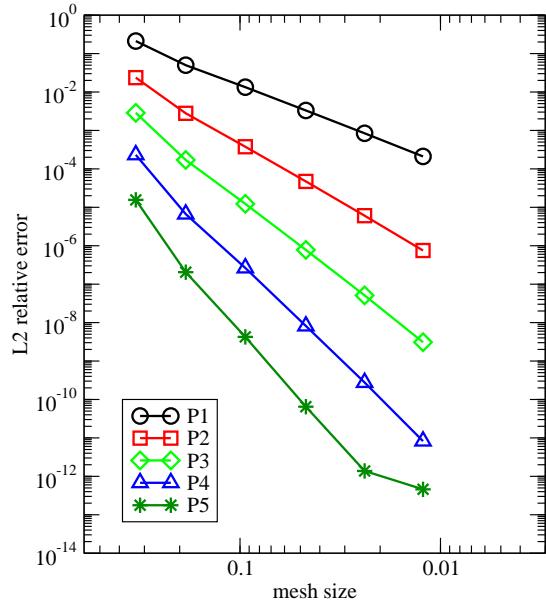


Fig. 269. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular hexagons.

**Internal/External VEM - Variable Coeffs.**  
 Test C -- Two-Layers Solution -- Randomized Quadrilaterals Mesh



**Internal/External VEM - Variable Coeffs.**  
 Test C -- Two-Layers Solution -- Randomized Quadrilaterals Mesh

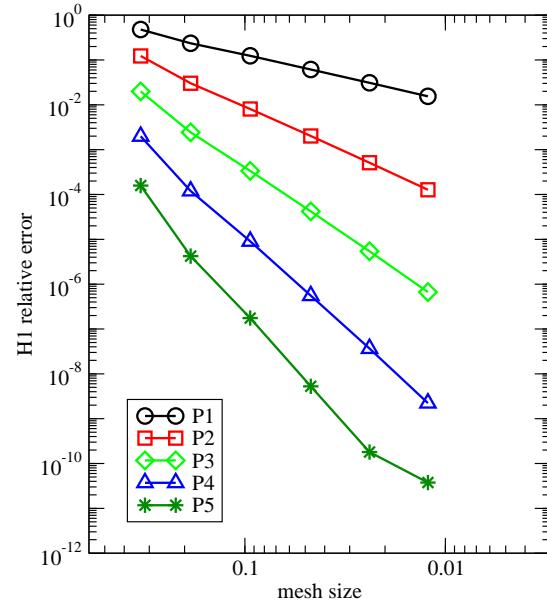
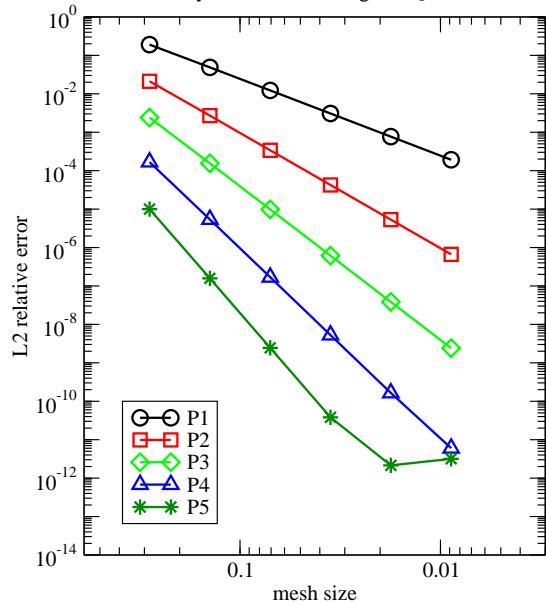


Fig. 270. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

**Internal/External VEM - Variable Coeffs.**  
 Test C -- Two-Layers Solution -- Regular Quadrilaterals Mesh



**Internal/External VEM - Variable Coeffs.**  
 Test C -- Two-Layers Solution -- Regular Quadrilaterals Mesh

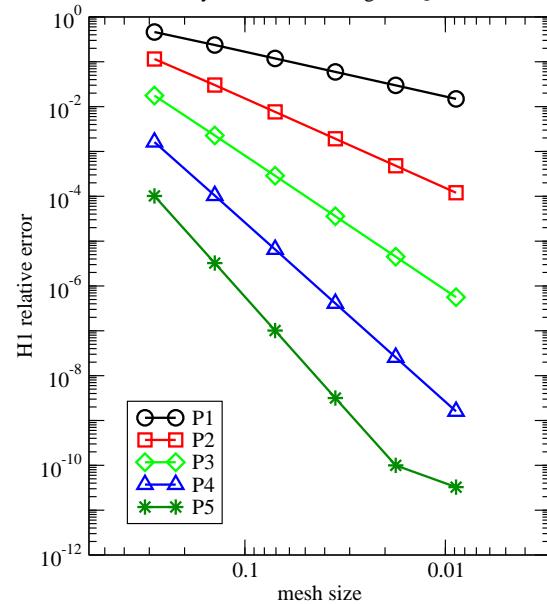
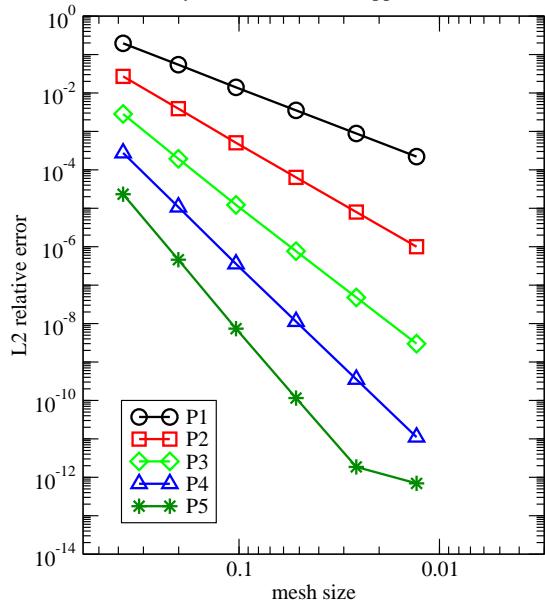


Fig. 271. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

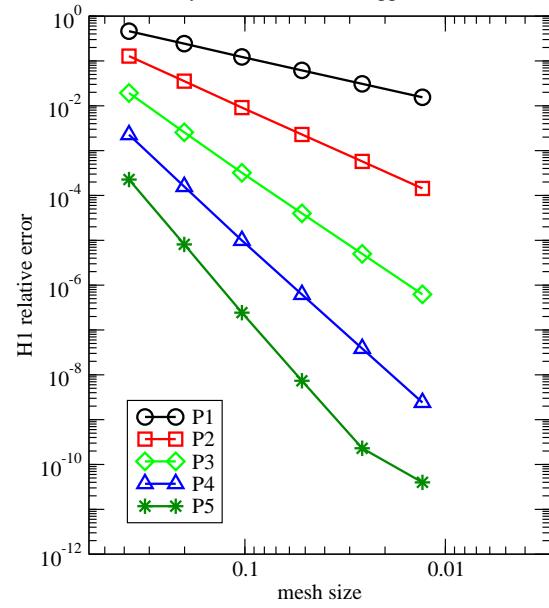
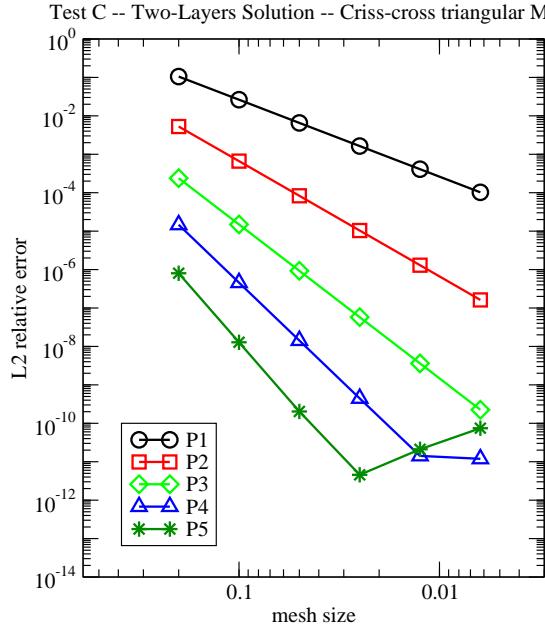


Fig. 272. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test C -- Two-Layers Solution -- Criss-cross triangular Mesh

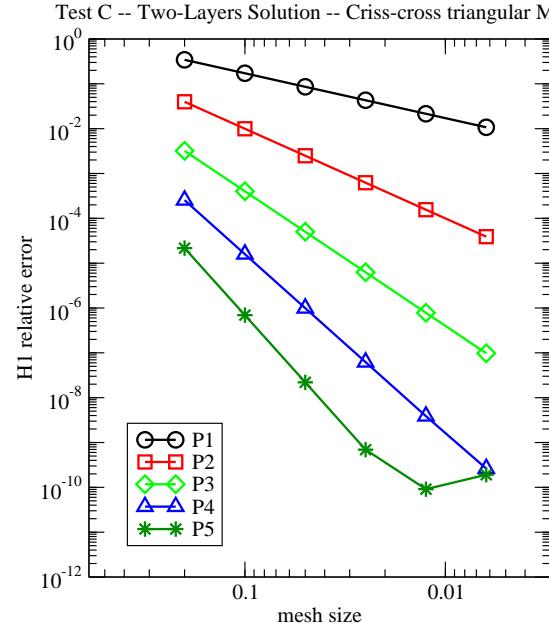
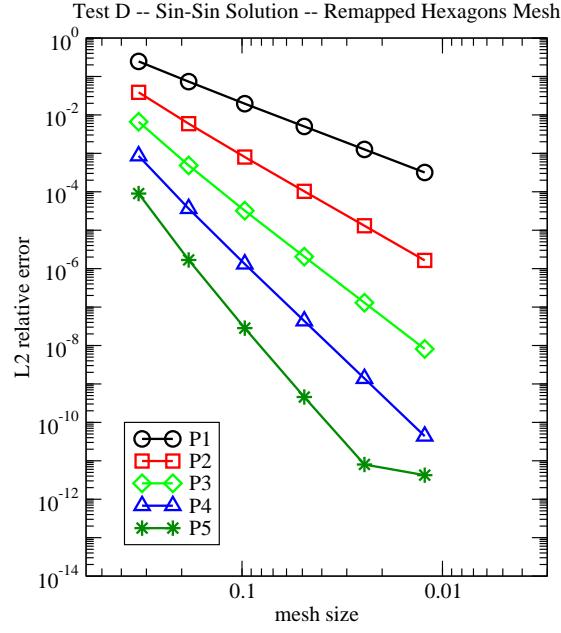


Fig. 273. Internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

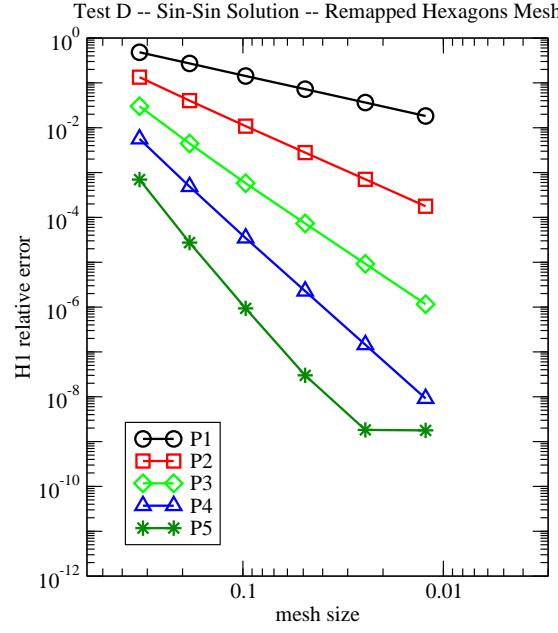
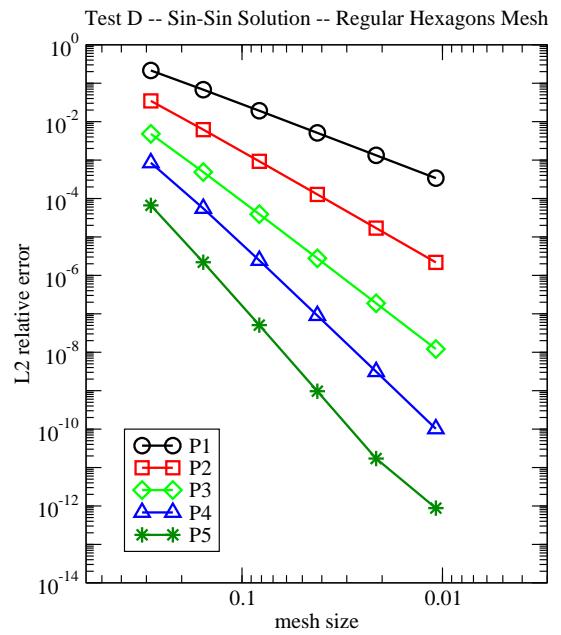


Fig. 274. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

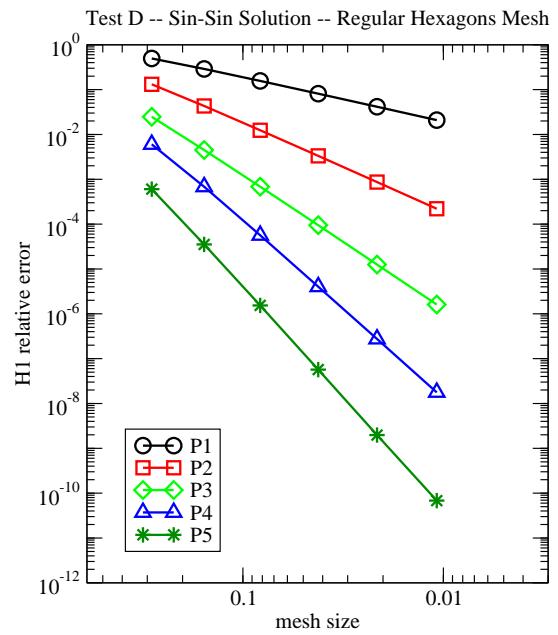
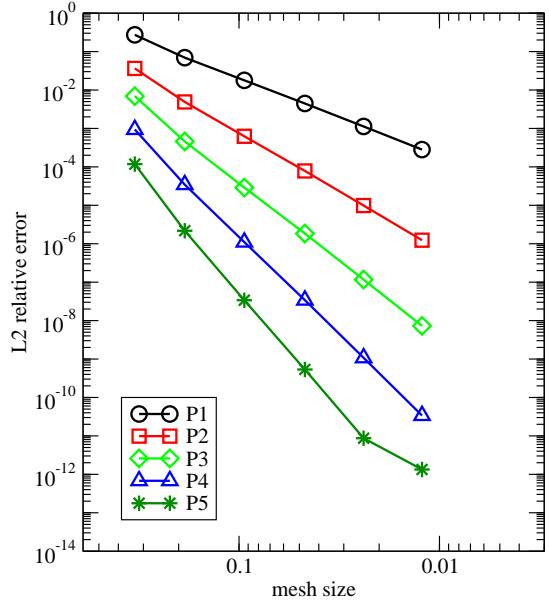


Fig. 275. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Randomized Quadrilaterals Mesh

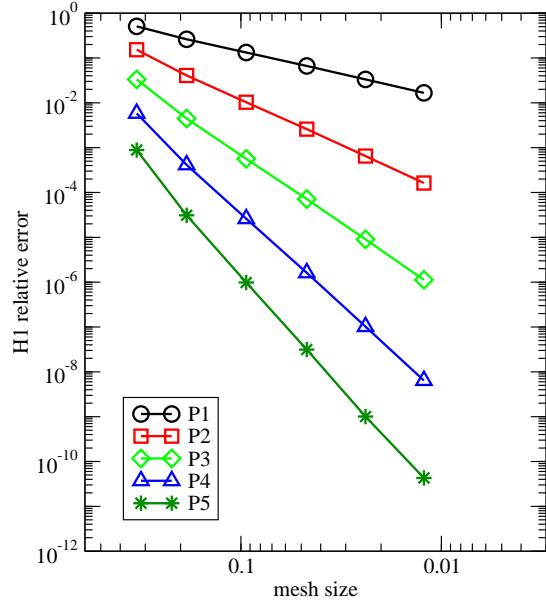
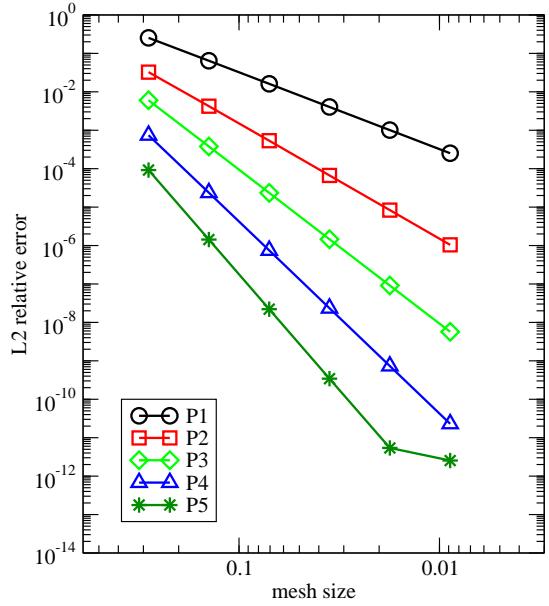


Fig. 276. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Regular Quadrilaterals Mesh

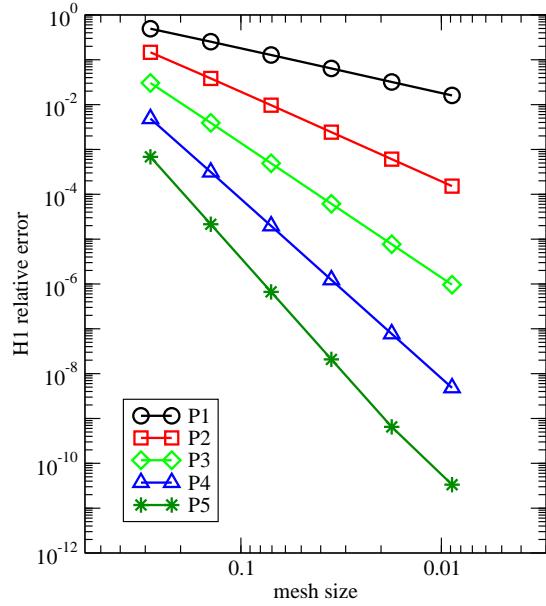
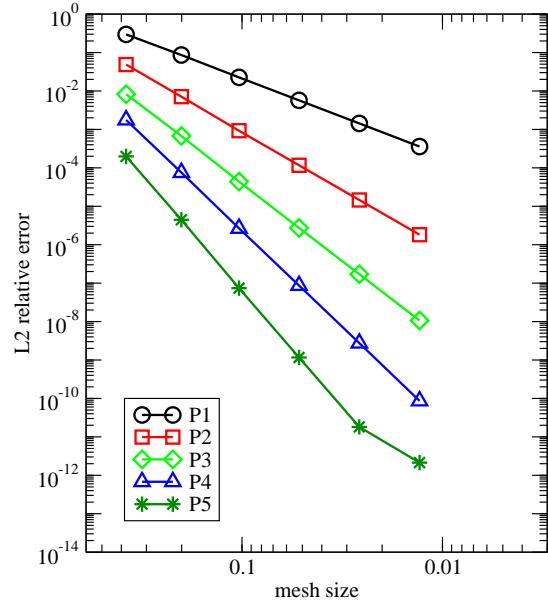


Fig. 277. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Remapped Quadrilaterals Mesh

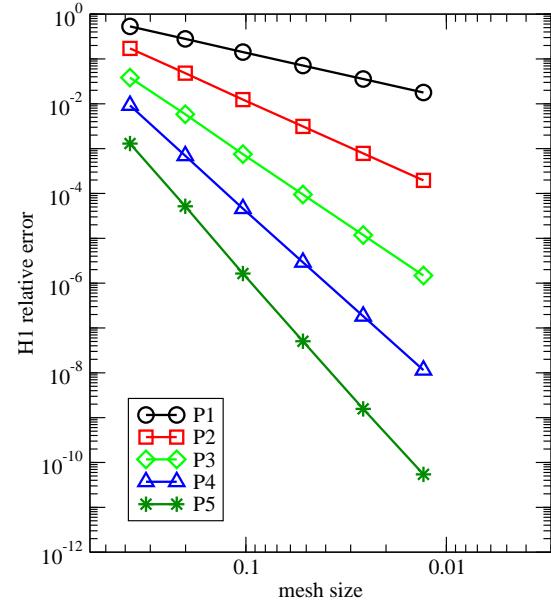
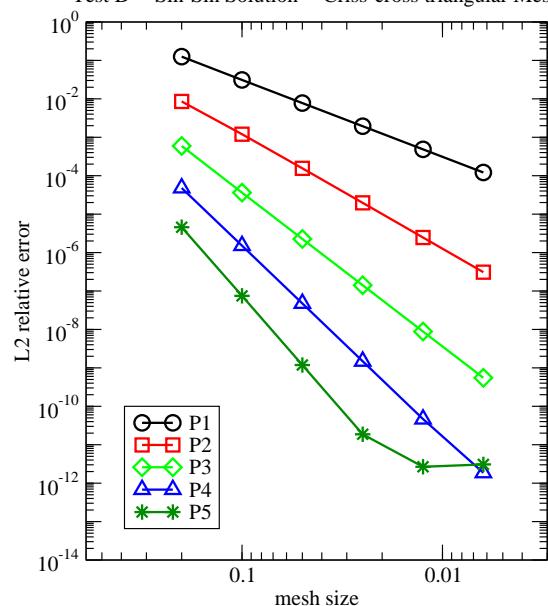


Fig. 278. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Sin Solution -- Criss-cross triangular Mesh

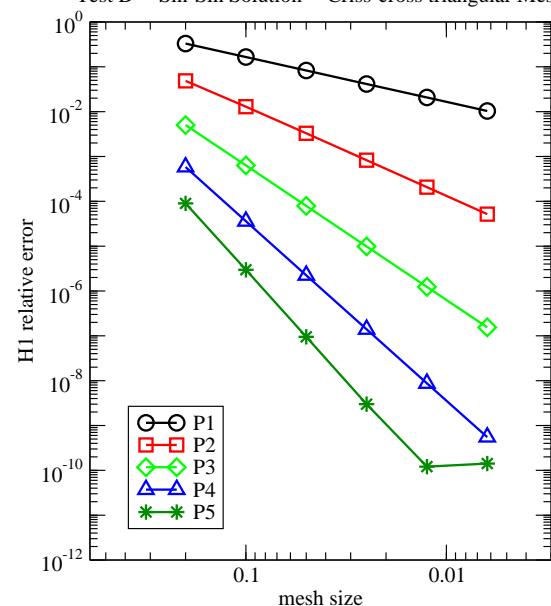
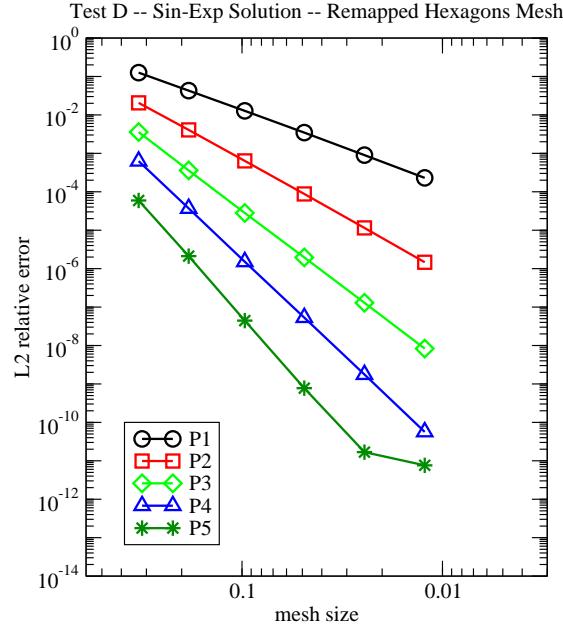


Fig. 279. Internal VEM formulation with variable coefficients; Test D; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

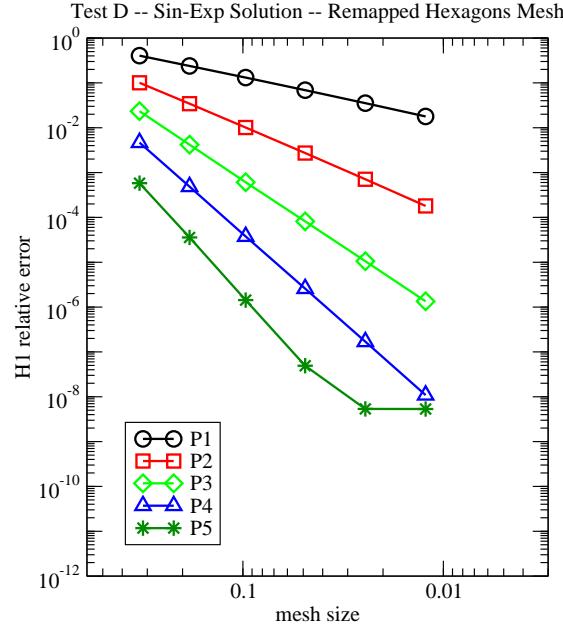
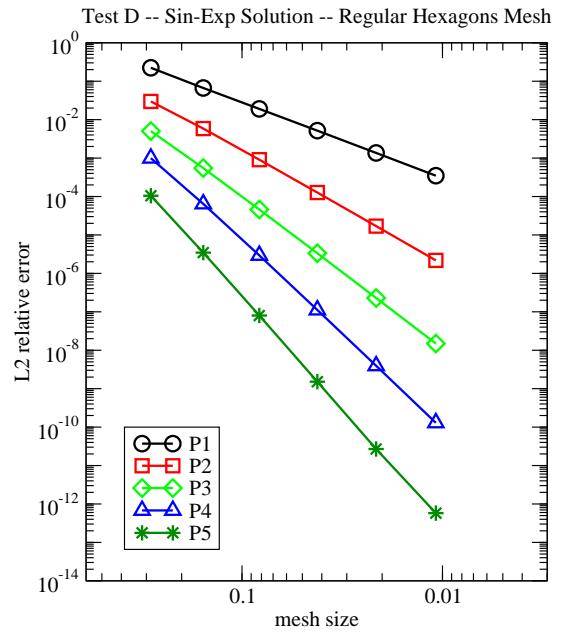


Fig. 280. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.



### Internal/External VEM - Variable Coeffs.

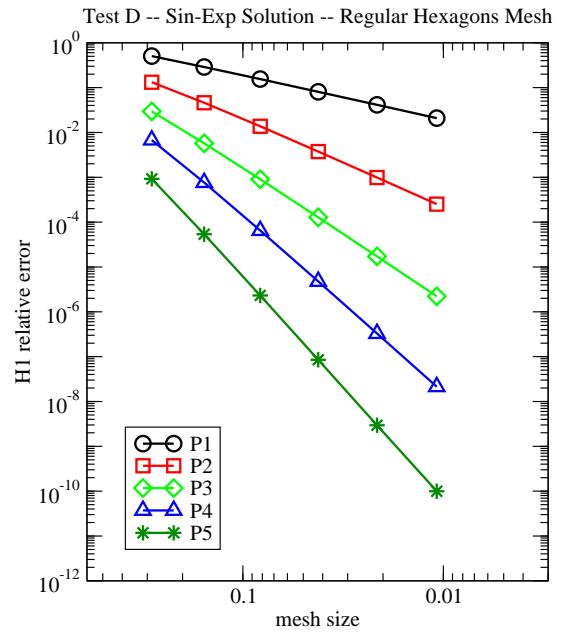
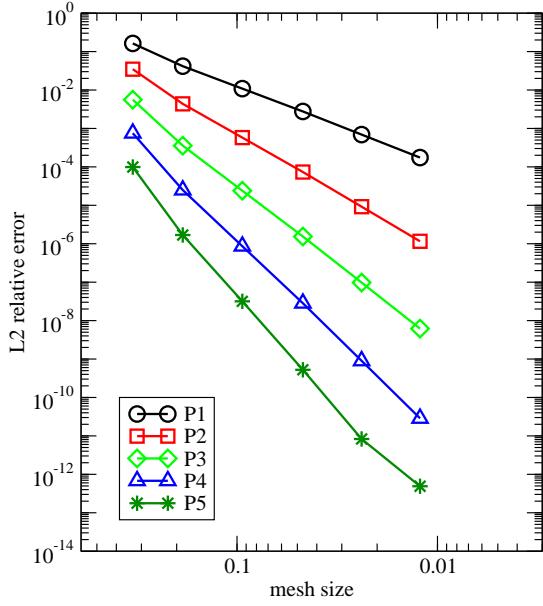


Fig. 281. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular hexagons.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Randomized Quadrilaterals Mesh

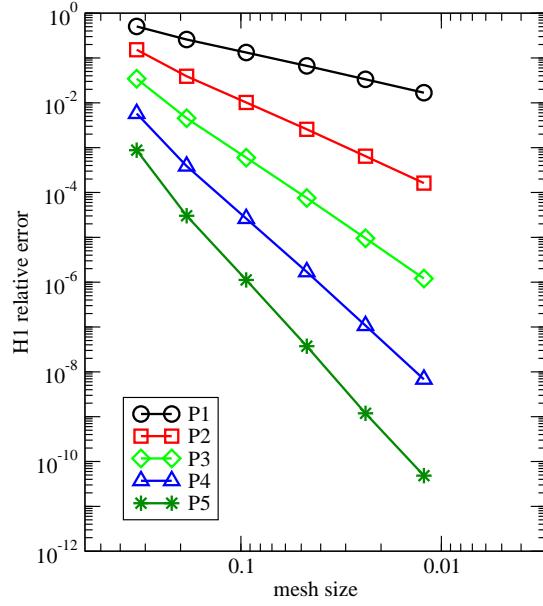
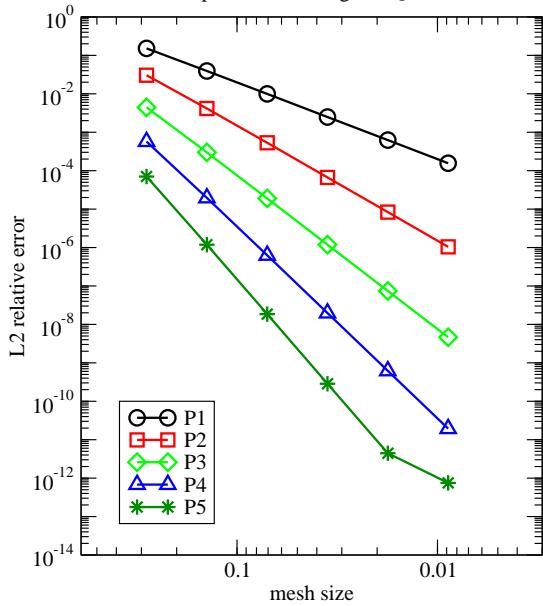


Fig. 282. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of randomized quadrilateral cells.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Regular Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Regular Quadrilaterals Mesh

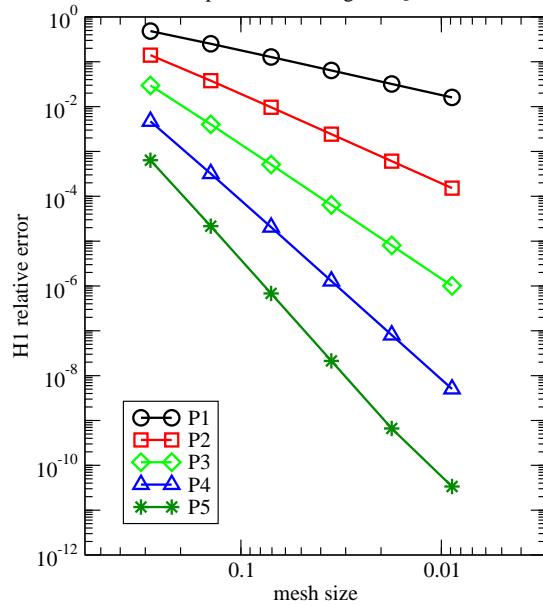
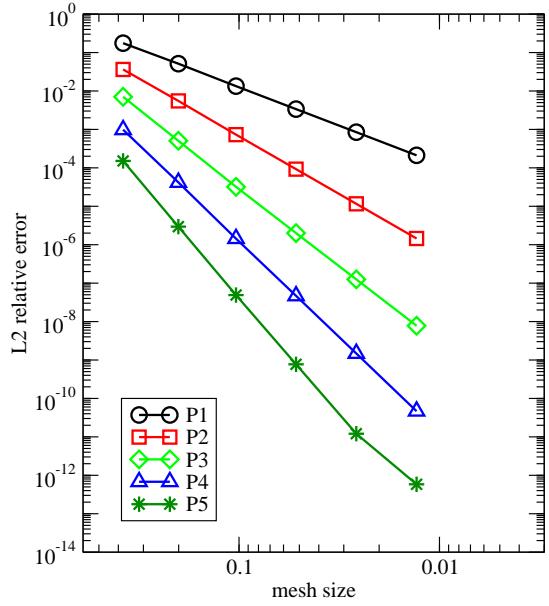


Fig. 283. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Remapped Quadrilaterals Mesh

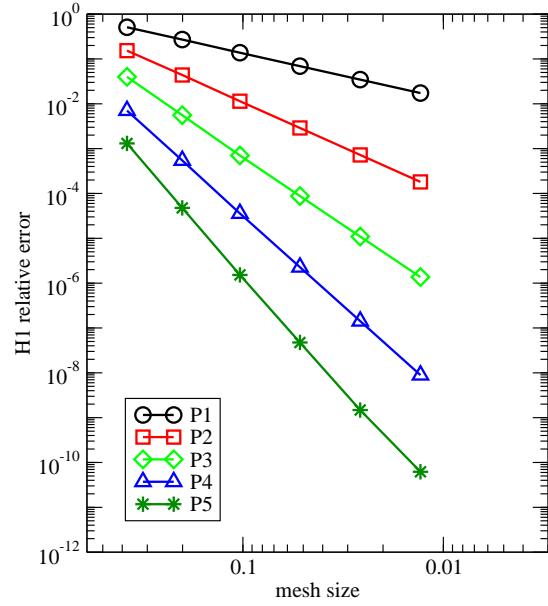
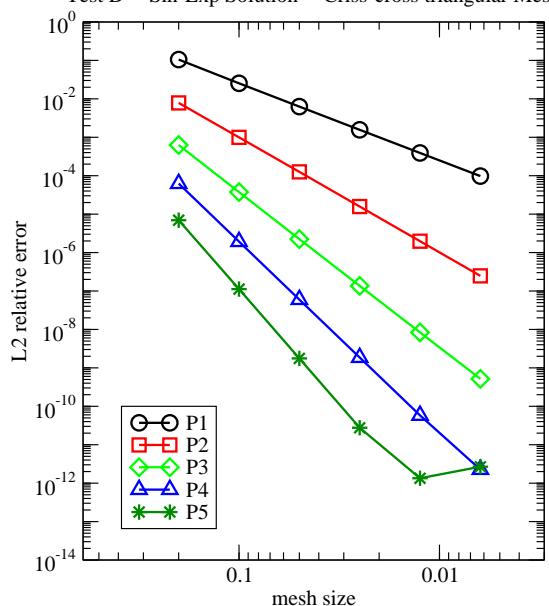


Fig. 284. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Sin-Exp Solution -- Criss-cross triangular Mesh

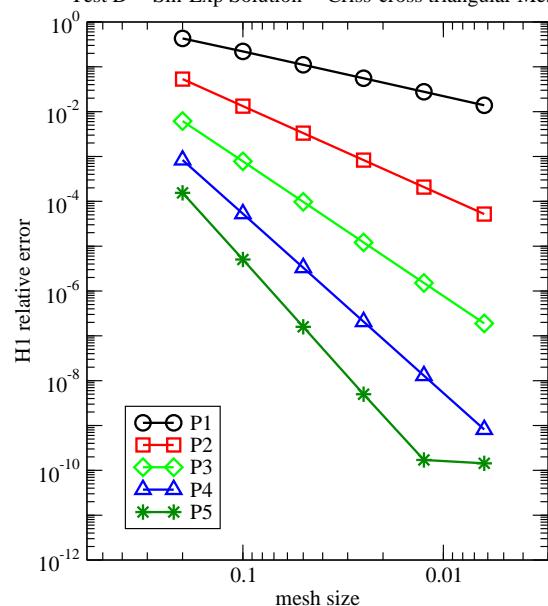
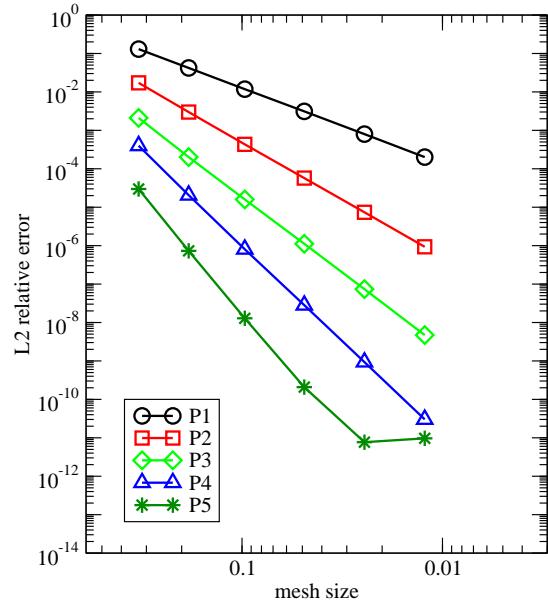


Fig. 285. Internal VEM formulation with variable coefficients; Test D; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Remapped Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Remapped Hexagons Mesh

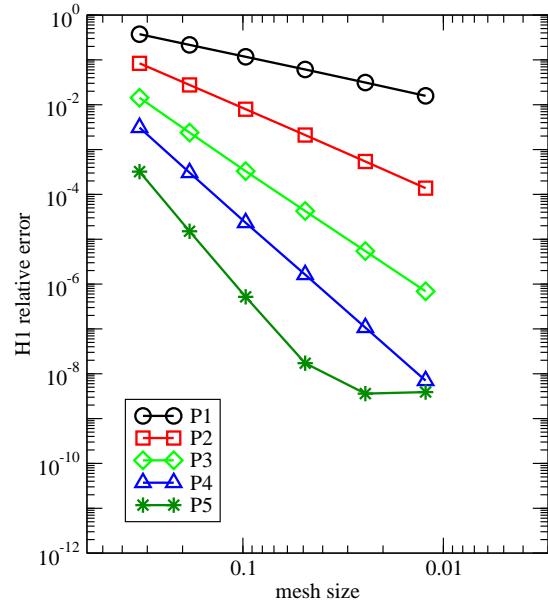
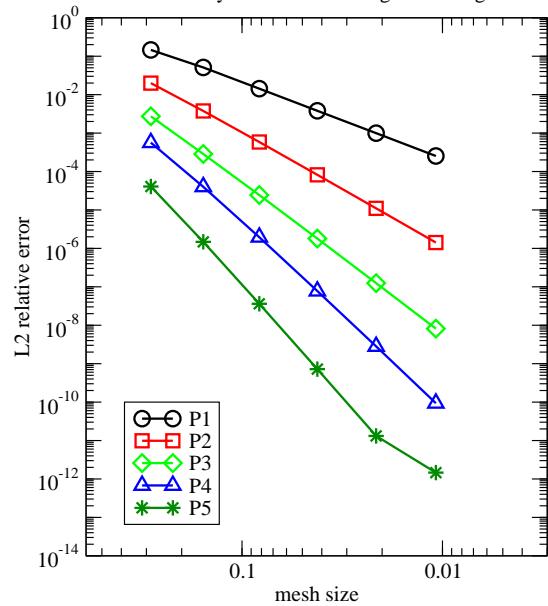


Fig. 286. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of smoothly remapped hexagons.

### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Regular Hexagons Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Regular Hexagons Mesh

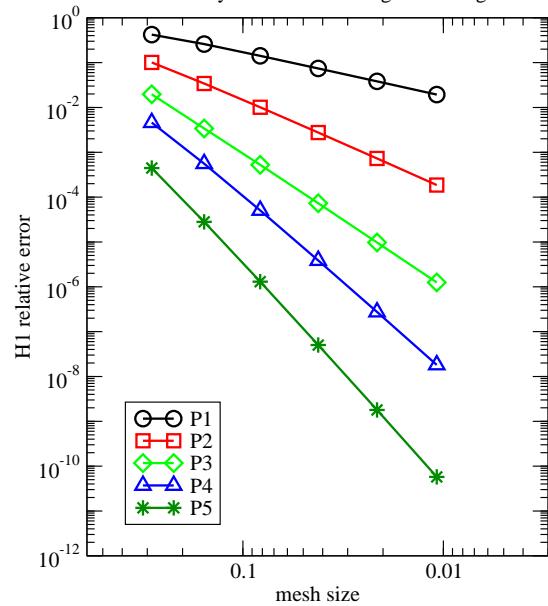


Fig. 287. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular hexagons.

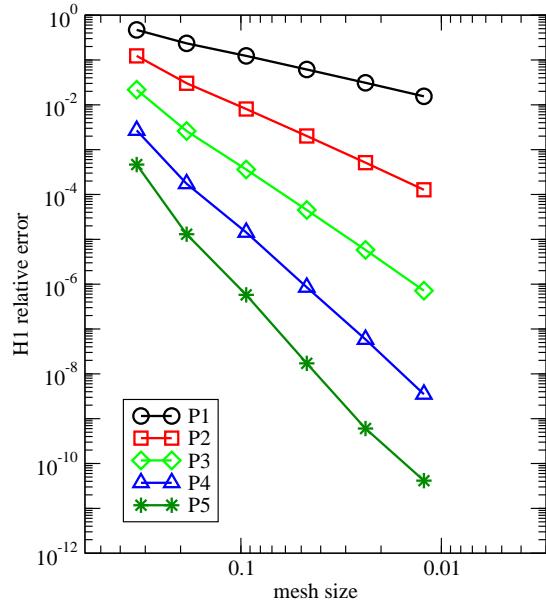
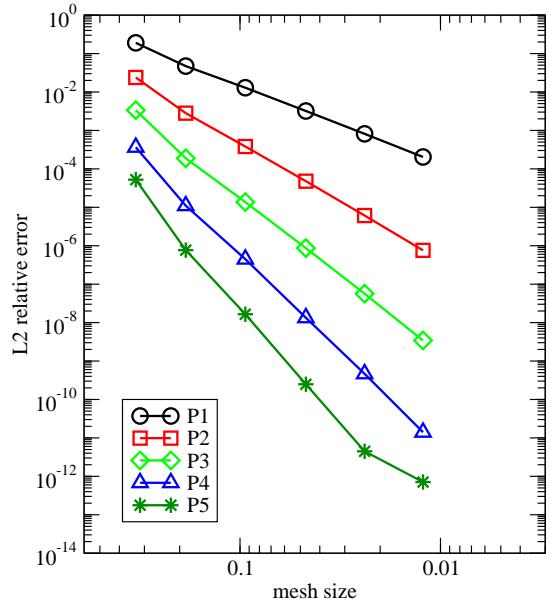


Fig. 288. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of randomized quadrilateral cells.

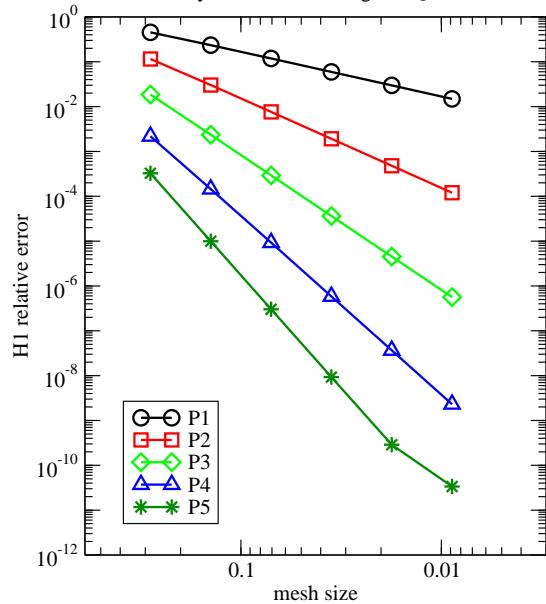
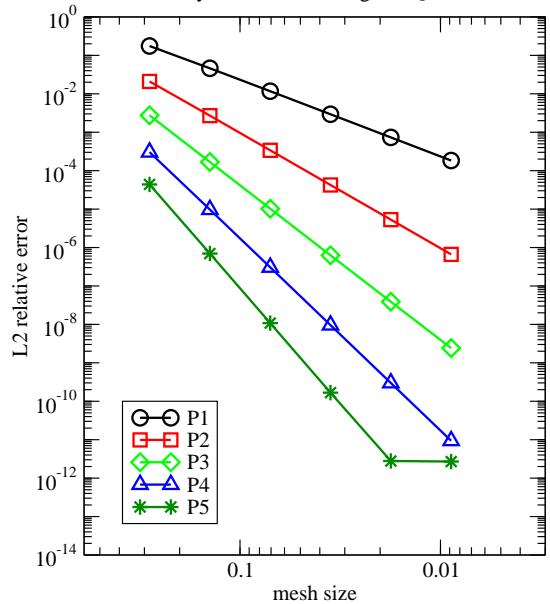
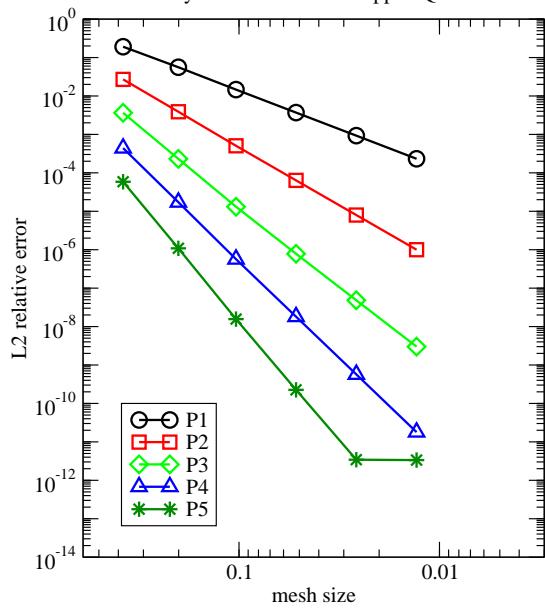


Fig. 289. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Remapped Quadrilaterals Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Remapped Quadrilaterals Mesh

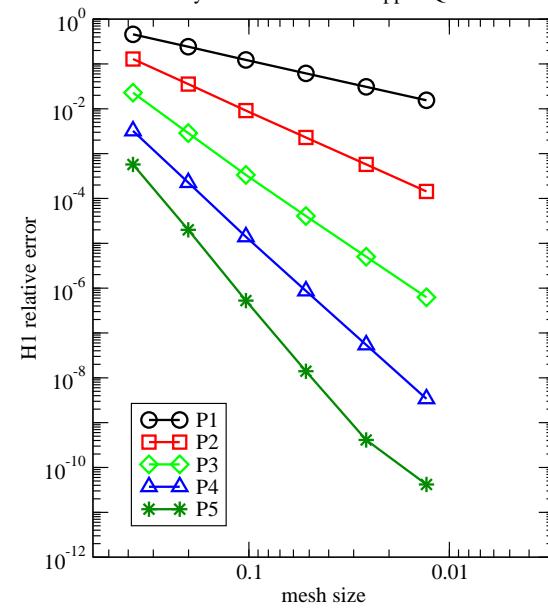
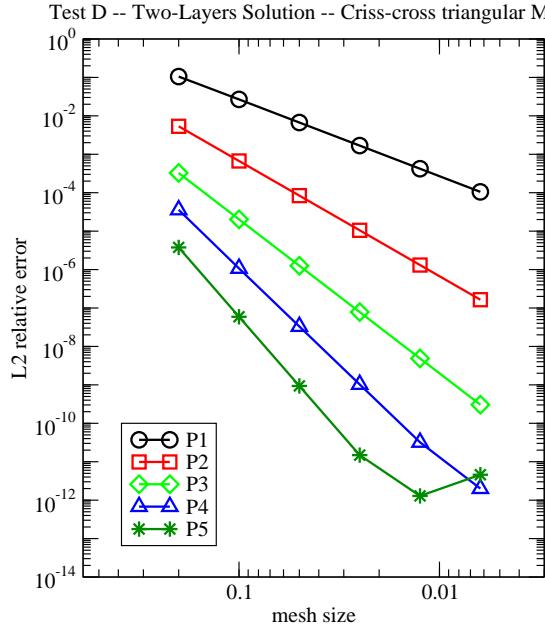


Fig. 290. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Criss-cross triangular Mesh



### Internal/External VEM - Variable Coeffs.

Test D -- Two-Layers Solution -- Criss-cross triangular Mesh

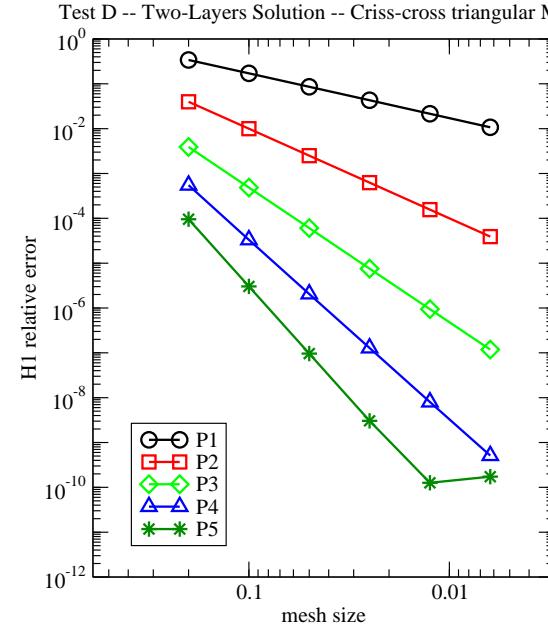
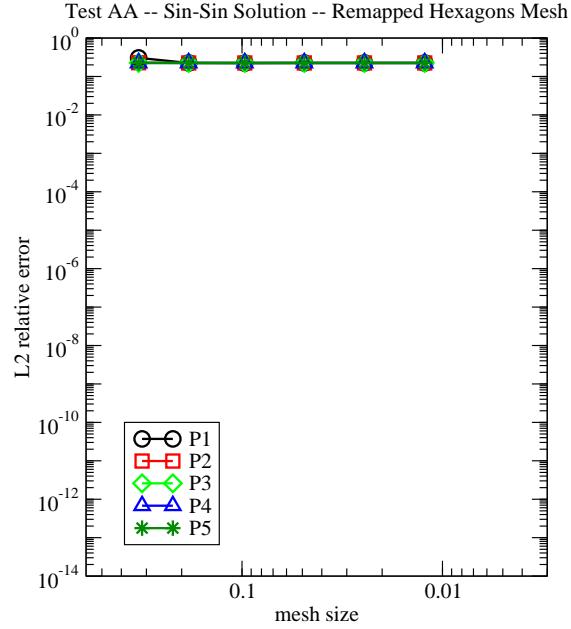


Fig. 291. Internal VEM formulation with variable coefficients; Test D; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

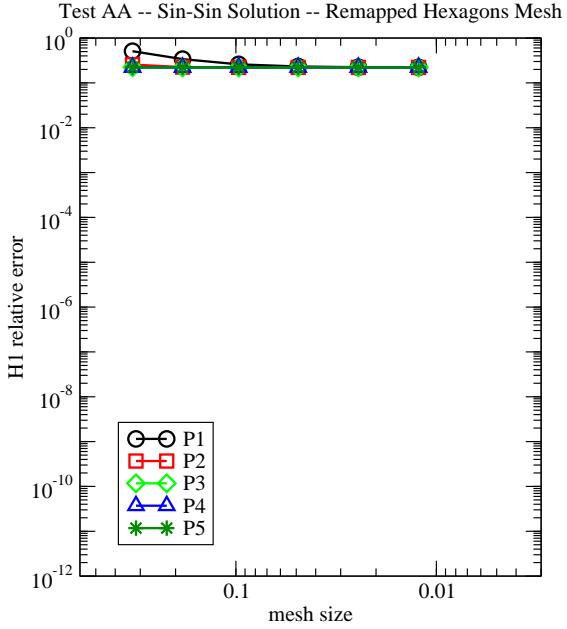
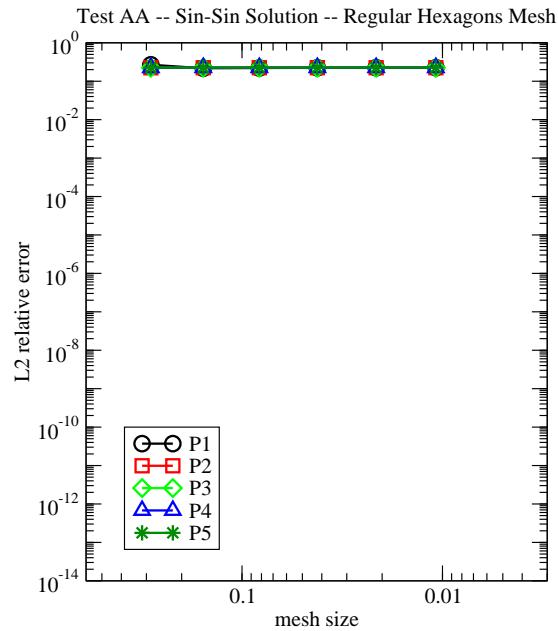


Fig. 292. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

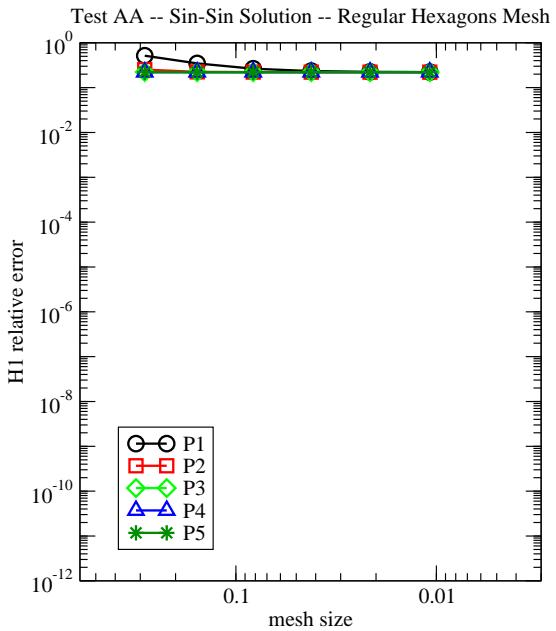


Fig. 293. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

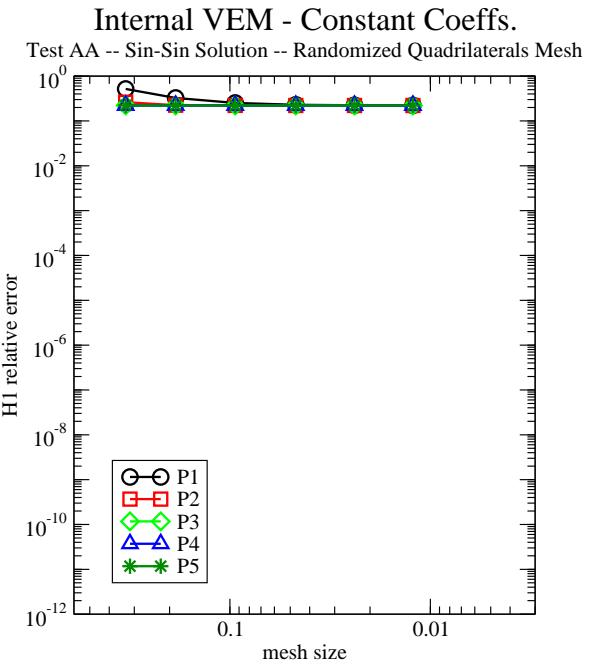
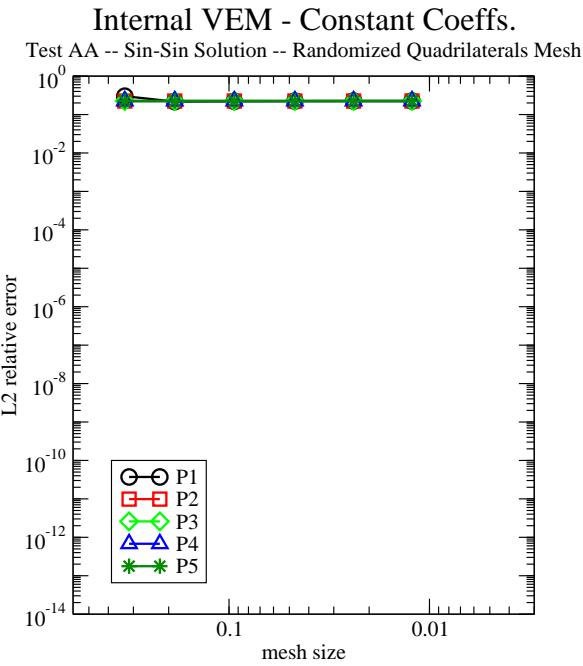


Fig. 294. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

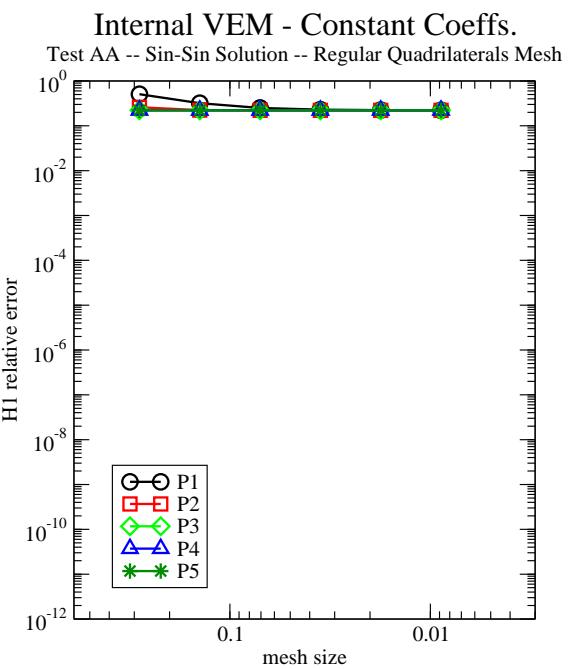
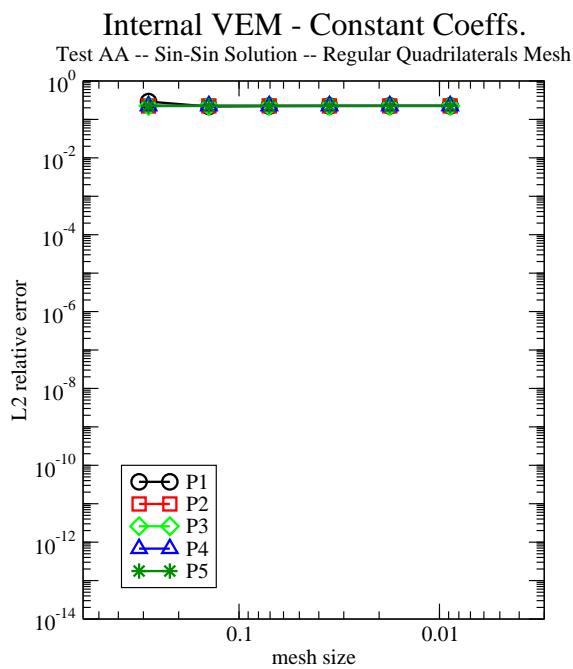


Fig. 295. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

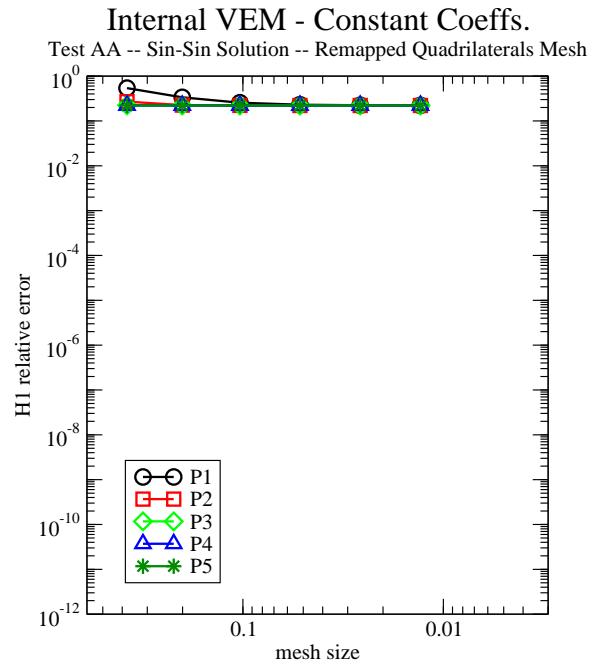
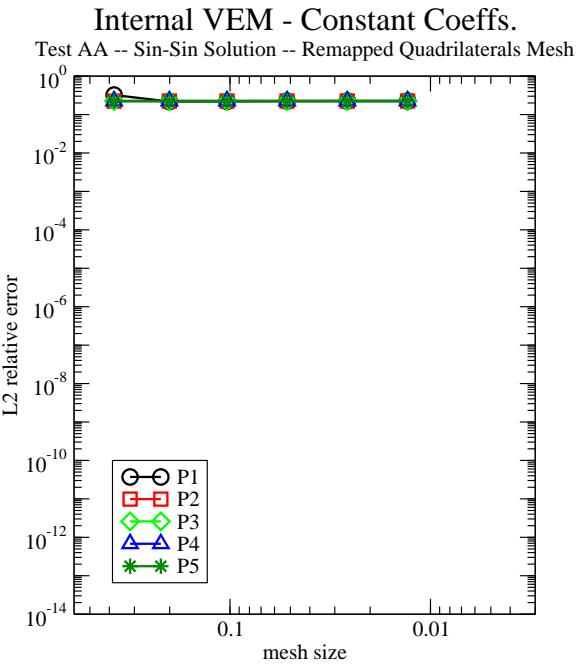


Fig. 296. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

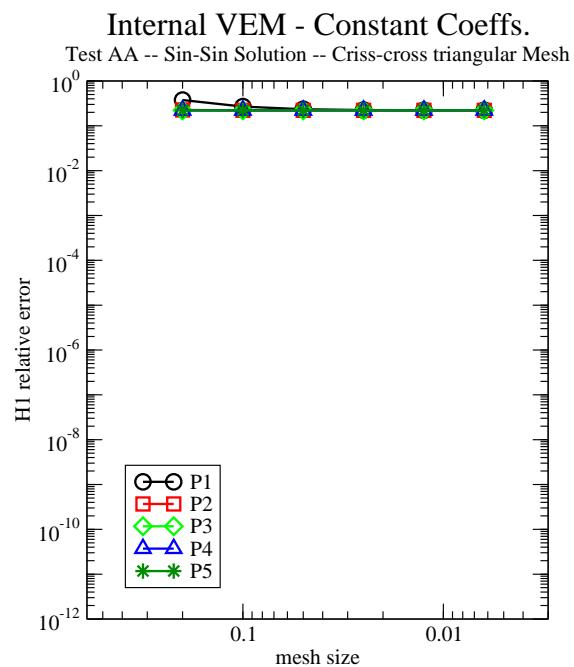
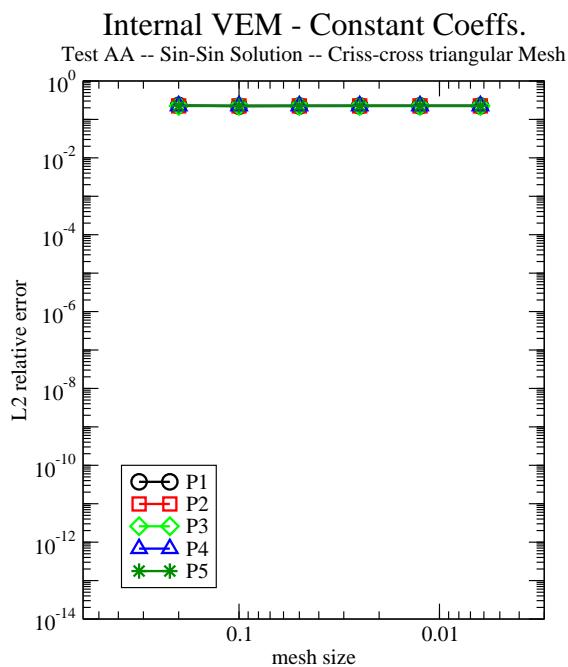
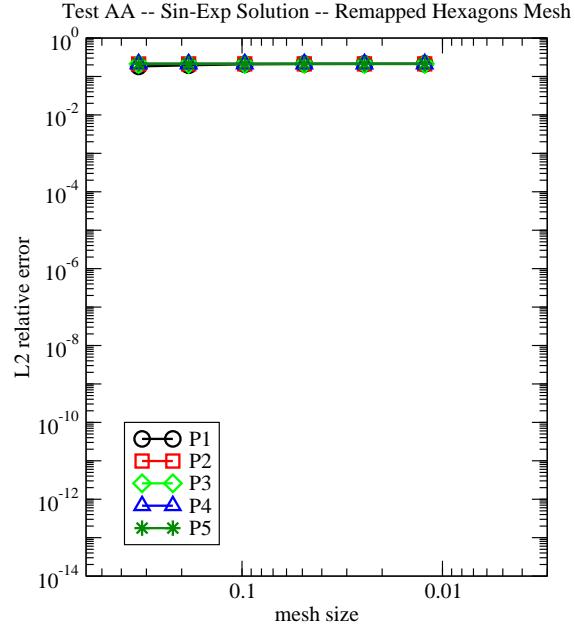


Fig. 297. Internal-internal VEM formulation with constant coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

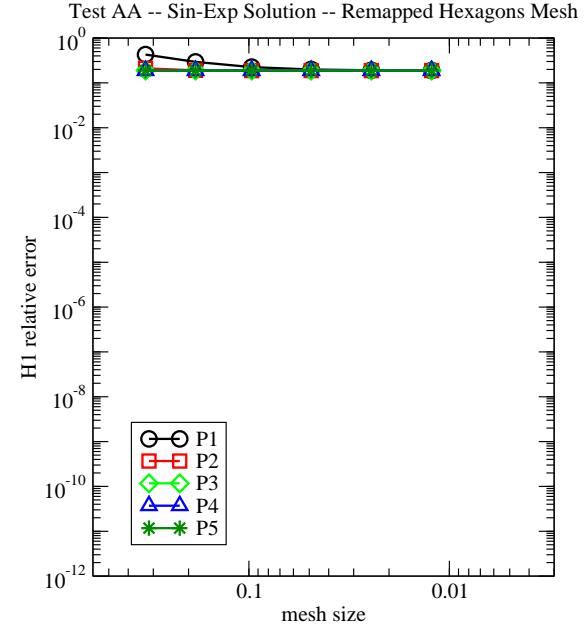
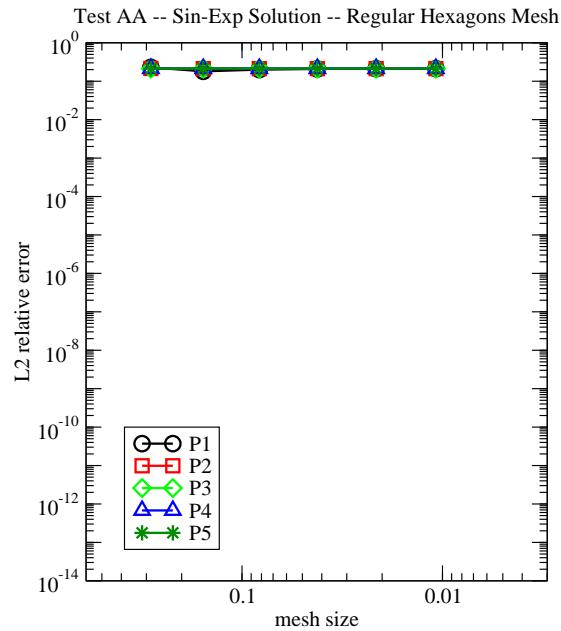


Fig. 298. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

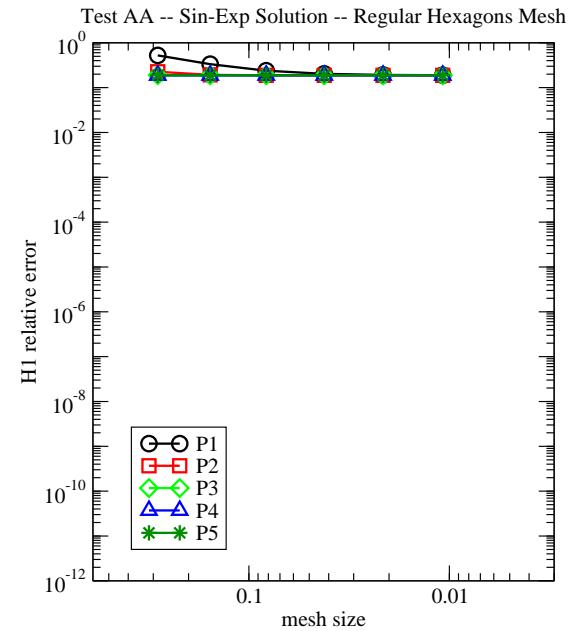


Fig. 299. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

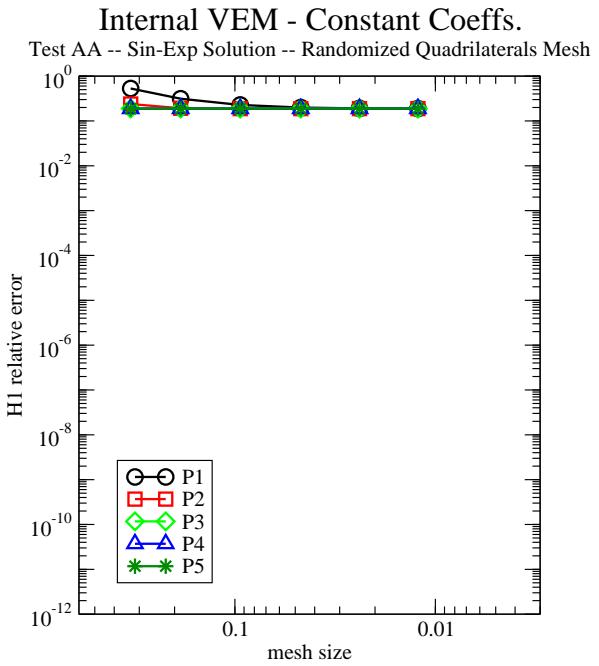
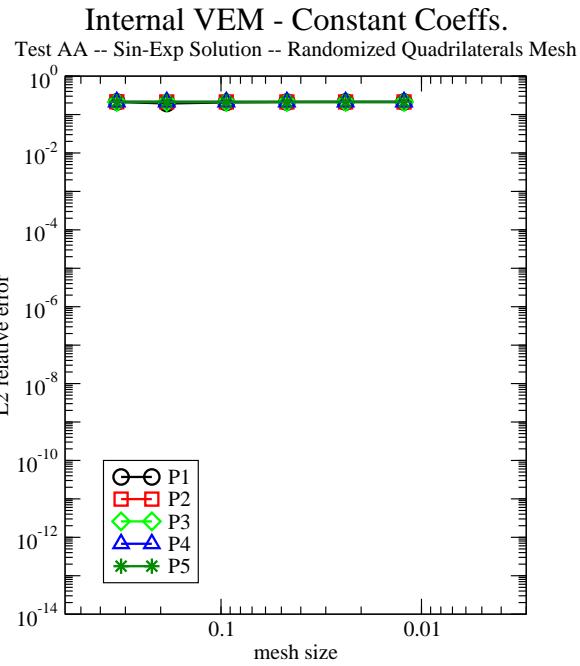


Fig. 300. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

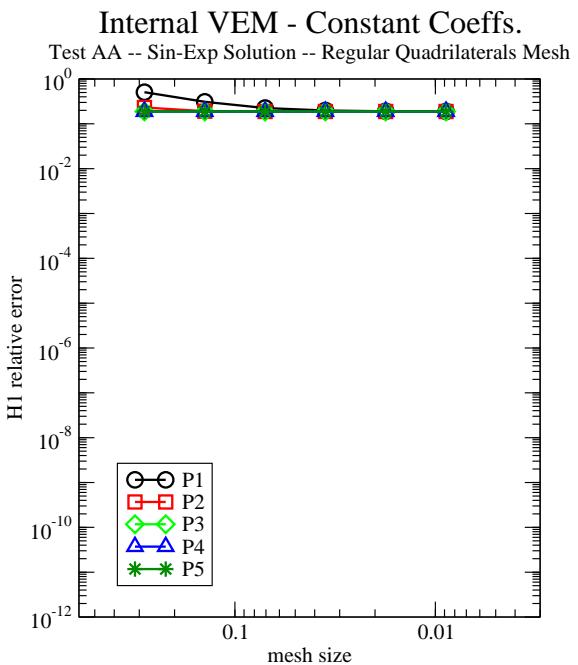
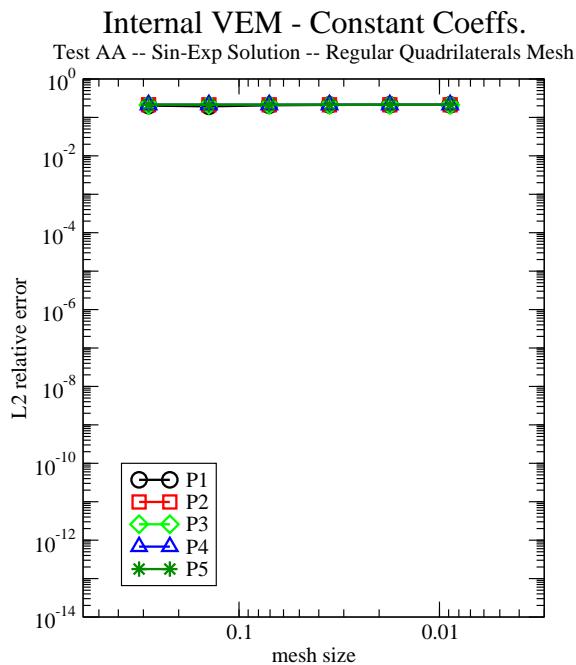


Fig. 301. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

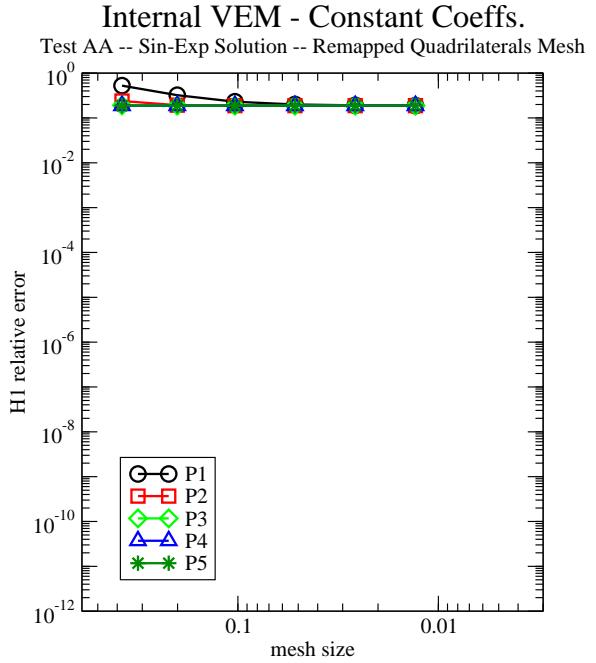
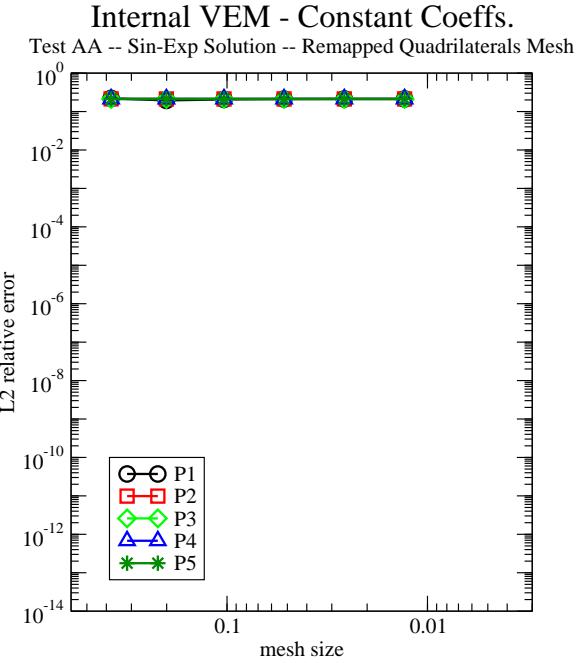


Fig. 302. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

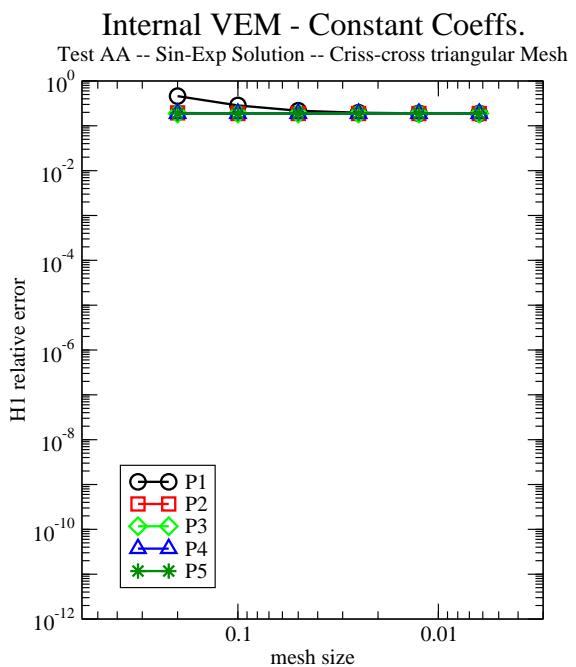
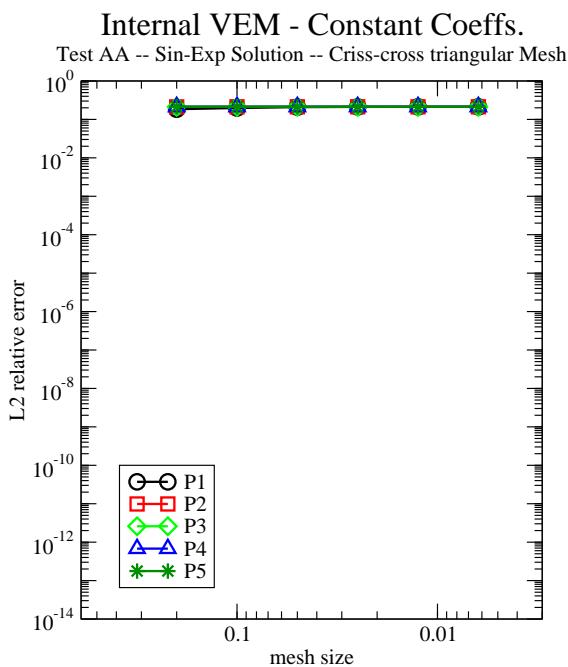


Fig. 303. Internal-internal VEM formulation with constant coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

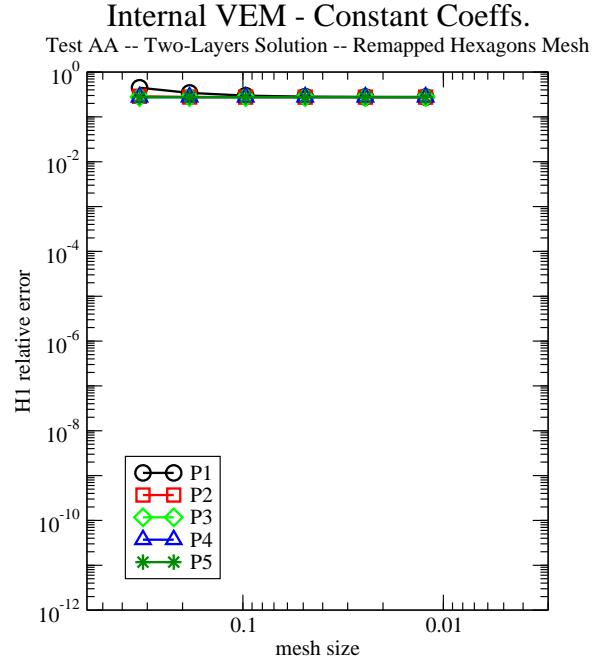
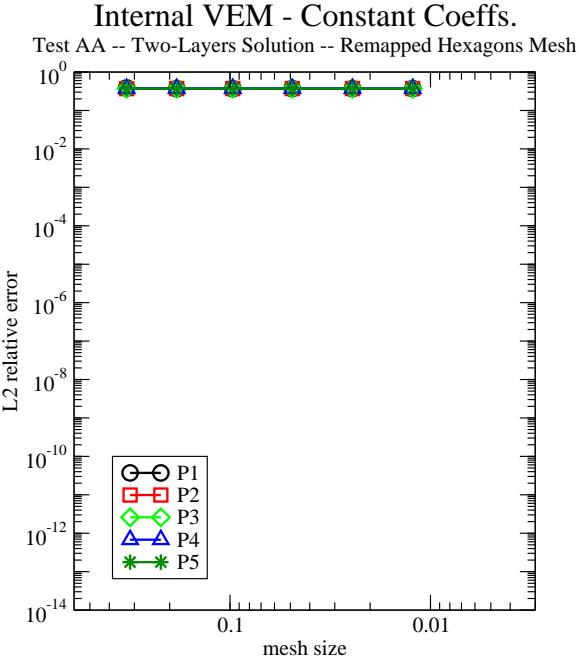


Fig. 304. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

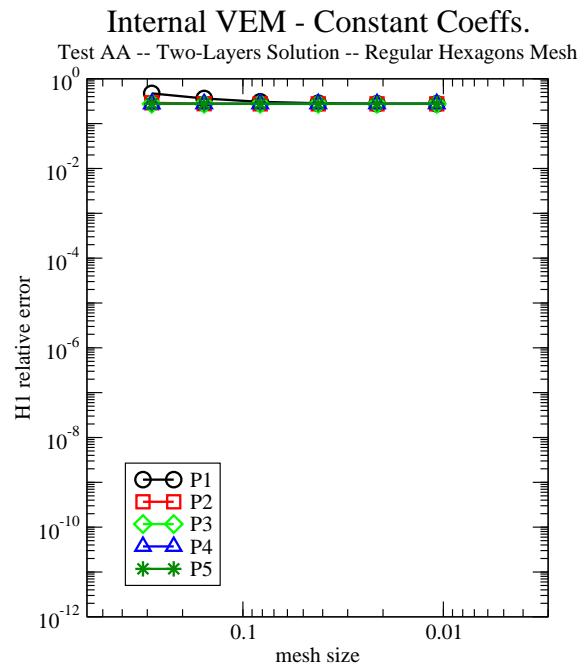
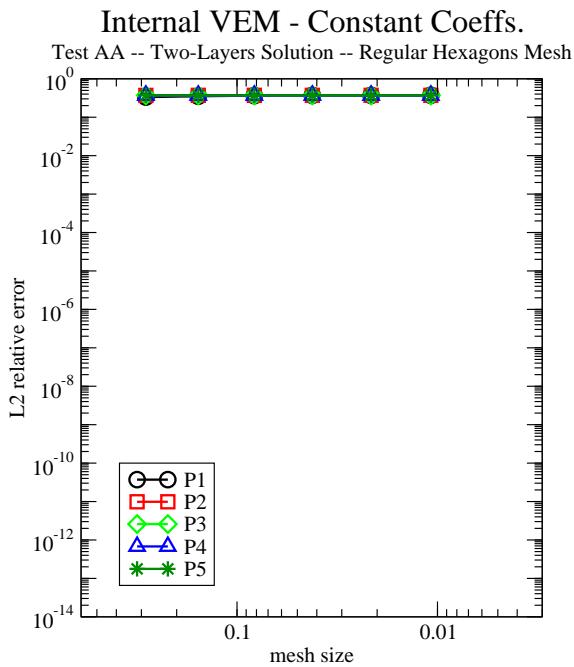


Fig. 305. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular hexagons.

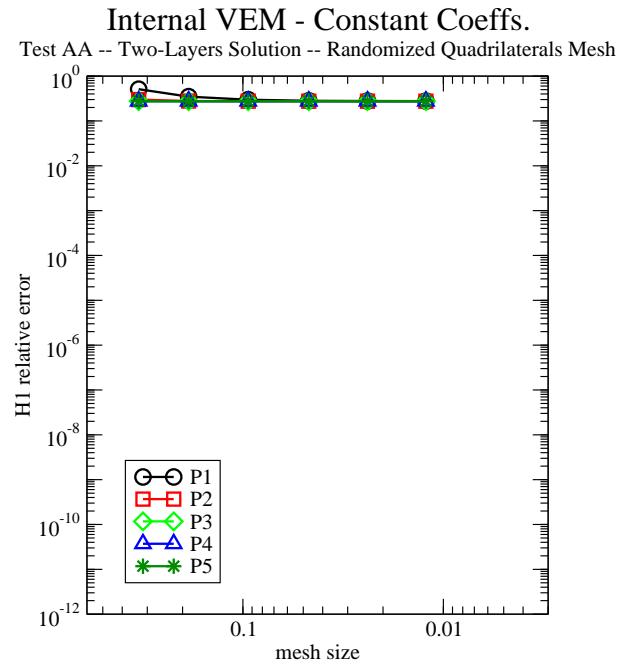
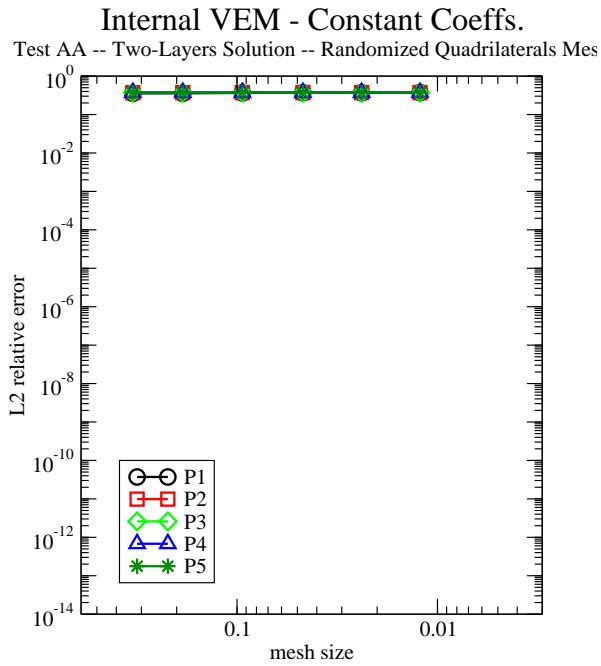


Fig. 306. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

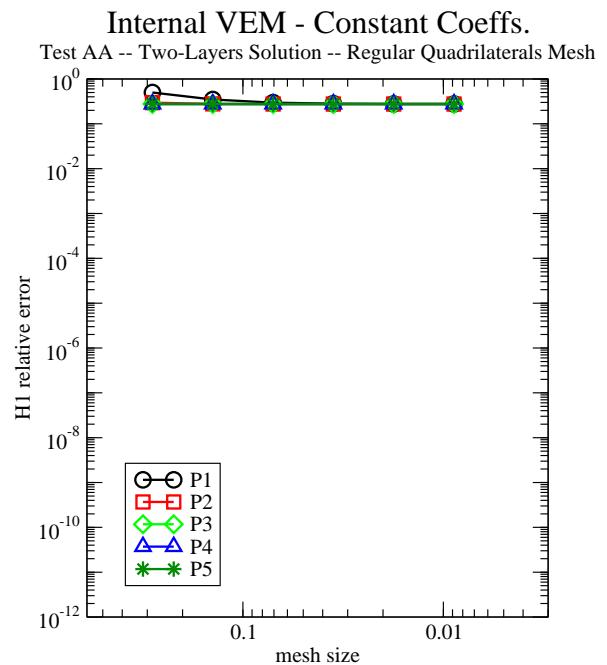
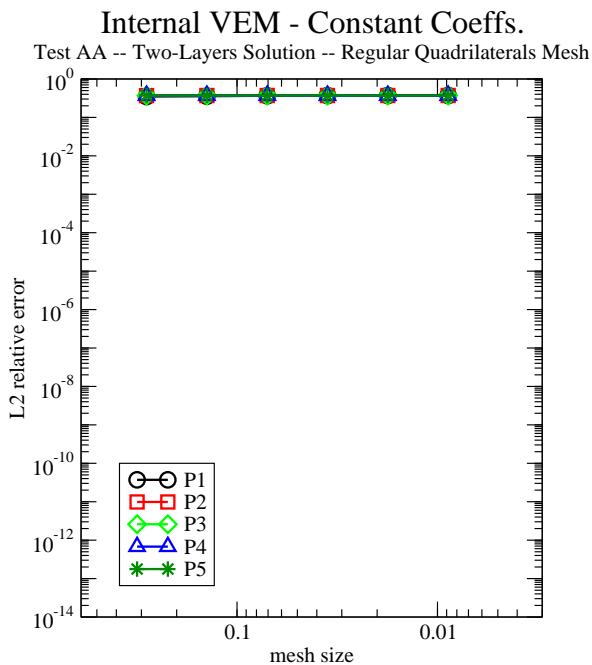


Fig. 307. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

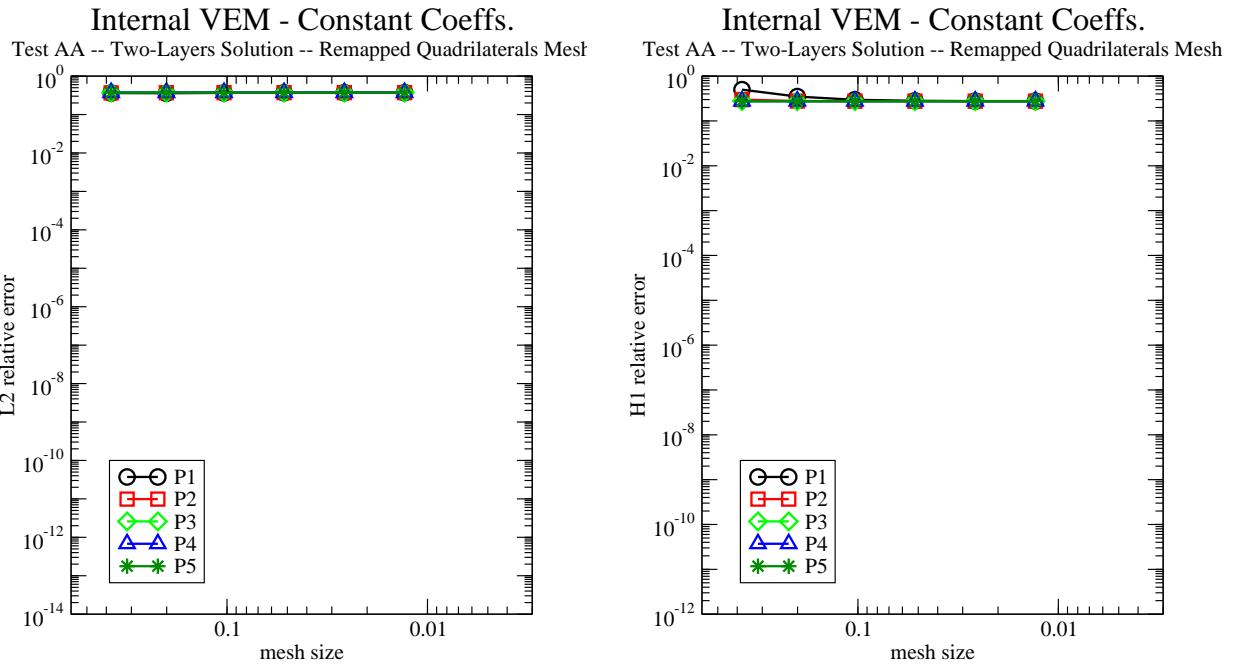


Fig. 308. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

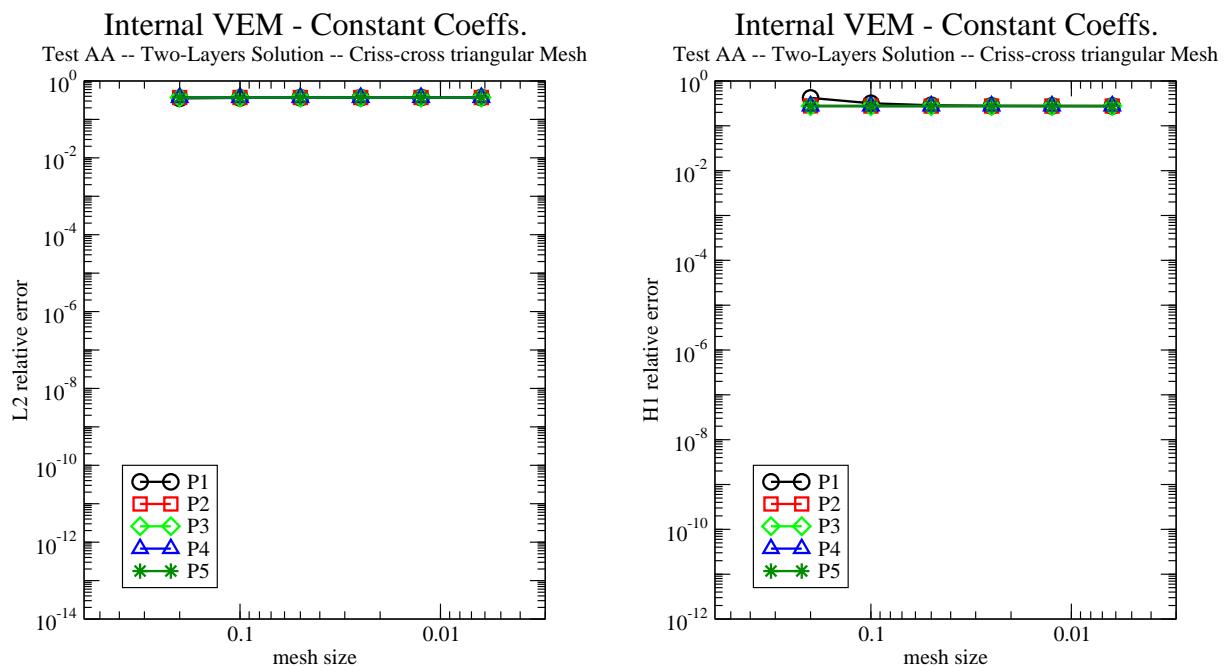
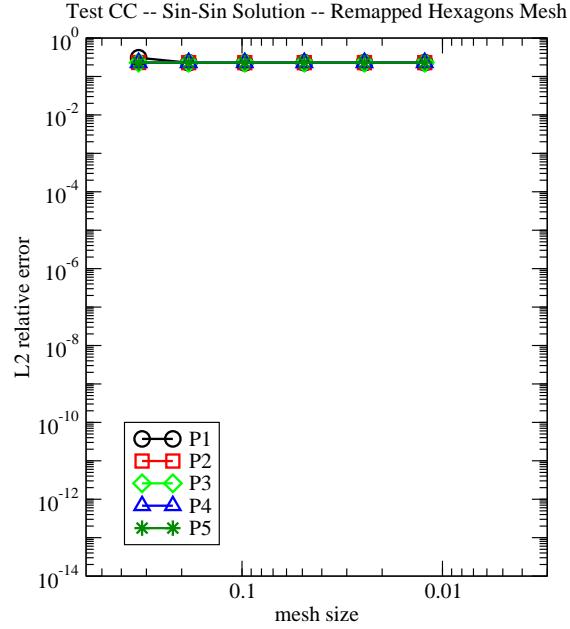


Fig. 309. Internal-internal VEM formulation with constant coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

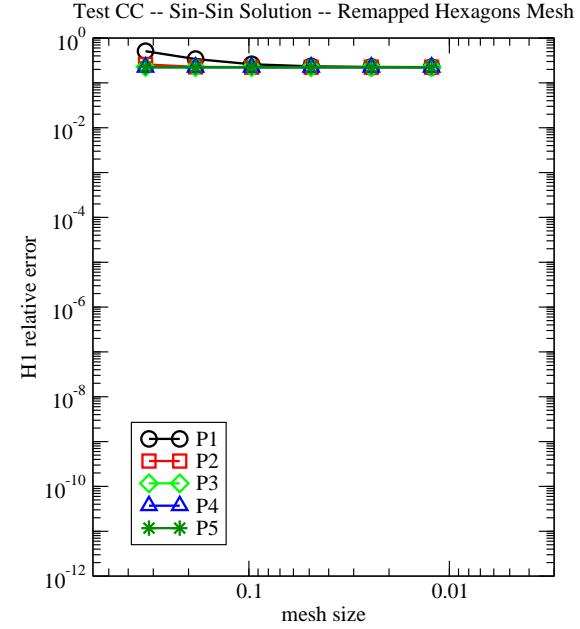
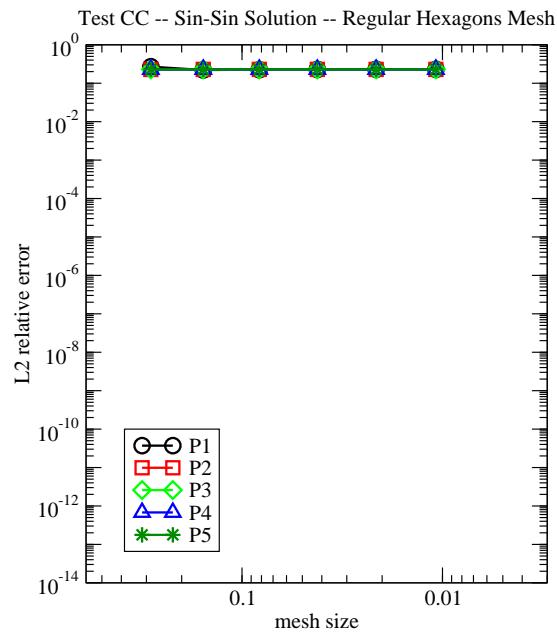


Fig. 310. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

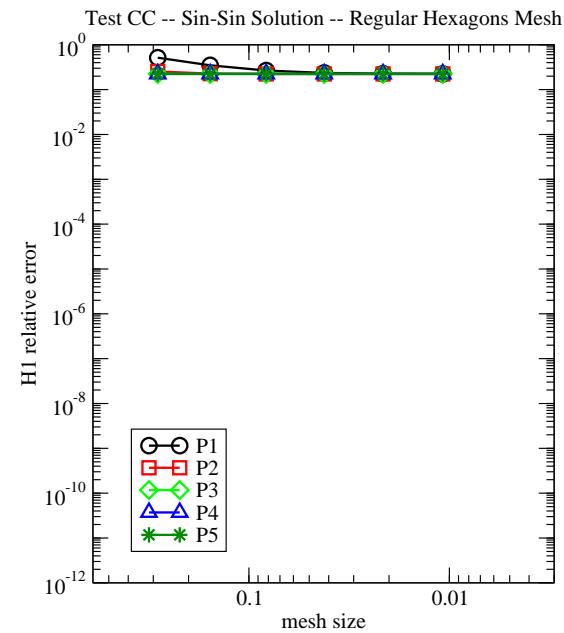


Fig. 311. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

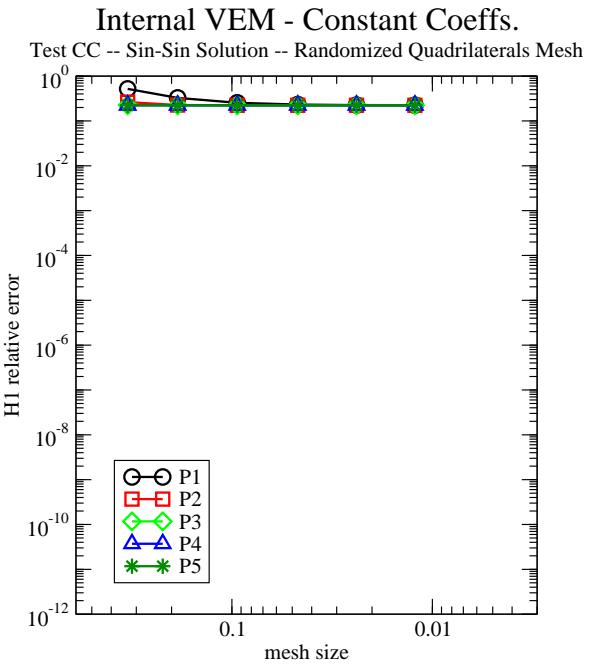
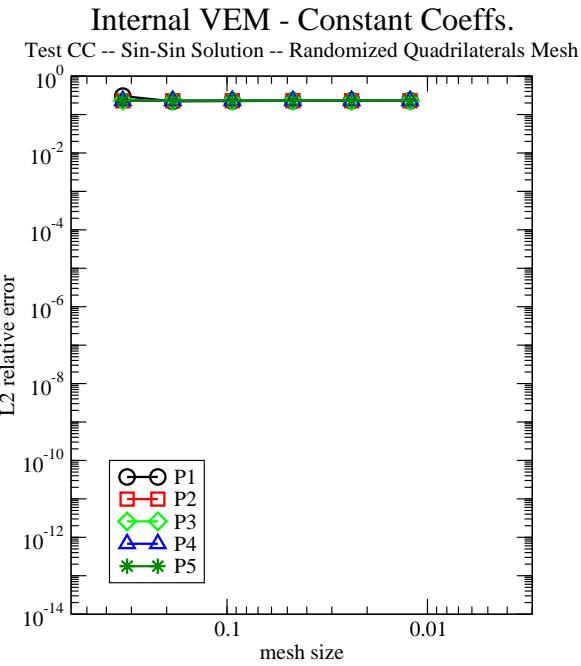


Fig. 312. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

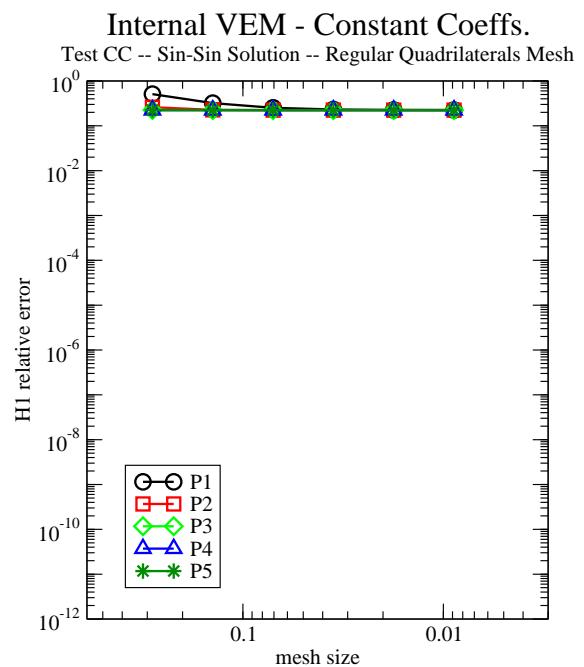
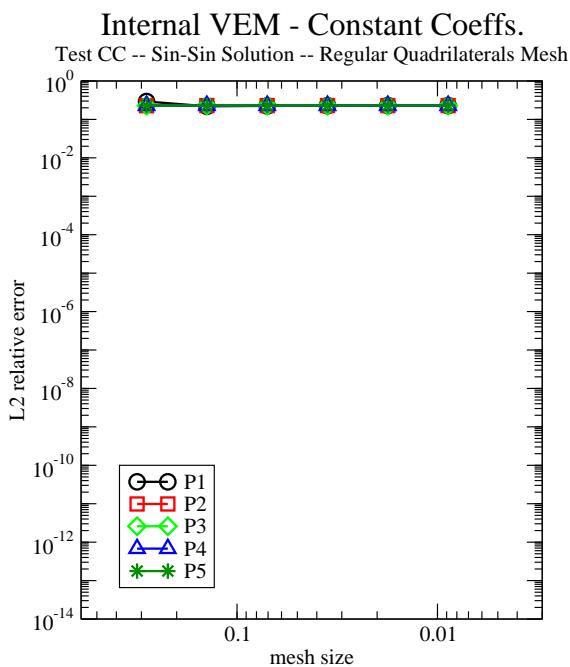


Fig. 313. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

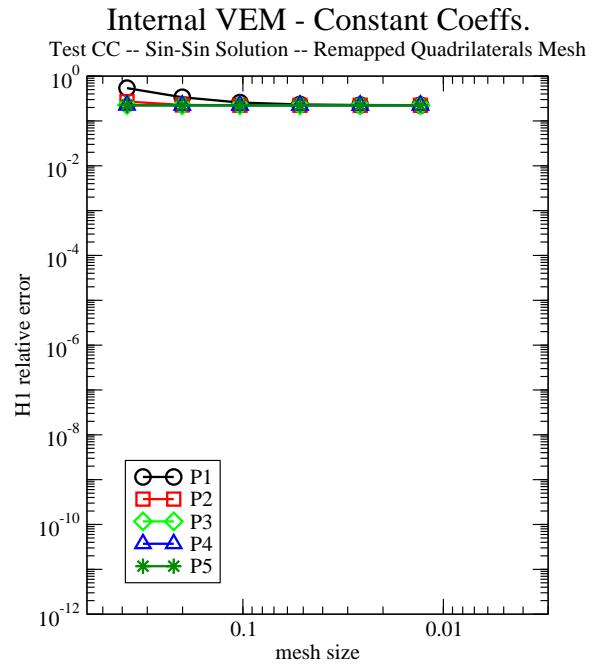
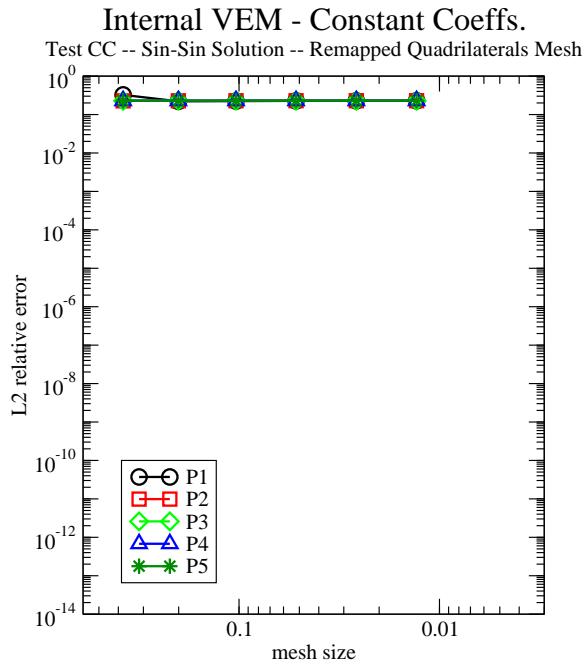


Fig. 314. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

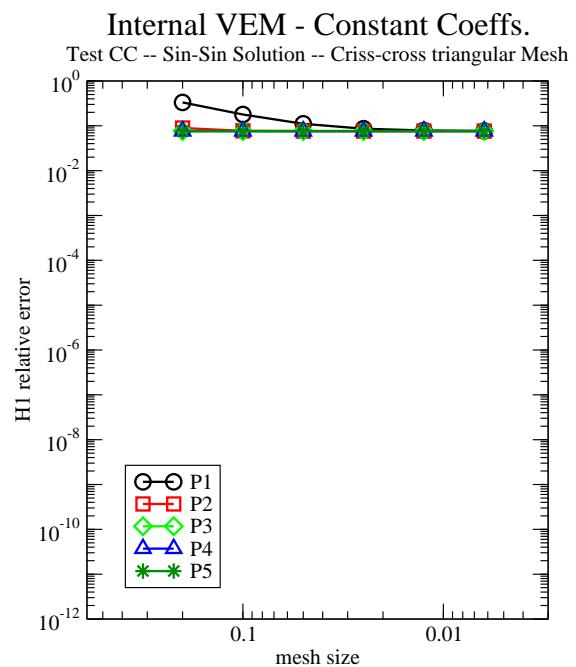
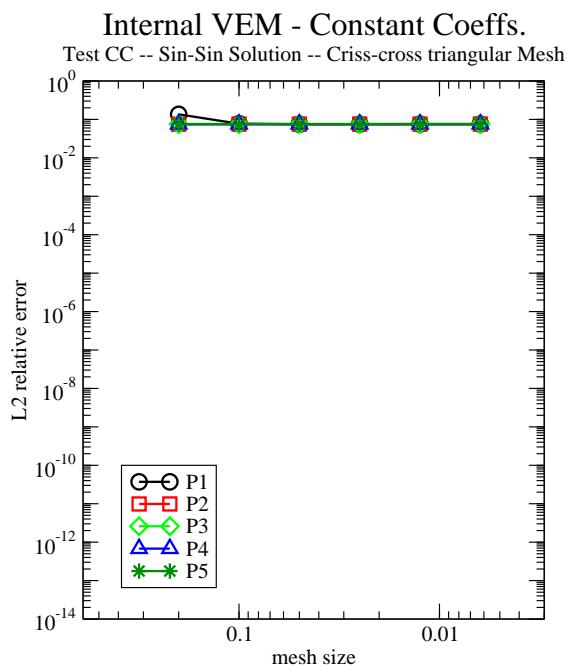
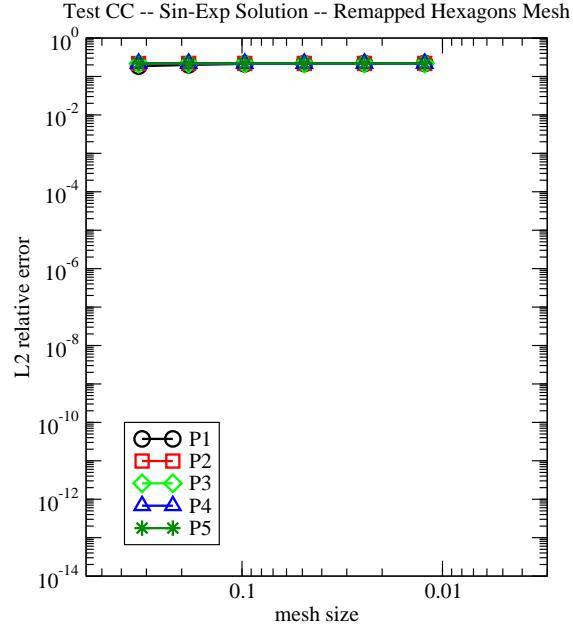


Fig. 315. Internal-internal VEM formulation with constant coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

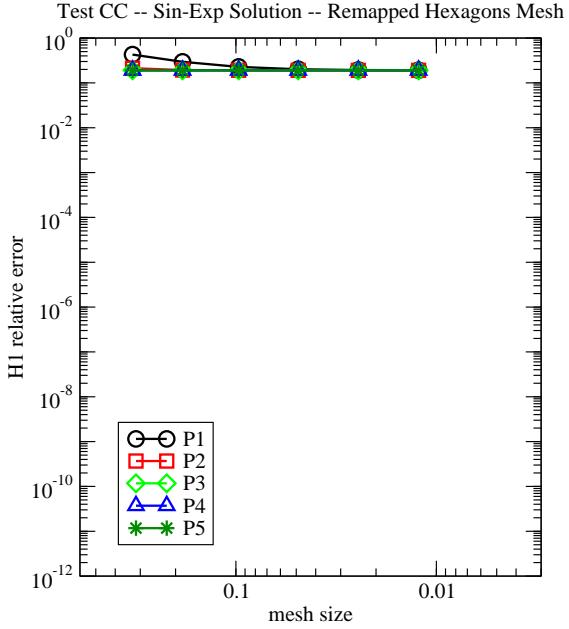
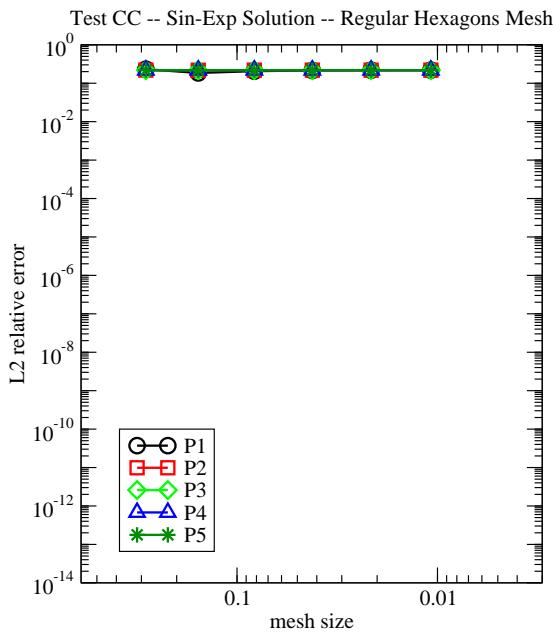


Fig. 316. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

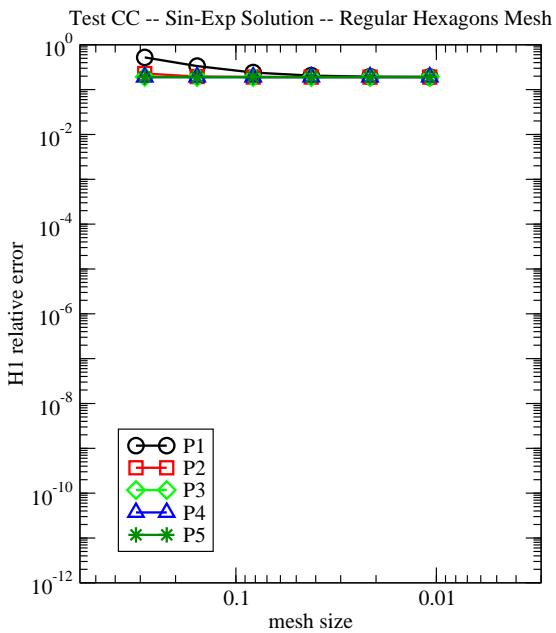


Fig. 317. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

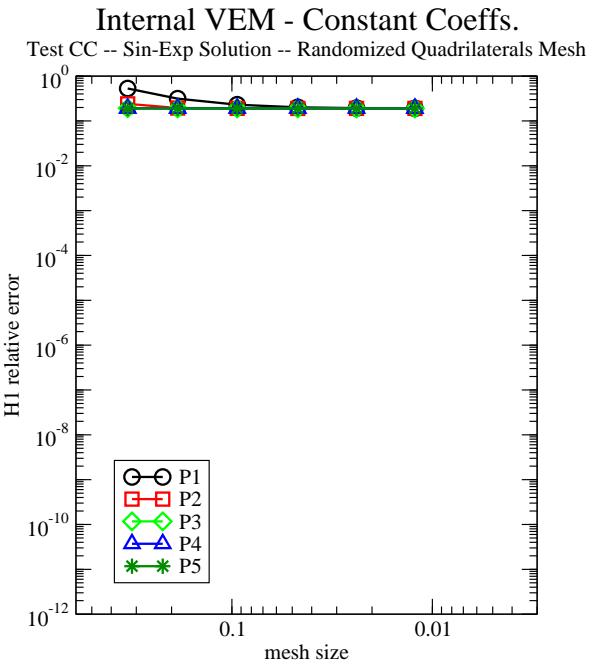
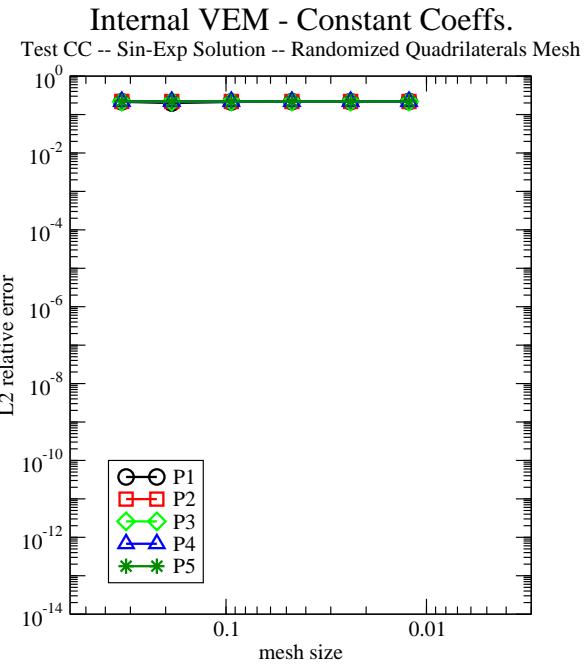


Fig. 318. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

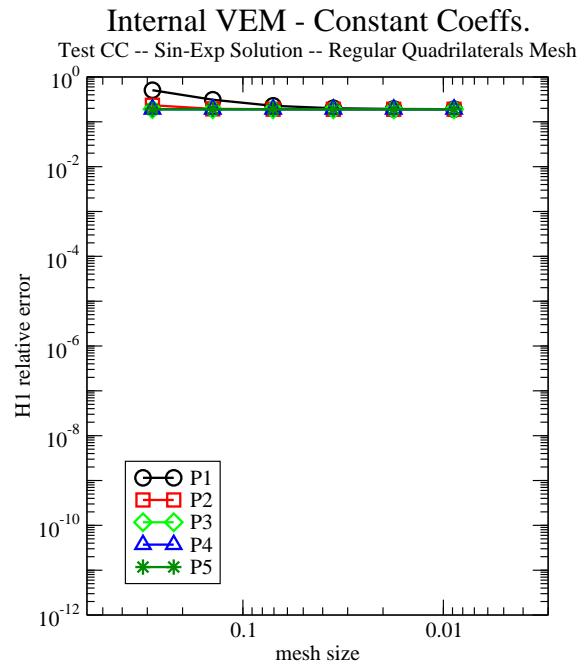
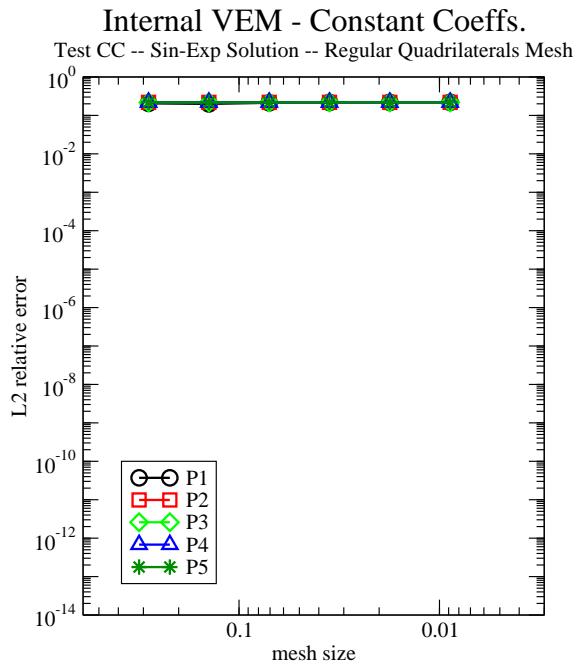


Fig. 319. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

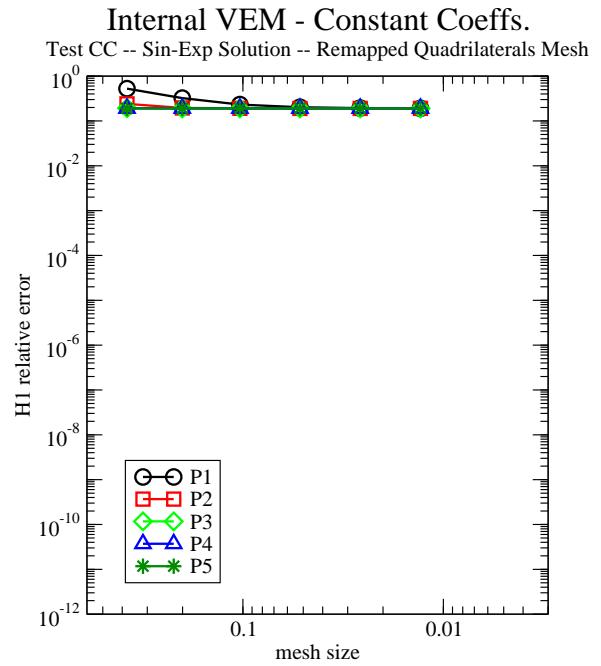
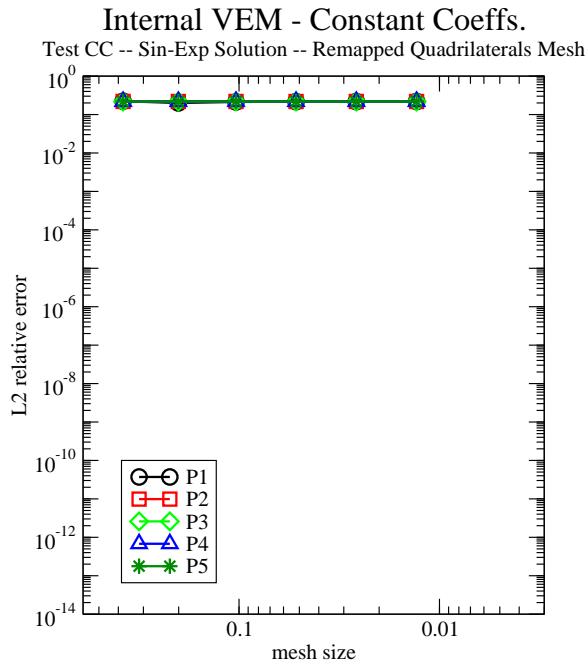


Fig. 320. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

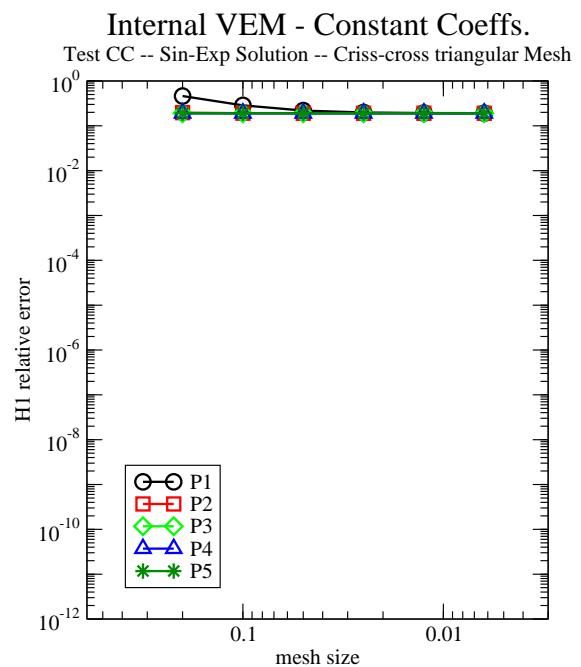
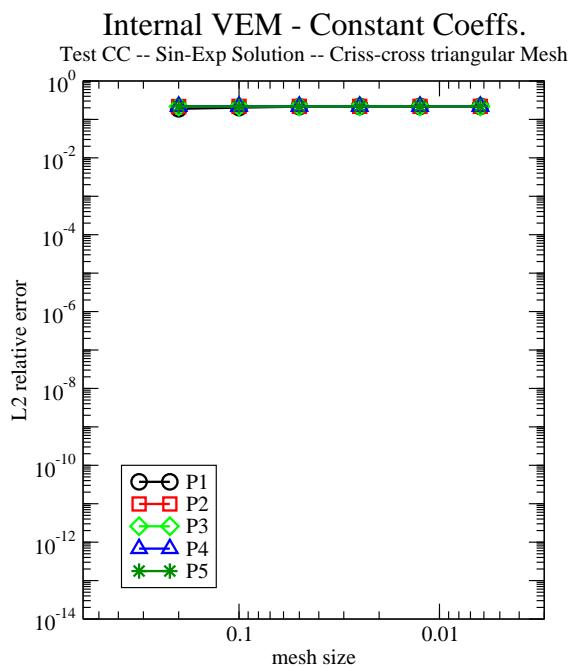


Fig. 321. Internal-internal VEM formulation with constant coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

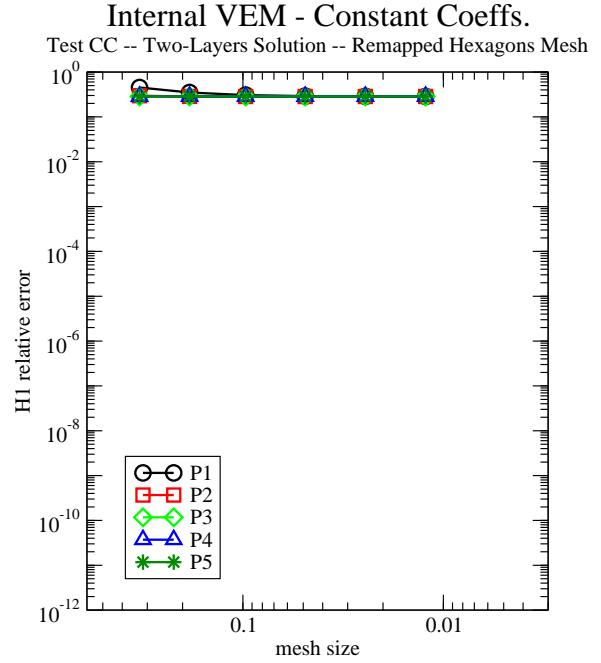
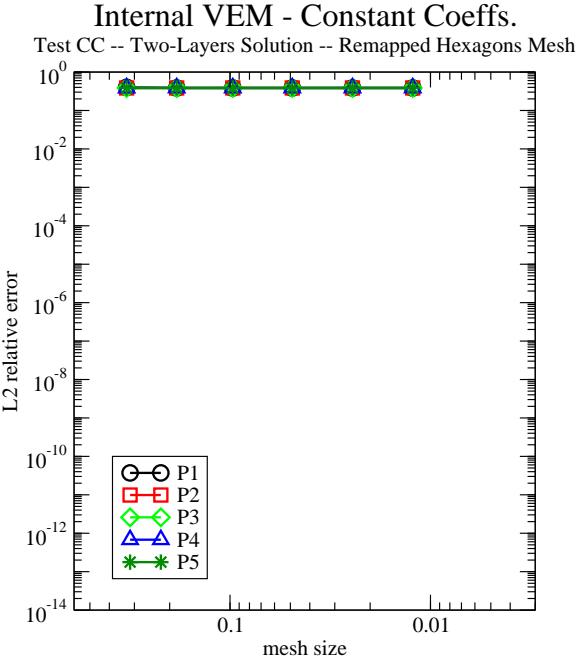


Fig. 322. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

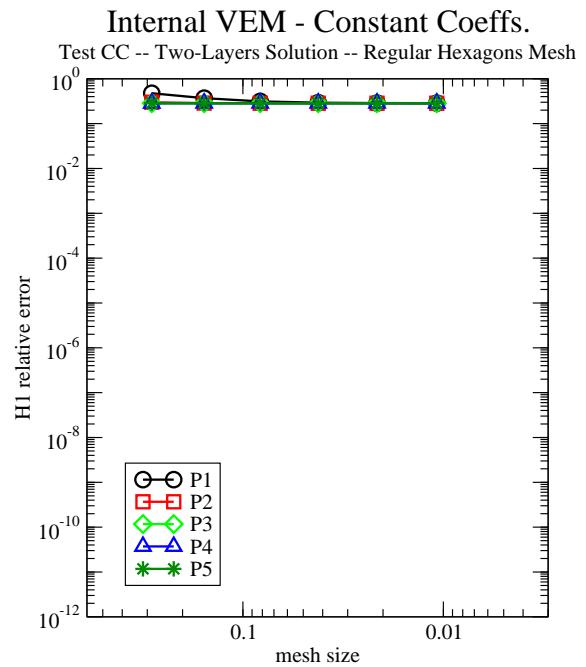
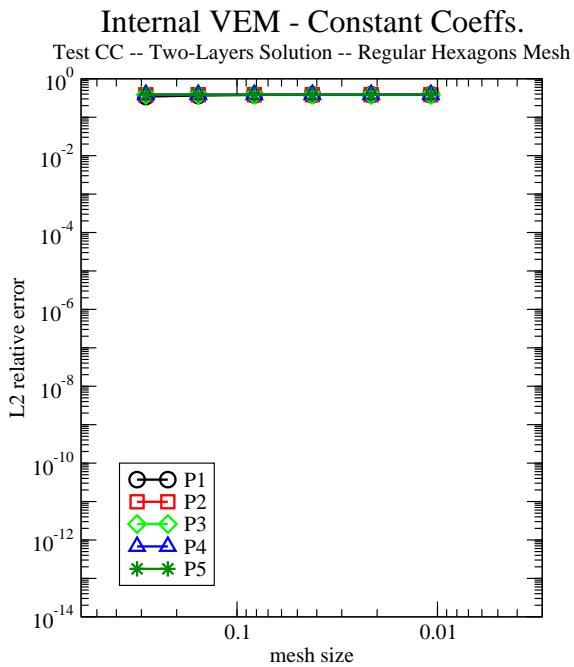


Fig. 323. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular hexagons.

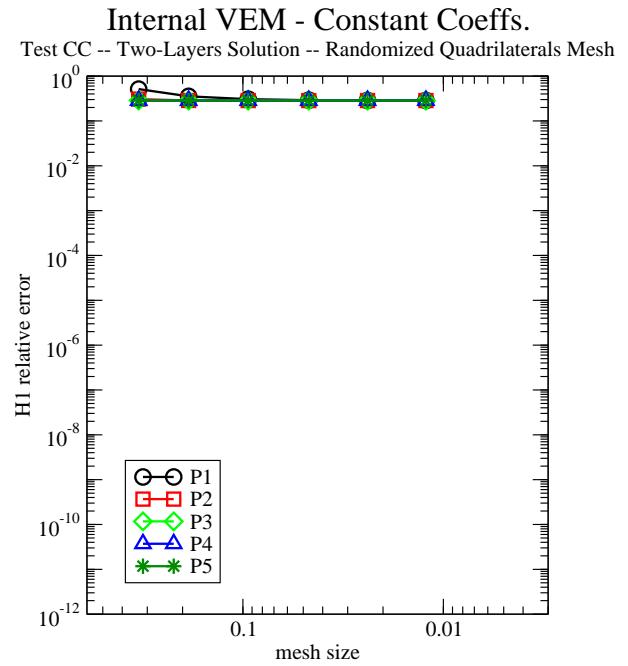
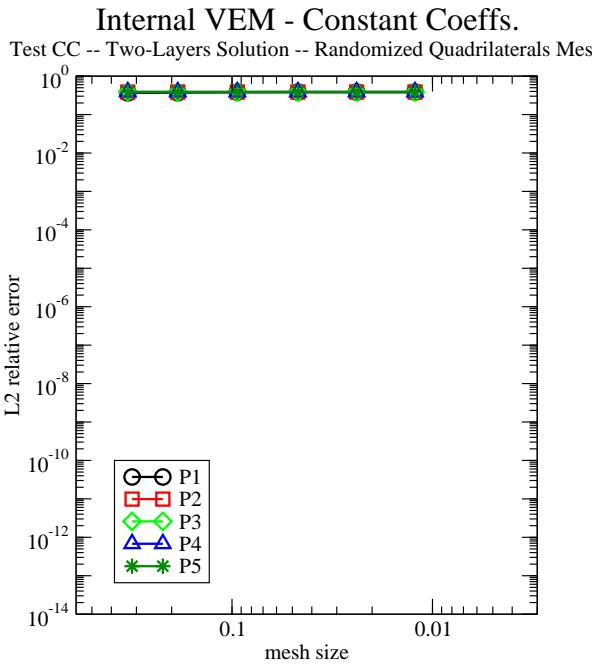


Fig. 324. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

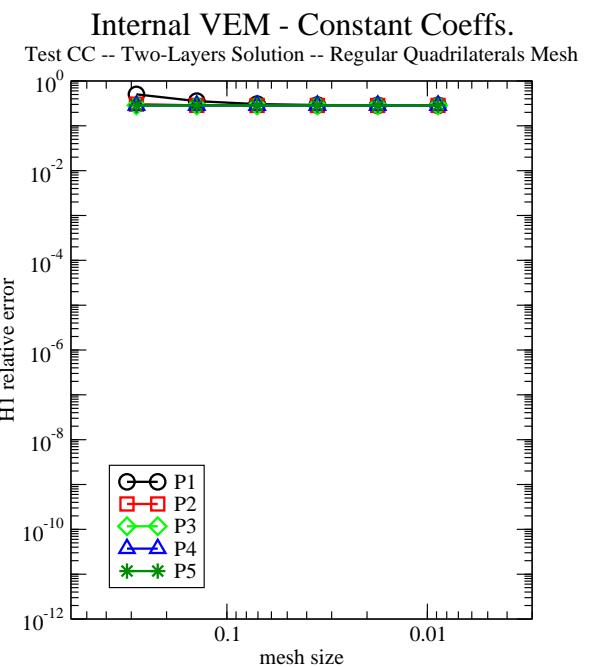
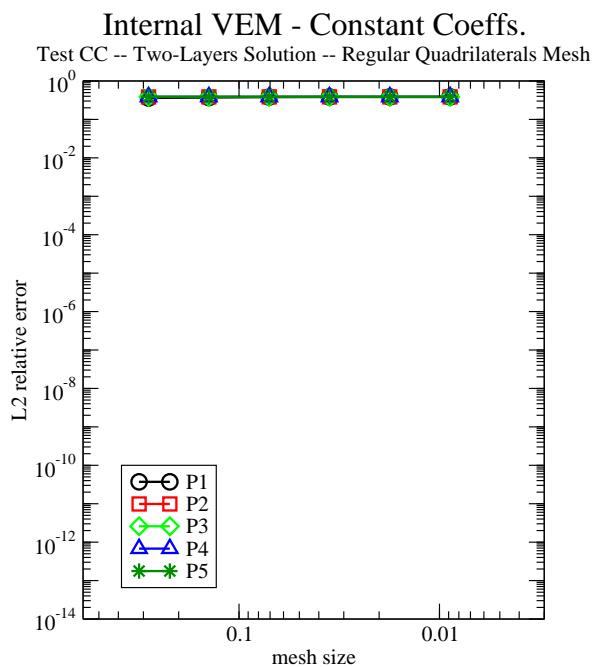
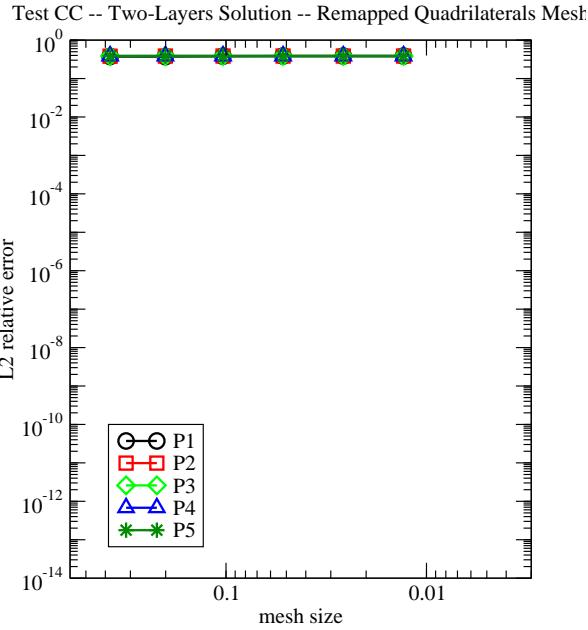


Fig. 325. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

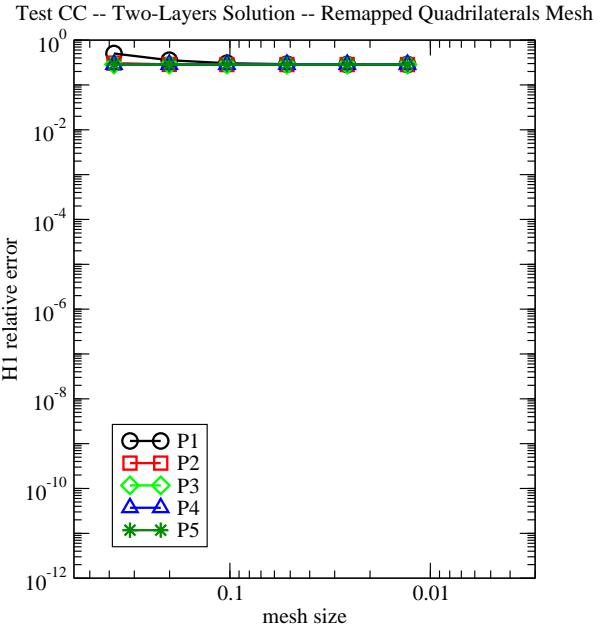
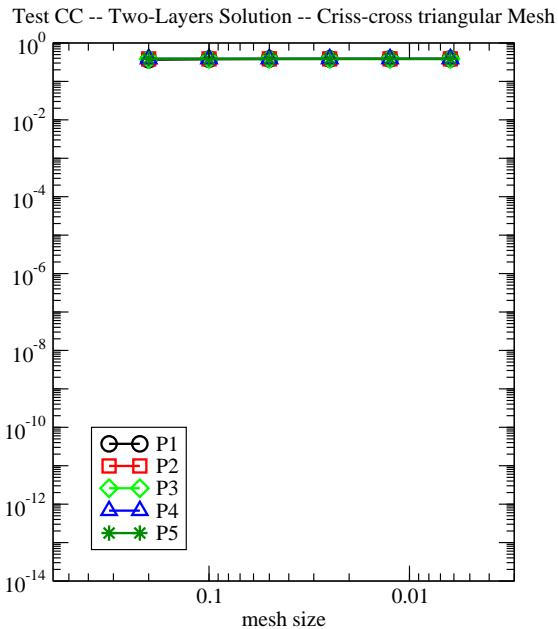


Fig. 326. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

### Internal VEM - Constant Coeffs.



### Internal VEM - Constant Coeffs.

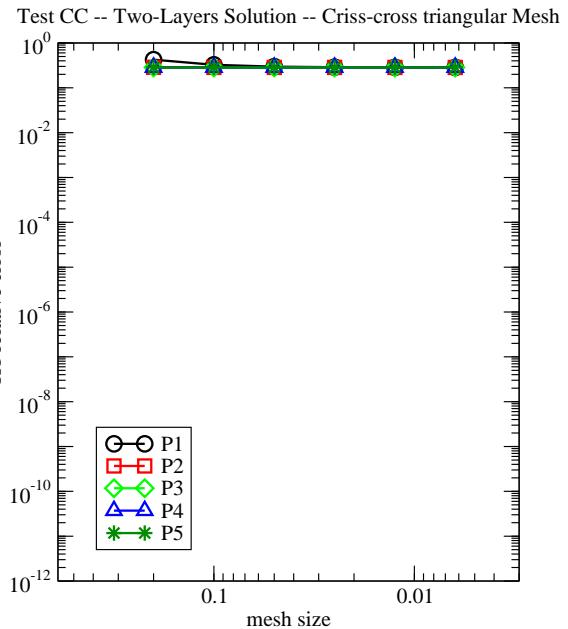
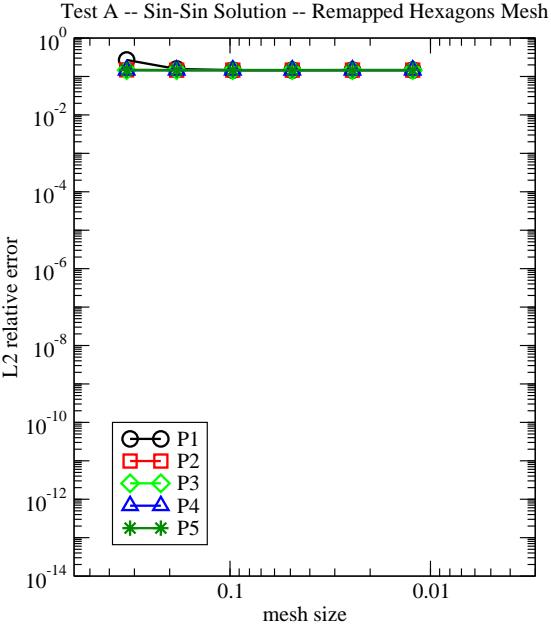


Fig. 327. Internal-internal VEM formulation with constant coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Variable Coeffs.



### Internal VEM - Variable Coeffs.

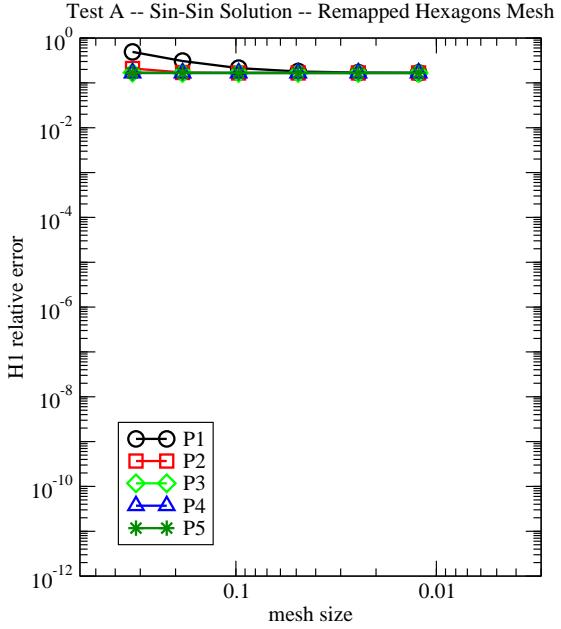
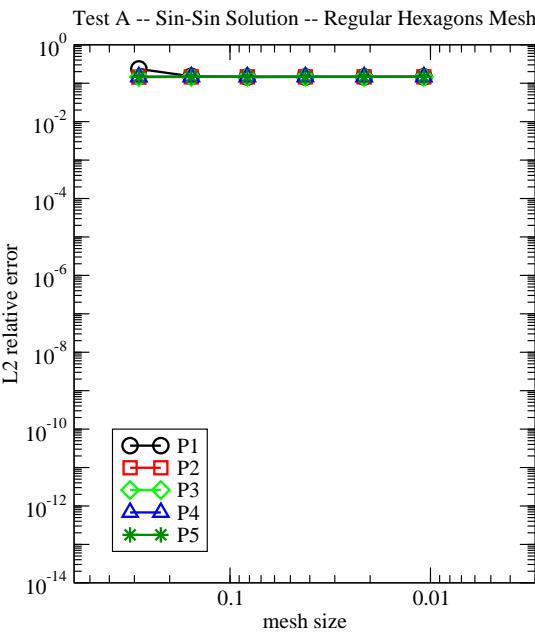


Fig. 328. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Variable Coeffs.



### Internal VEM - Variable Coeffs.

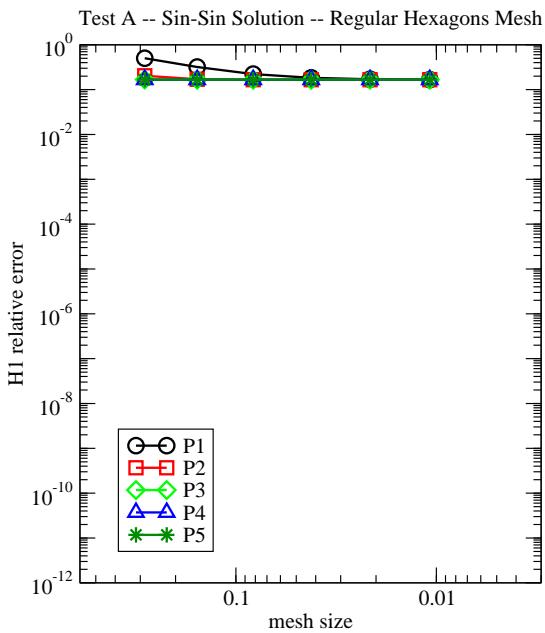


Fig. 329. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular hexagons.

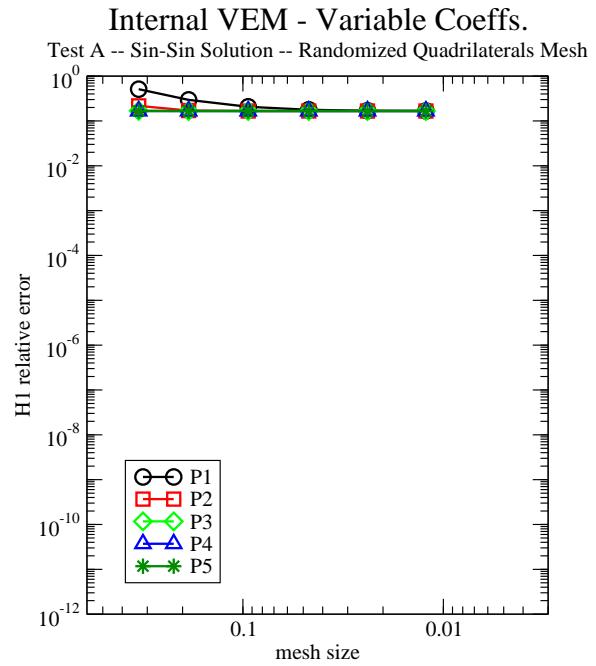
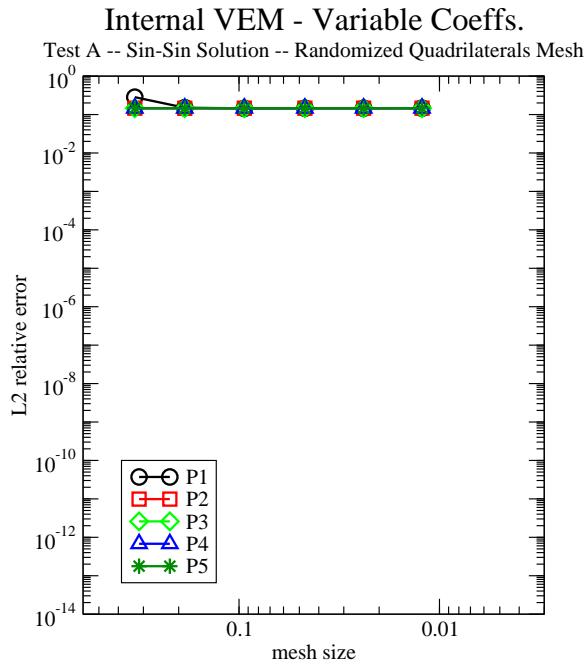


Fig. 330. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of randomized quadrilateral cells.

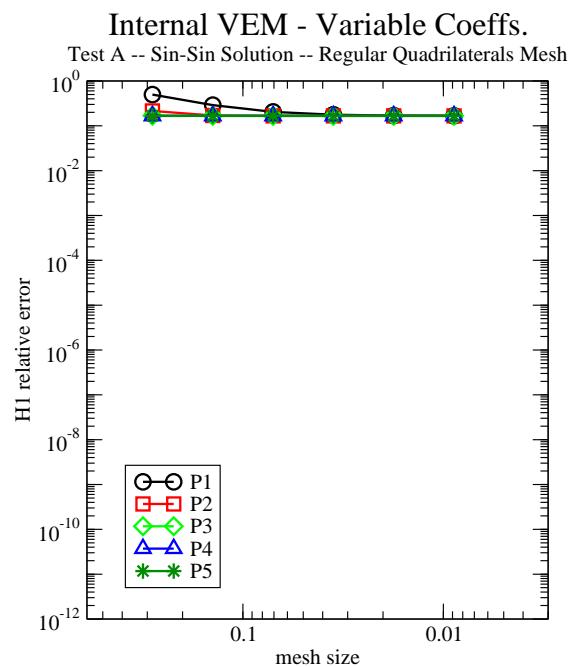
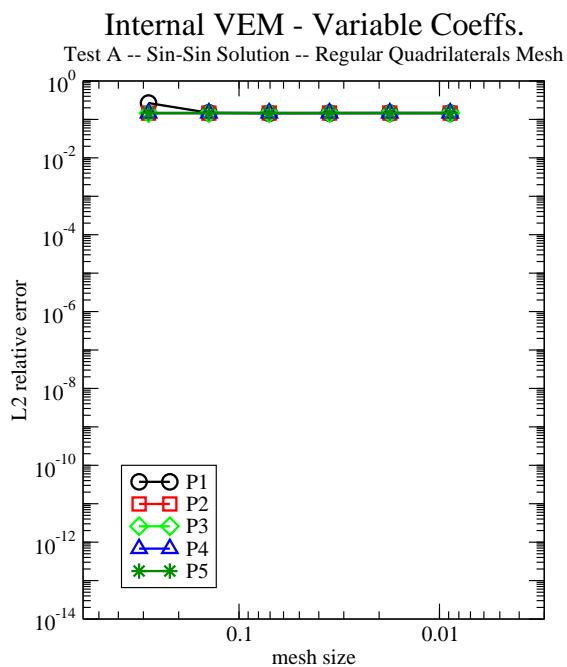


Fig. 331. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular quadrilateral cells (squares).

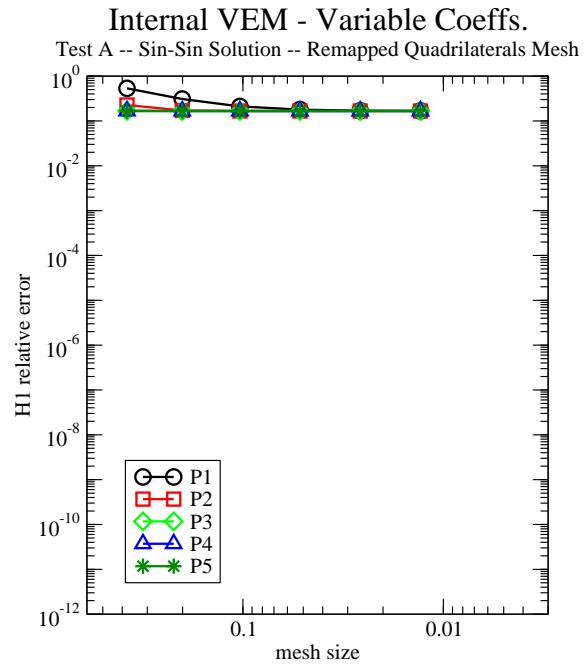
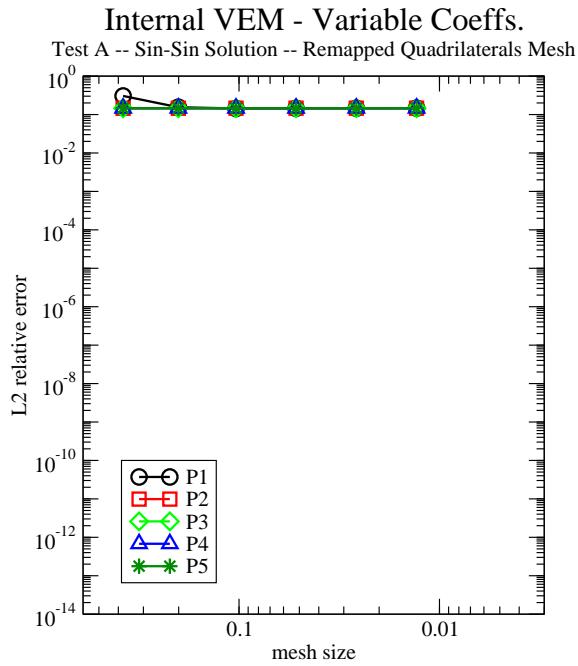


Fig. 332. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

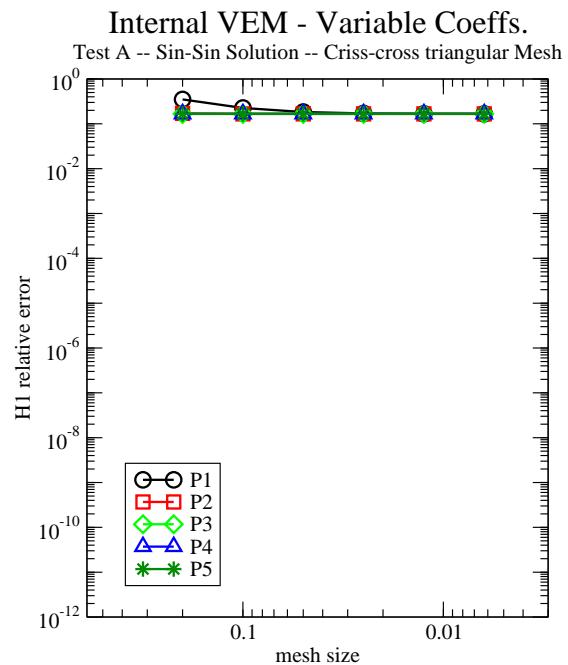
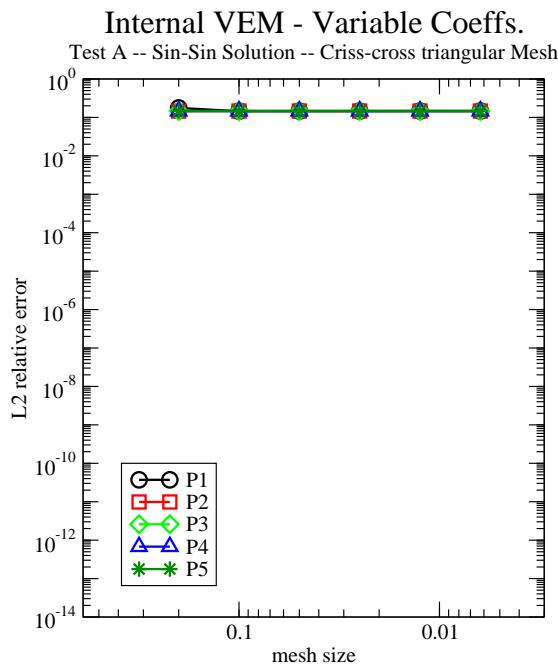
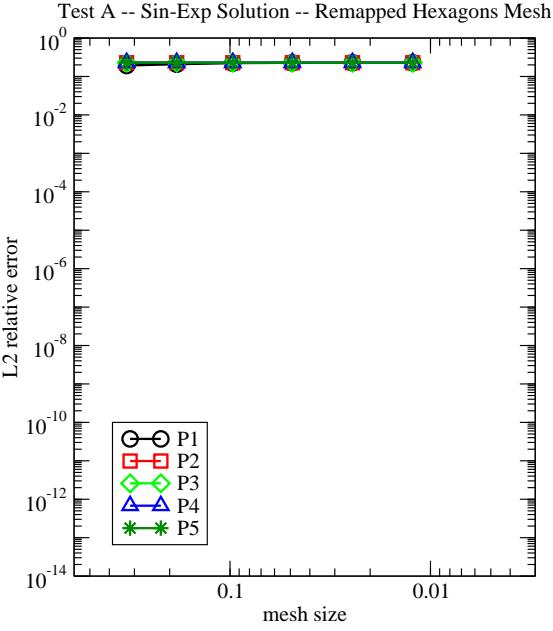


Fig. 333. Internal-internal VEM formulation with variable coefficients; Test A; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

### Internal VEM - Variable Coeffs.



### Internal VEM - Variable Coeffs.

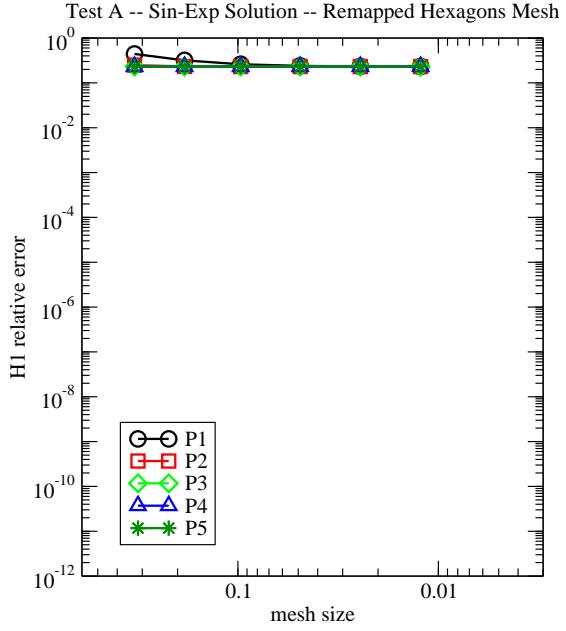
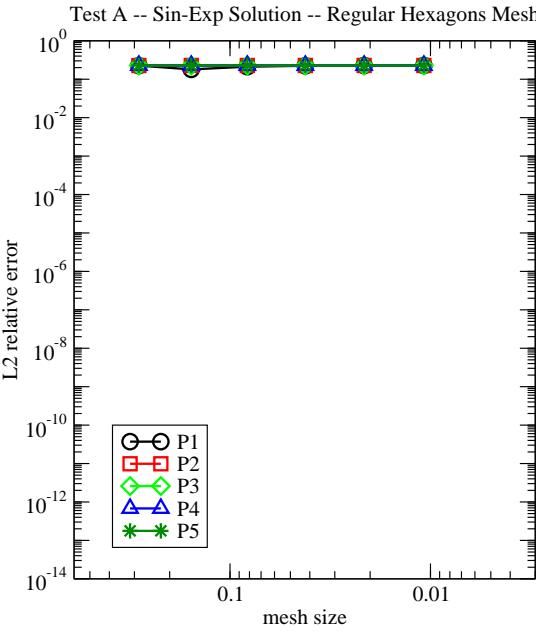


Fig. 334. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped hexagons.

### Internal VEM - Variable Coeffs.



### Internal VEM - Variable Coeffs.

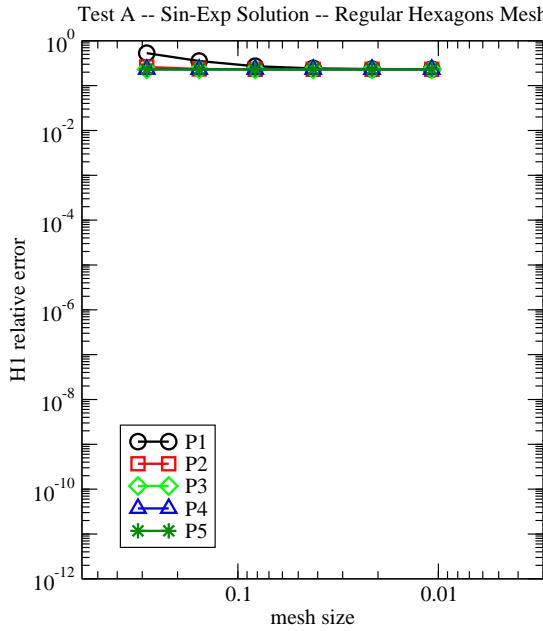


Fig. 335. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular hexagons.

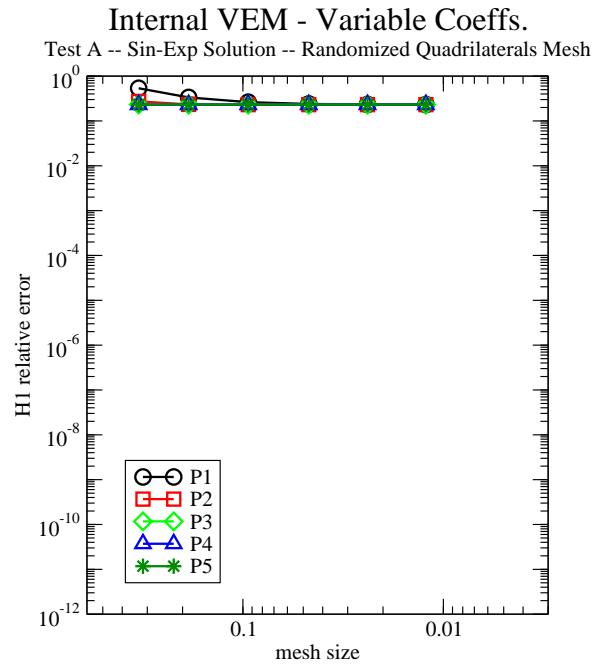
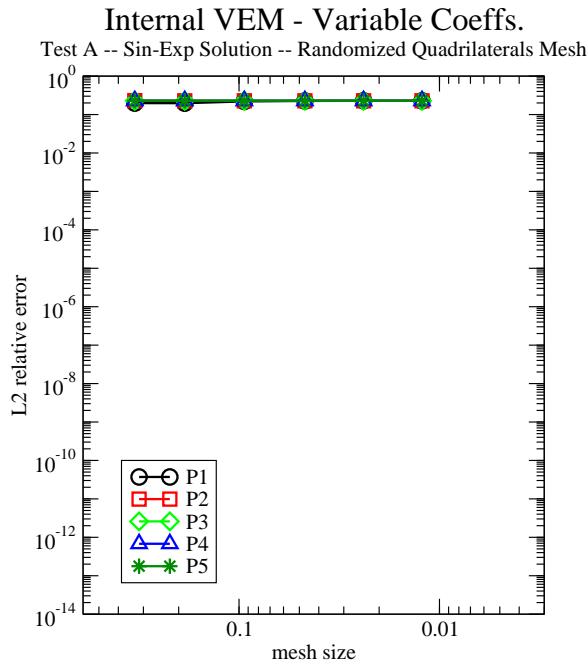


Fig. 336. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of randomized quadrilateral cells.

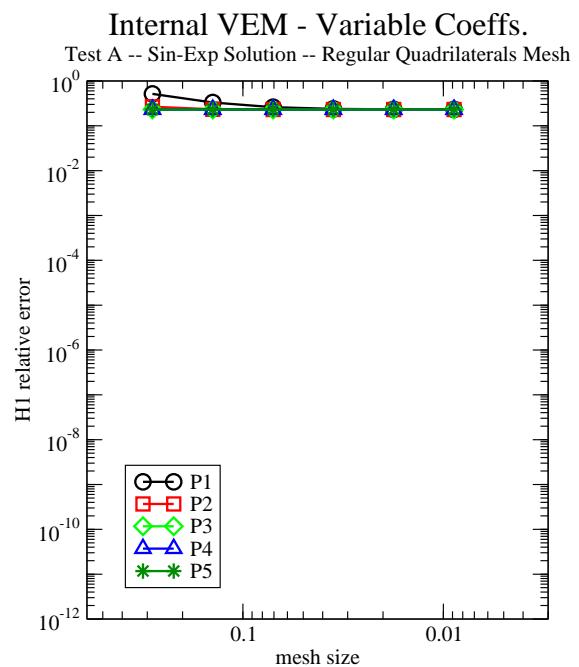
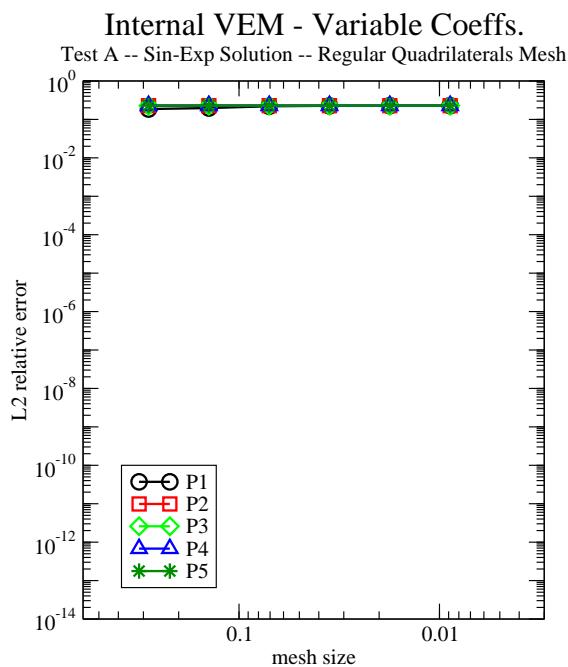


Fig. 337. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular quadrilateral cells (squares).

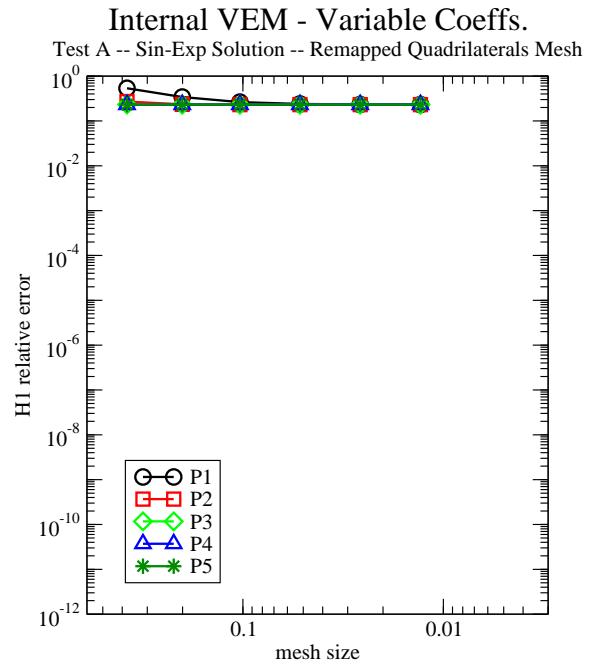
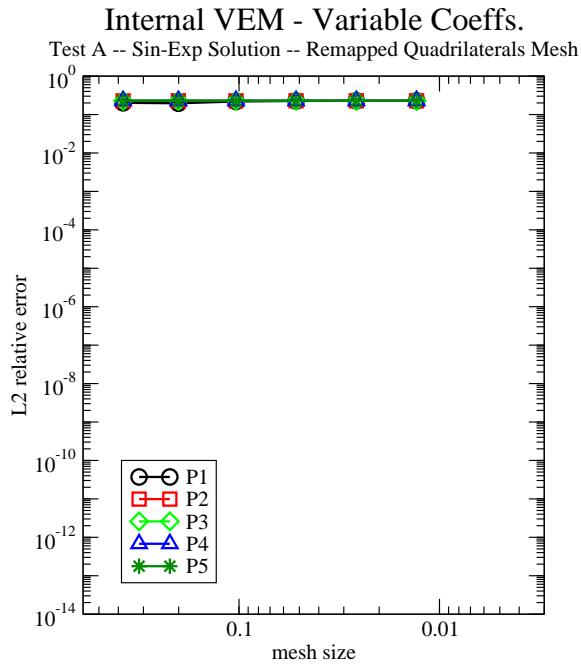


Fig. 338. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

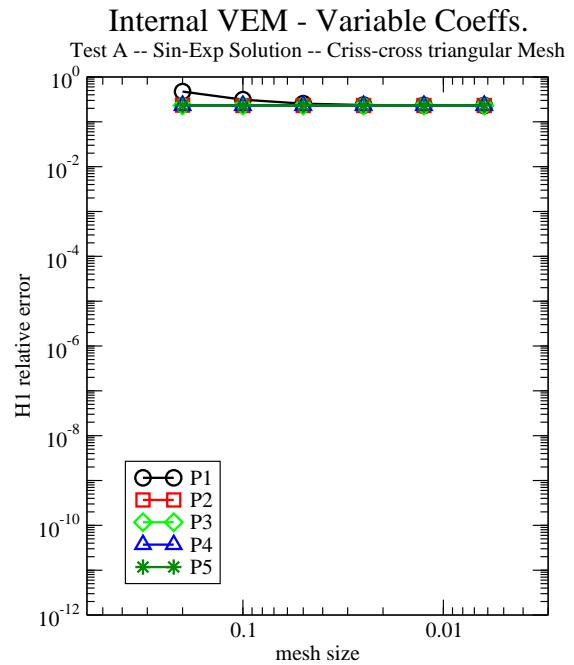
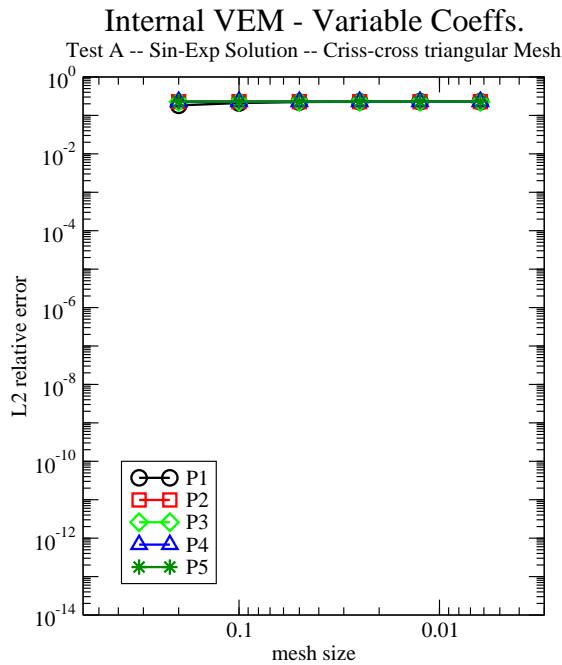


Fig. 339. Internal-internal VEM formulation with variable coefficients; Test A; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

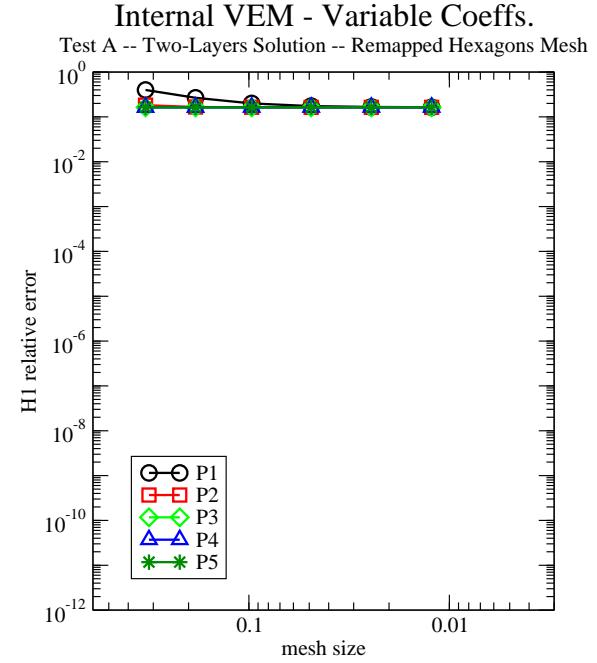
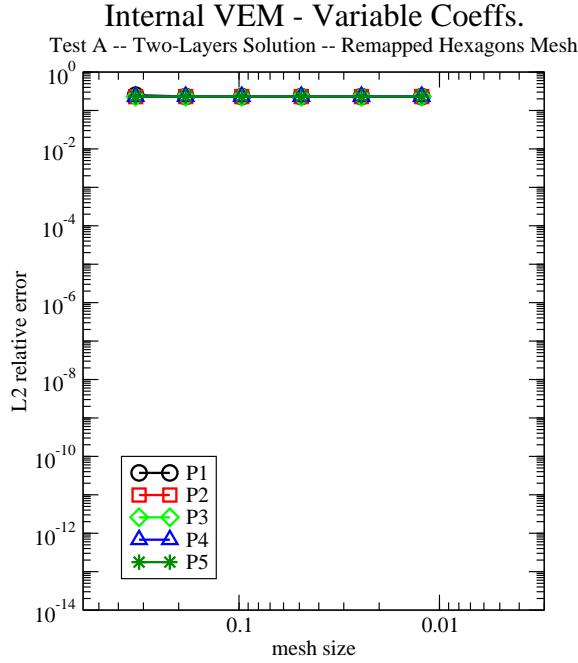


Fig. 340. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped hexagons.

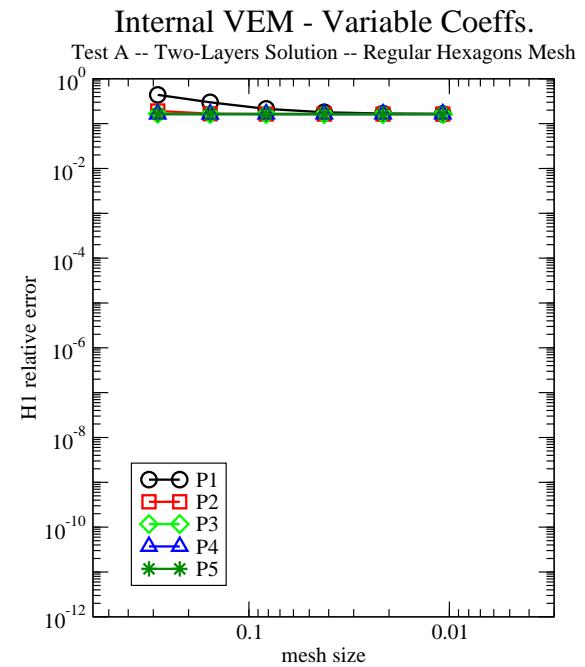
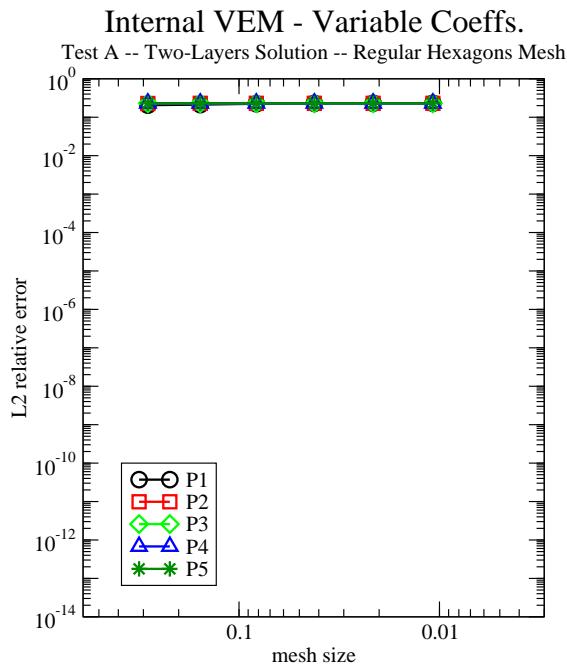


Fig. 341. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular hexagons.

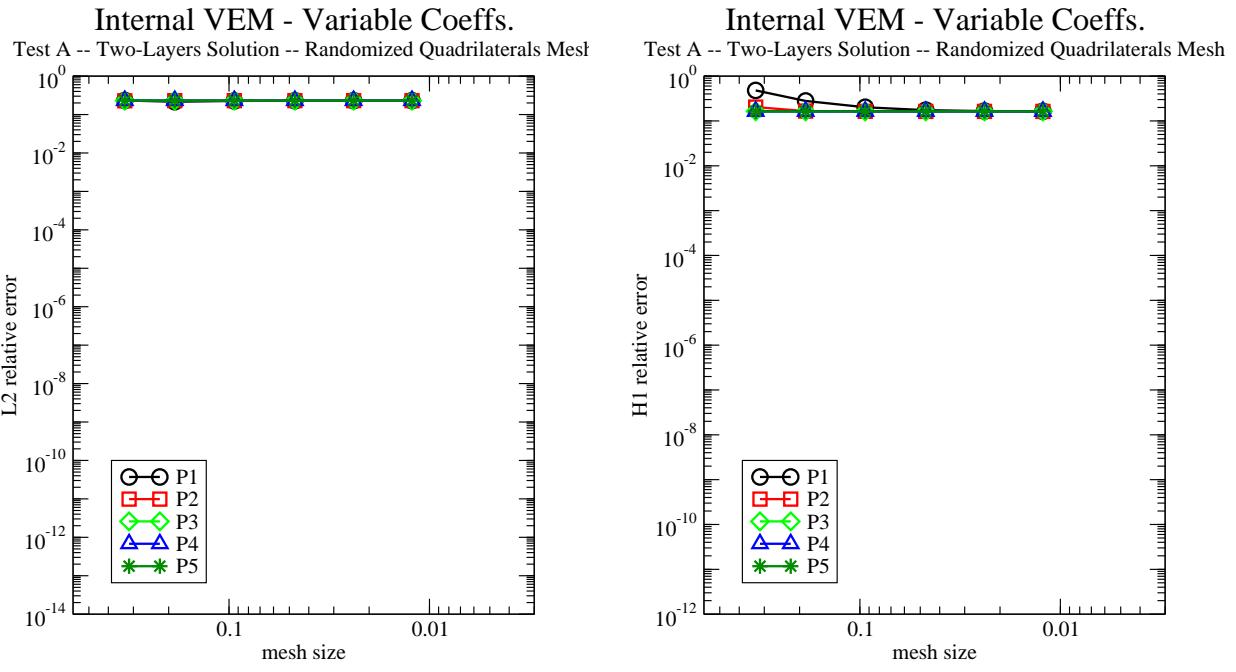


Fig. 342. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of randomized quadrilateral cells.

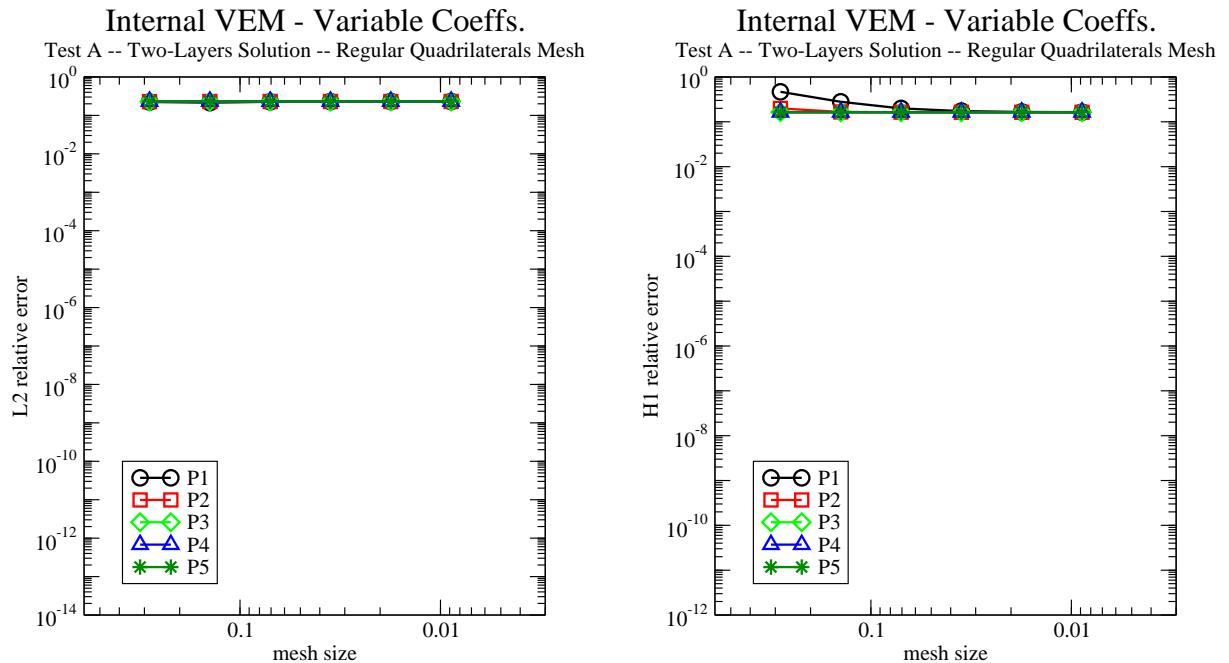


Fig. 343. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular quadrilateral cells (squares).

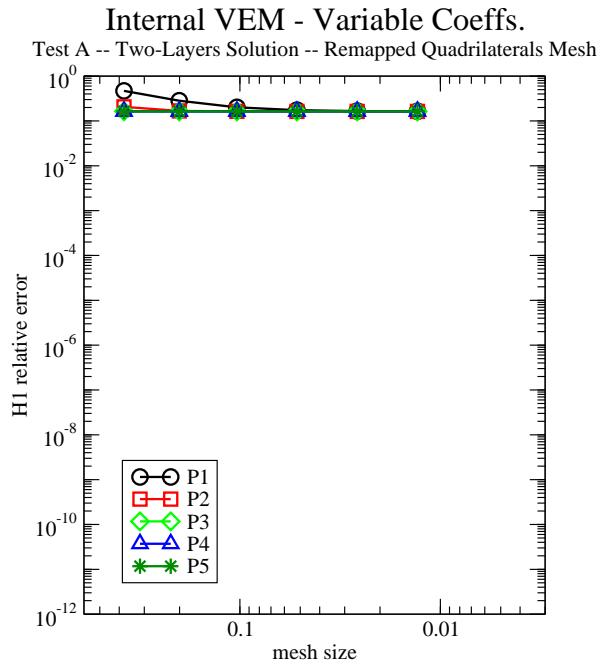
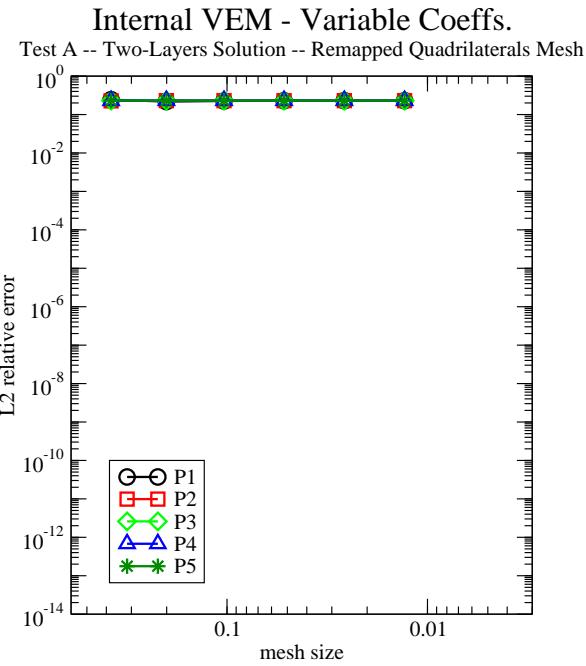


Fig. 344. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

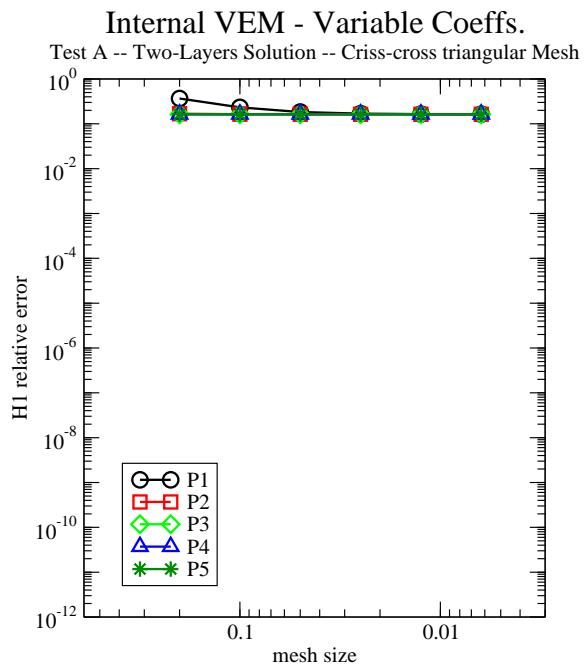
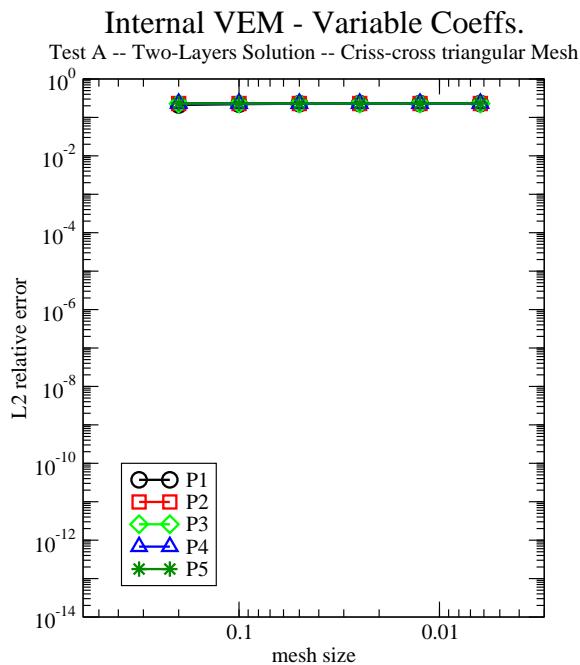


Fig. 345. Internal-internal VEM formulation with variable coefficients; Test A; two-layer solution on a mesh of regular triangular cells, (criss-cross).

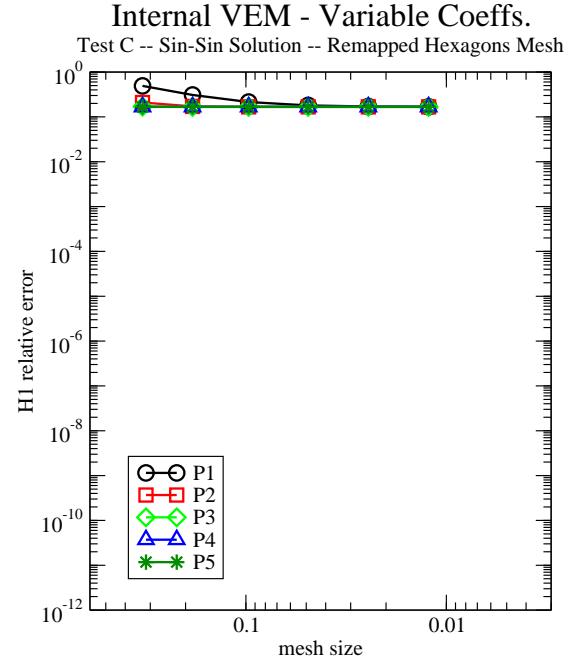
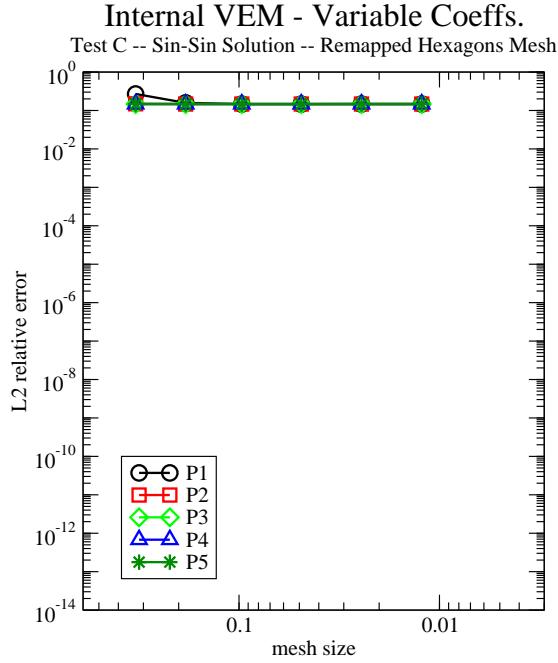


Fig. 346. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped hexagons.

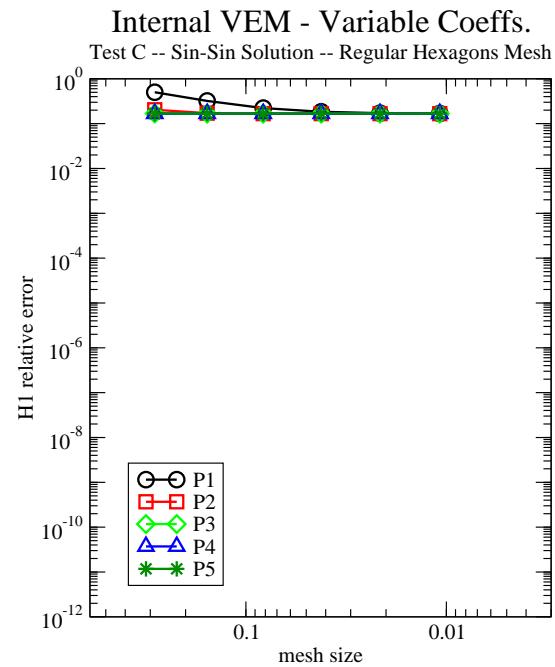
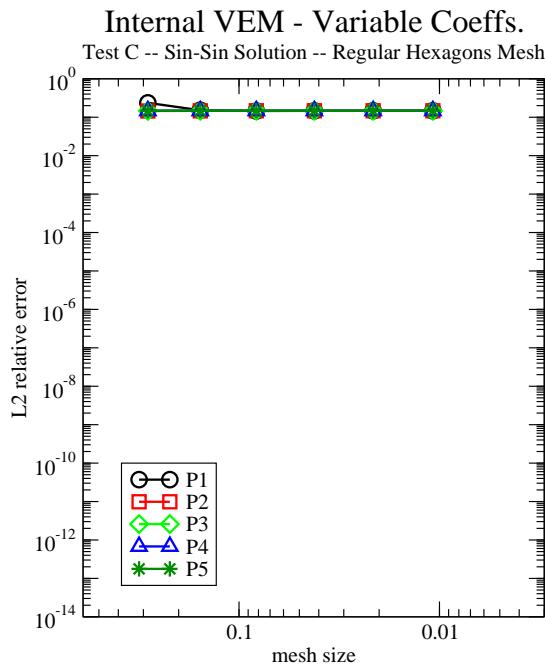


Fig. 347. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular hexagons.

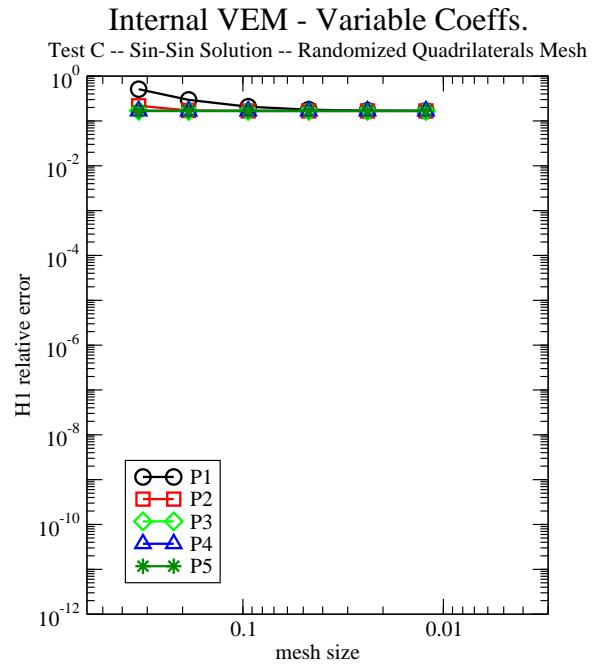
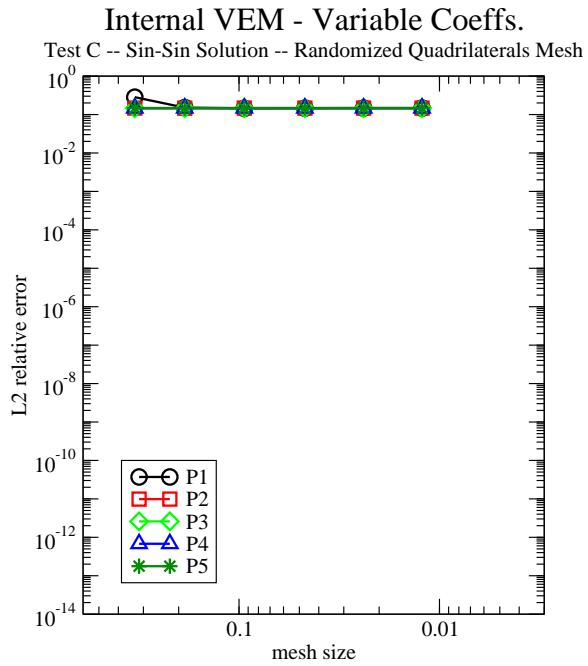


Fig. 348. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of randomized quadrilateral cells.

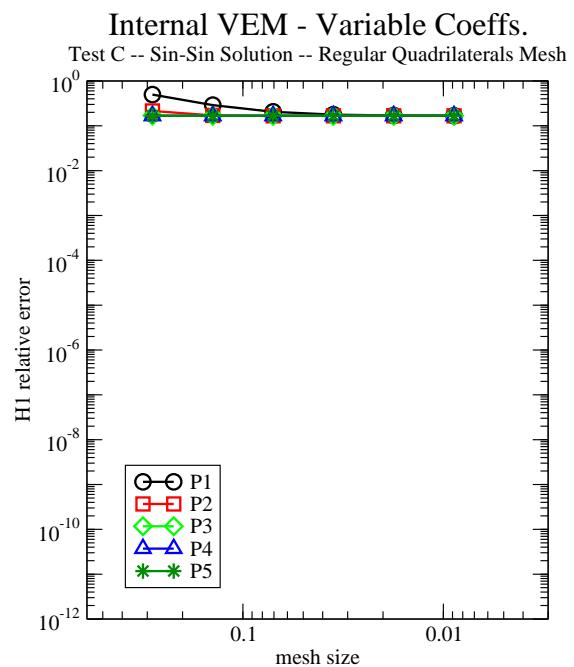
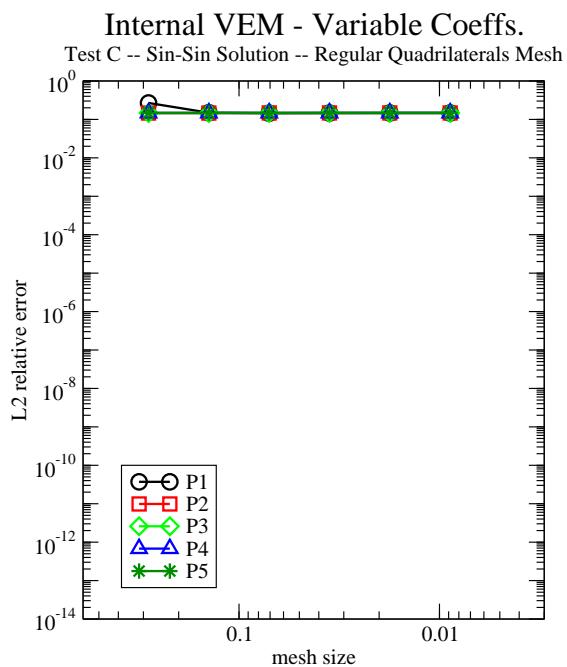


Fig. 349. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular quadrilateral cells (squares).

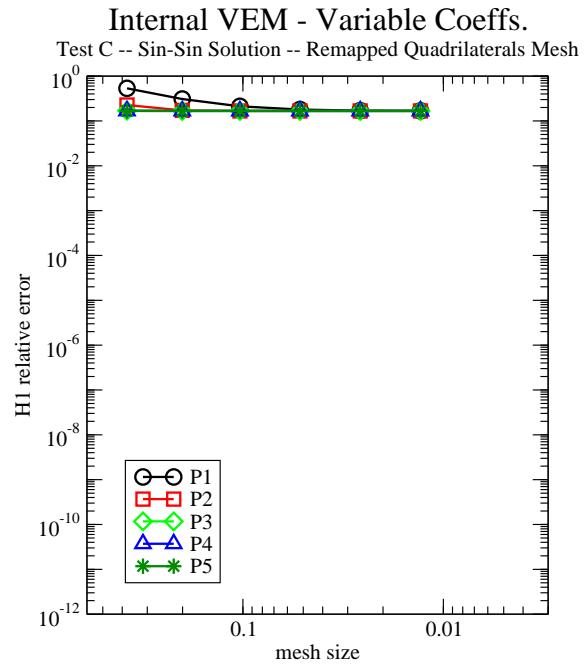
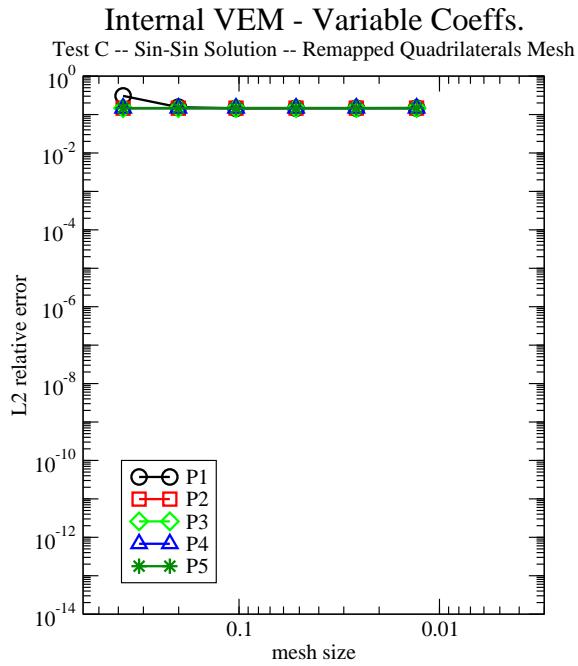


Fig. 350. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of smoothly remapped quadrilaterals cells.

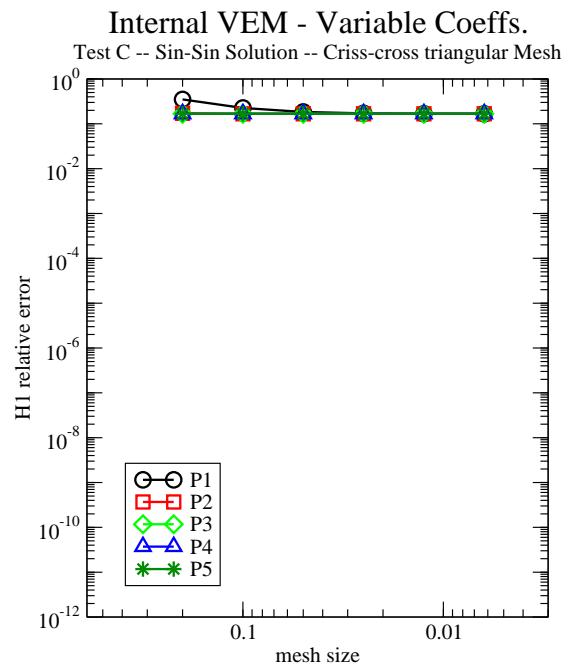
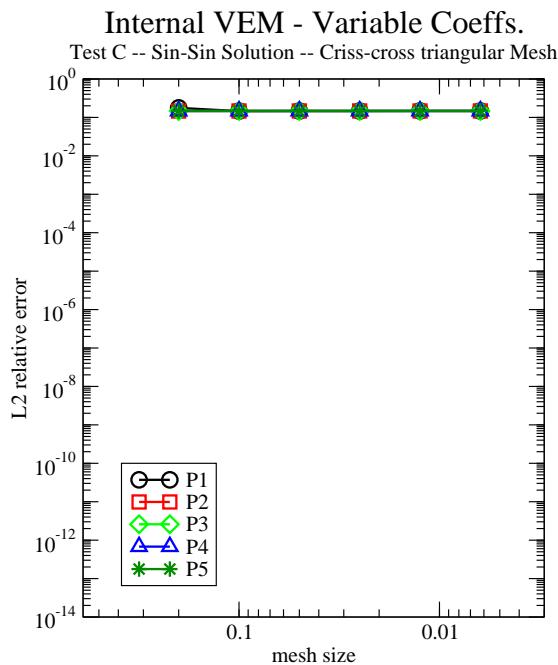


Fig. 351. Internal-internal VEM formulation with variable coefficients; Test C; sin-sin solution on a mesh of regular triangular cells, (criss-cross).

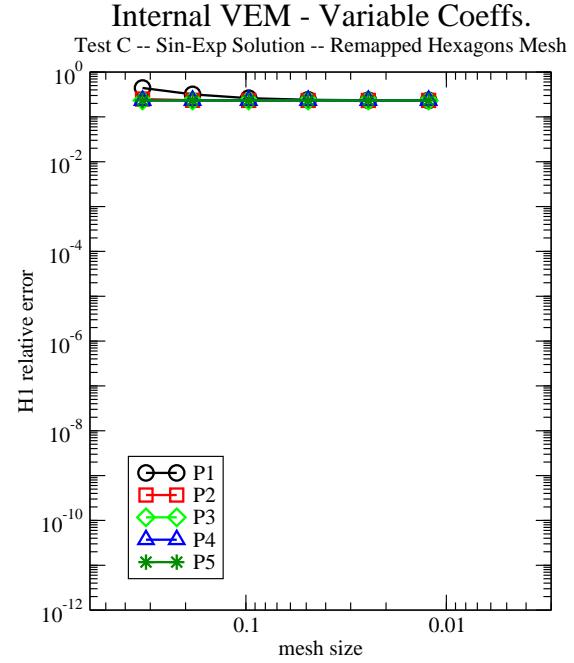
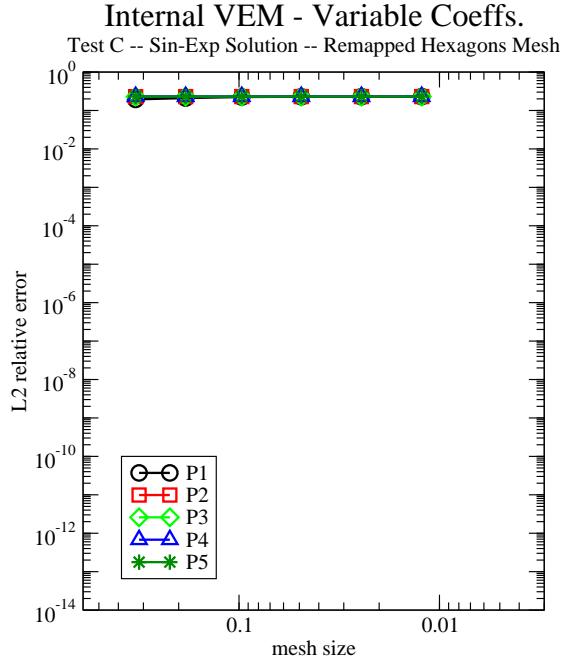


Fig. 352. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped hexagons.

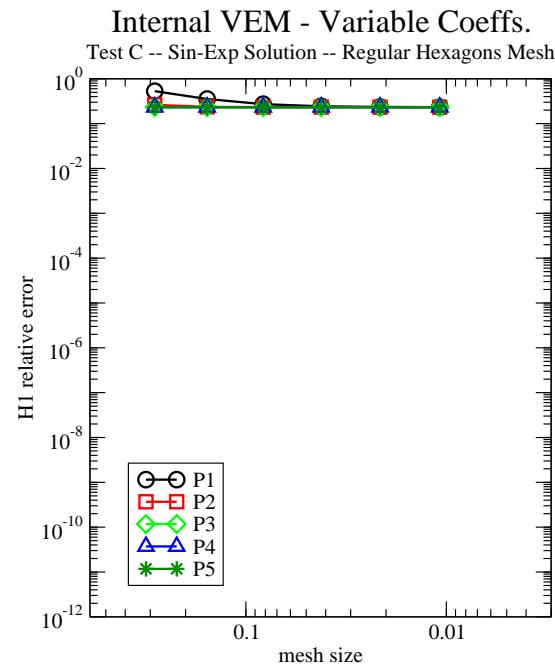
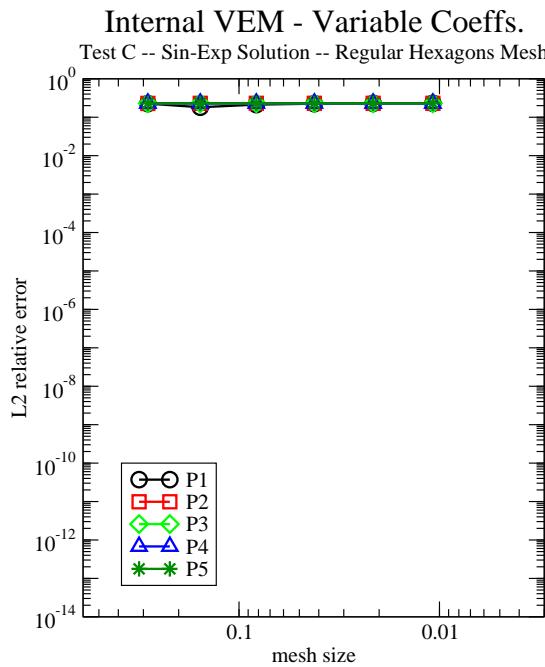


Fig. 353. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular hexagons.

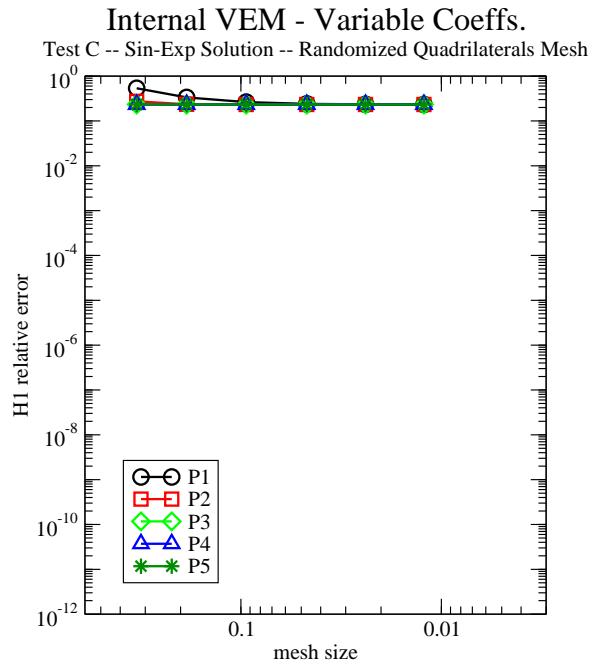
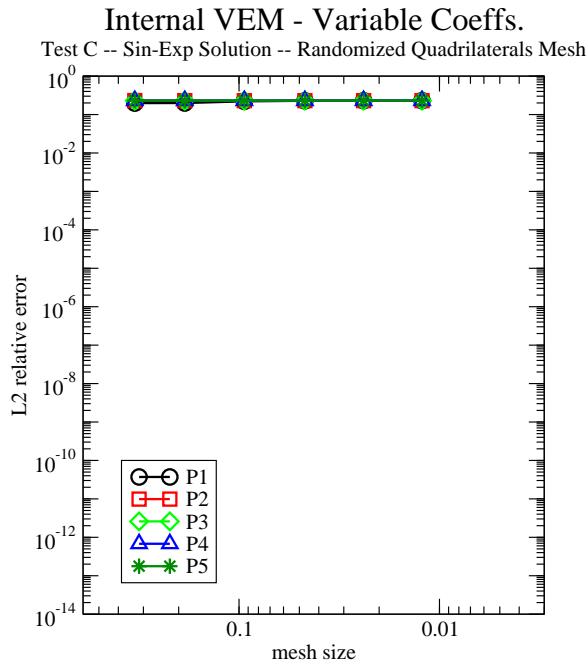


Fig. 354. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of randomized quadrilateral cells.

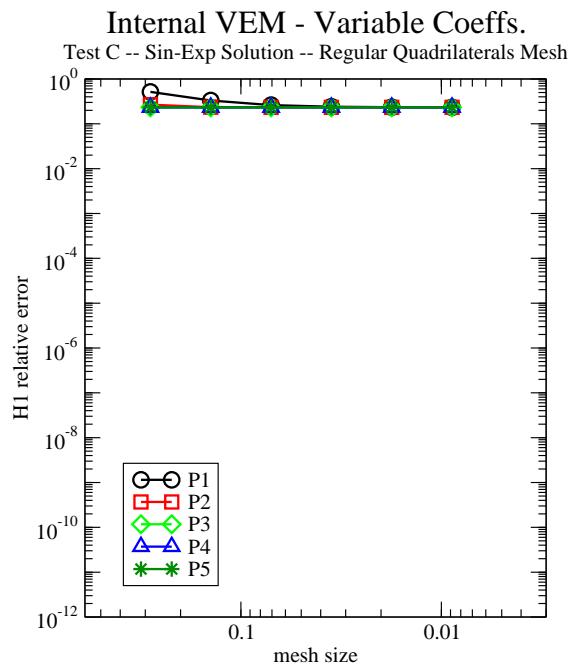
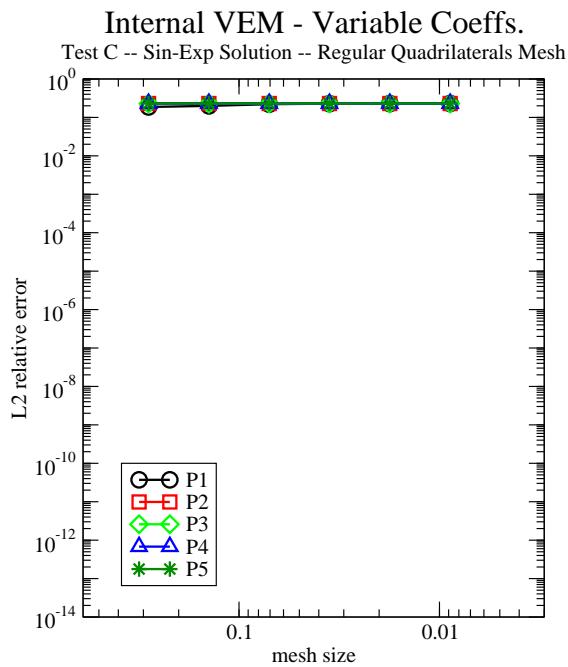


Fig. 355. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular quadrilateral cells (squares).

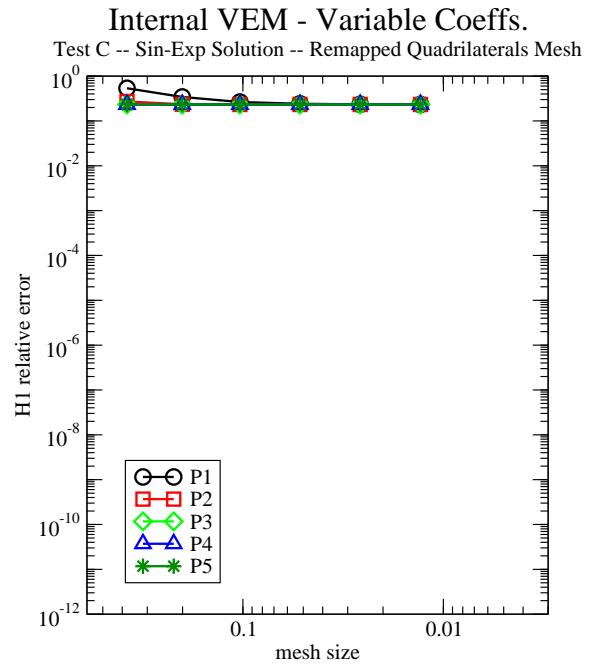
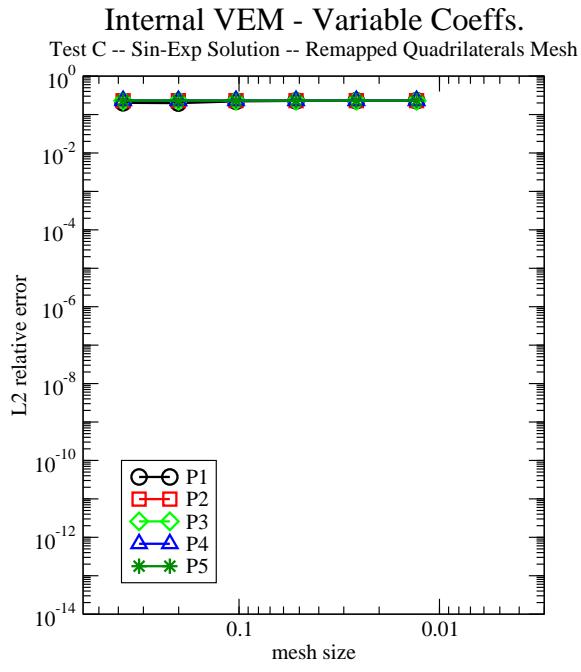


Fig. 356. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of smoothly remapped quadrilaterals cells.

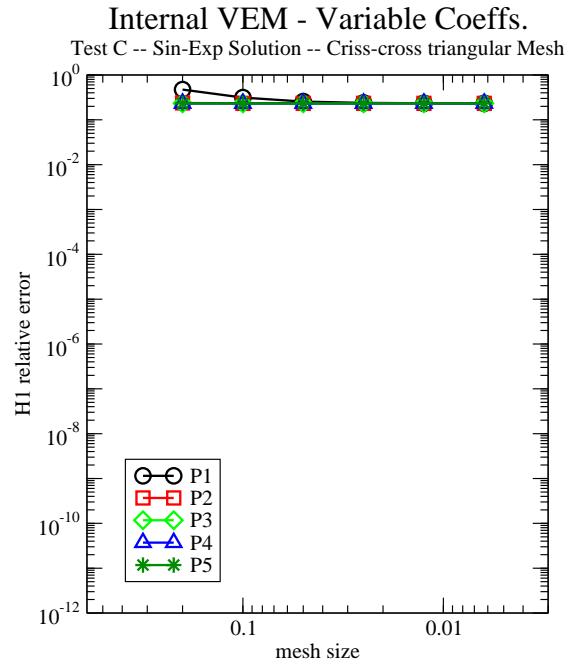
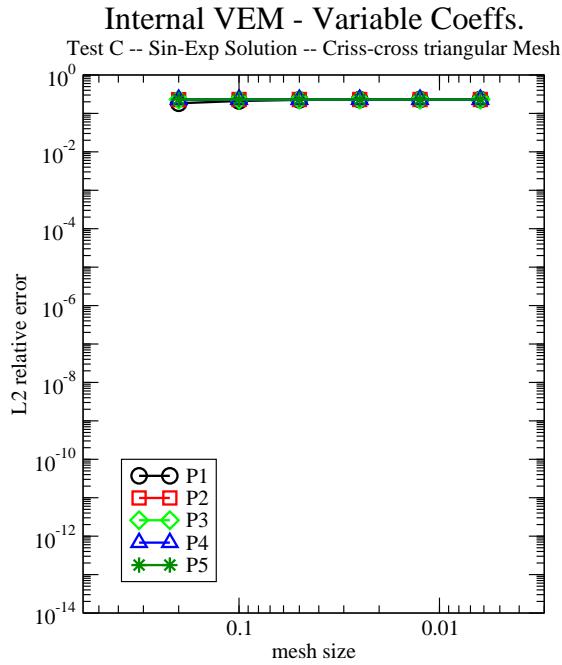


Fig. 357. Internal-internal VEM formulation with variable coefficients; Test C; sin-exp solution on a mesh of regular triangular cells, (criss-cross).

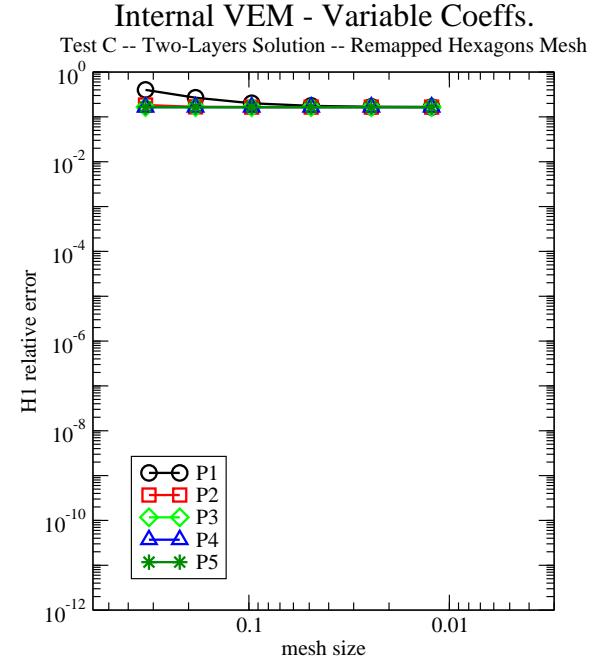
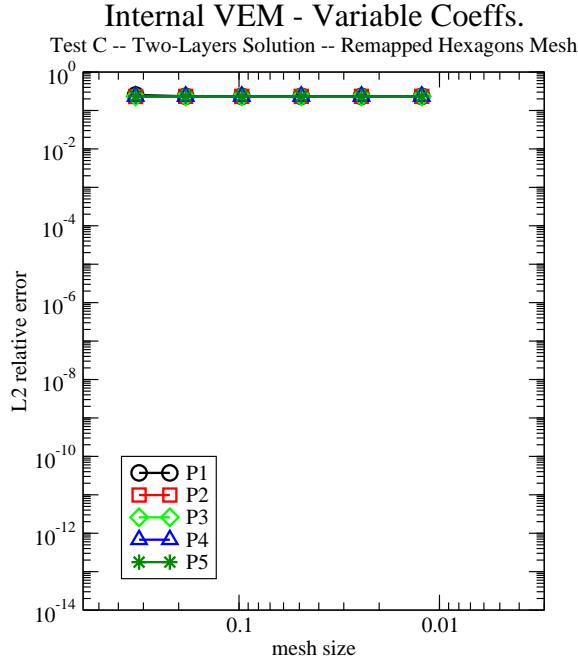


Fig. 358. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped hexagons.

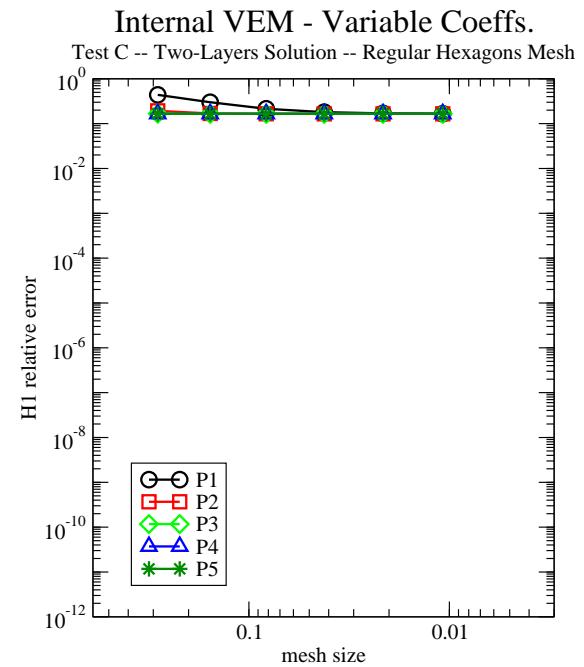
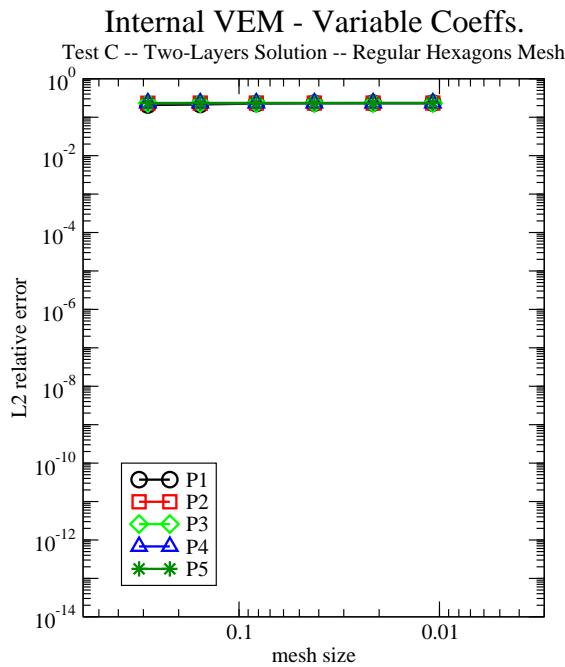


Fig. 359. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular hexagons.

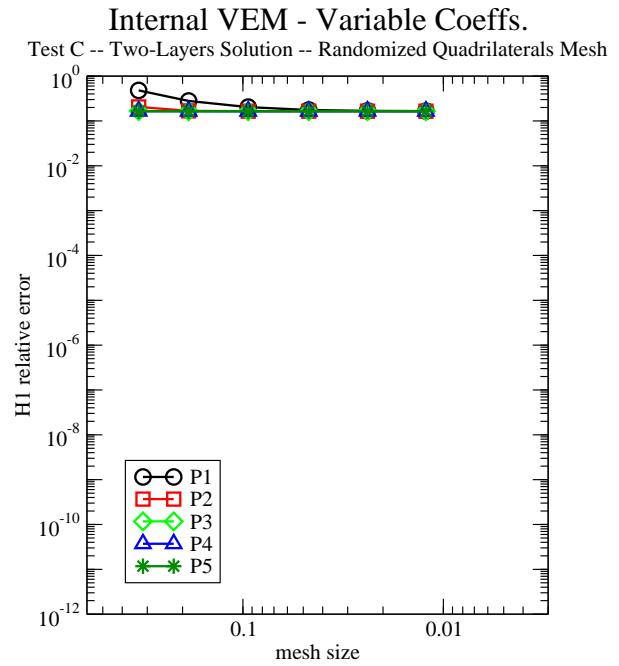
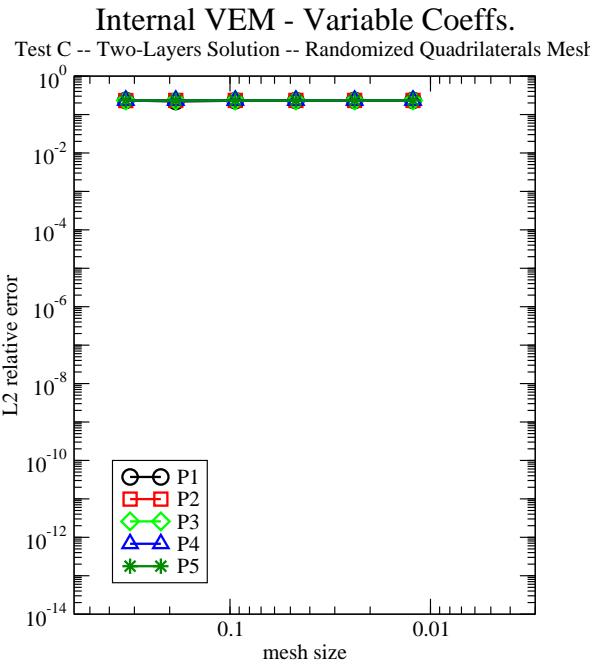


Fig. 360. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of randomized quadrilateral cells.

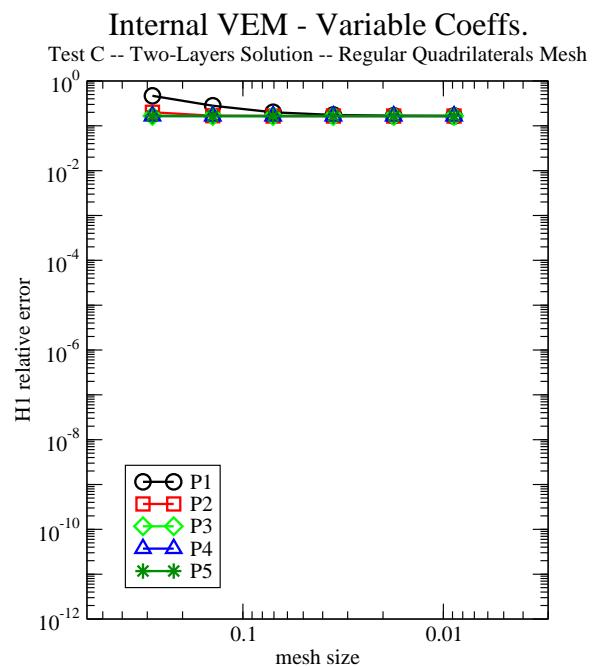
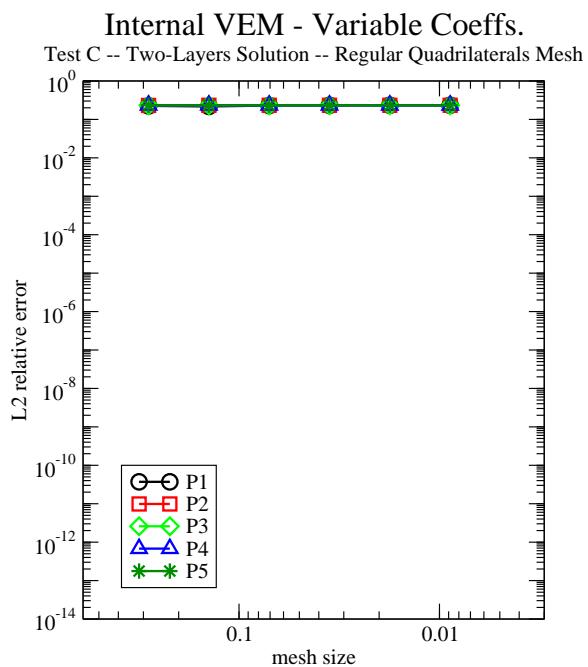


Fig. 361. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular quadrilateral cells (squares).

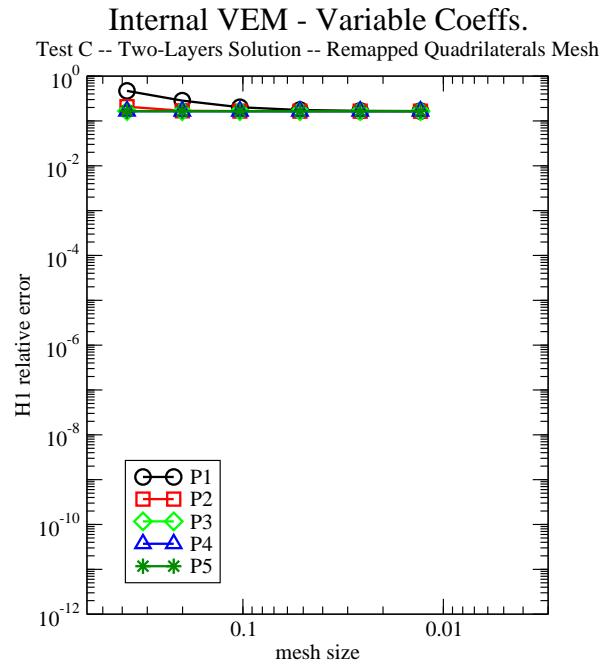
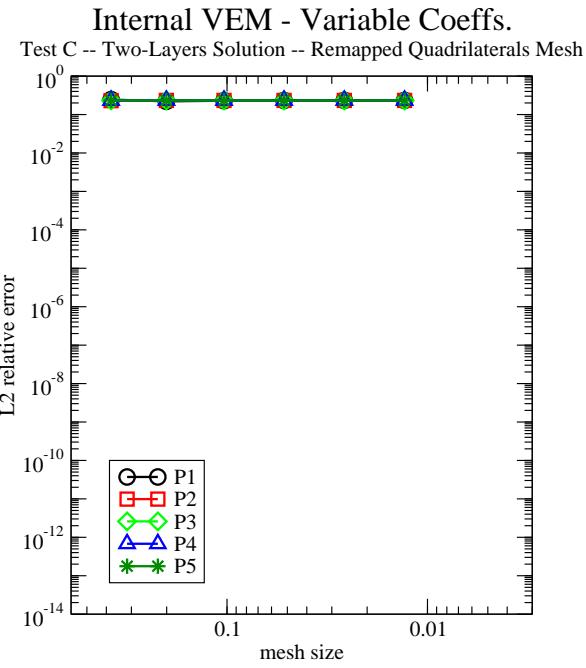


Fig. 362. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of smoothly remapped quadrilaterals cells.

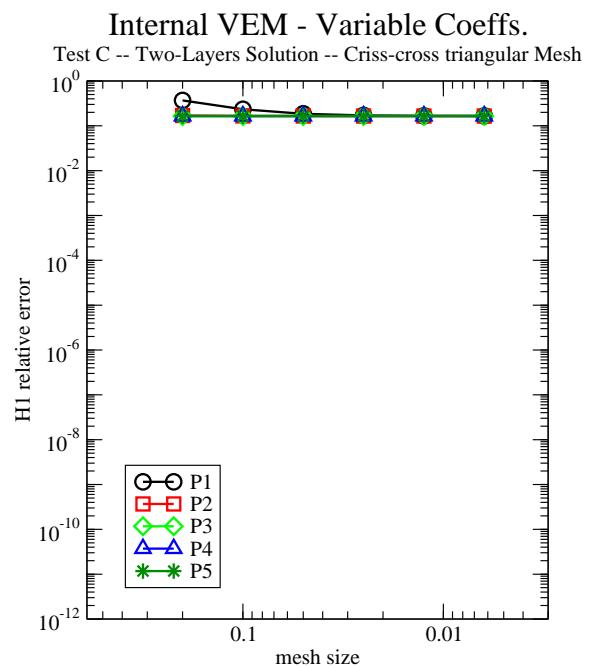
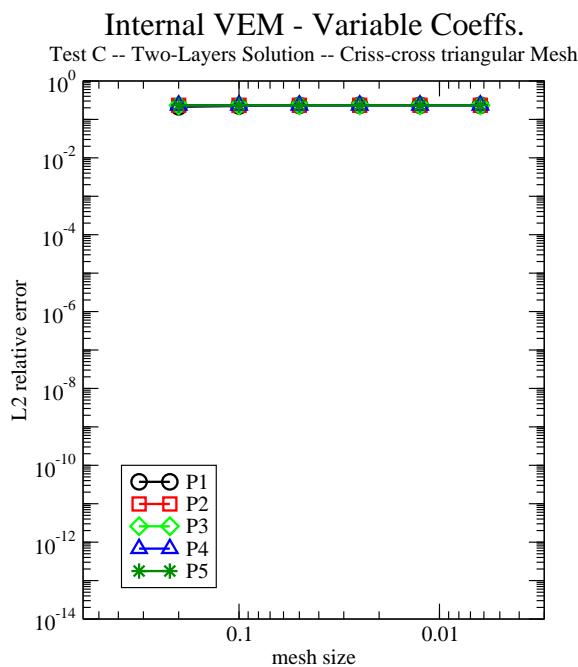


Fig. 363. Internal-internal VEM formulation with variable coefficients; Test C; two-layer solution on a mesh of regular triangular cells, (criss-cross).

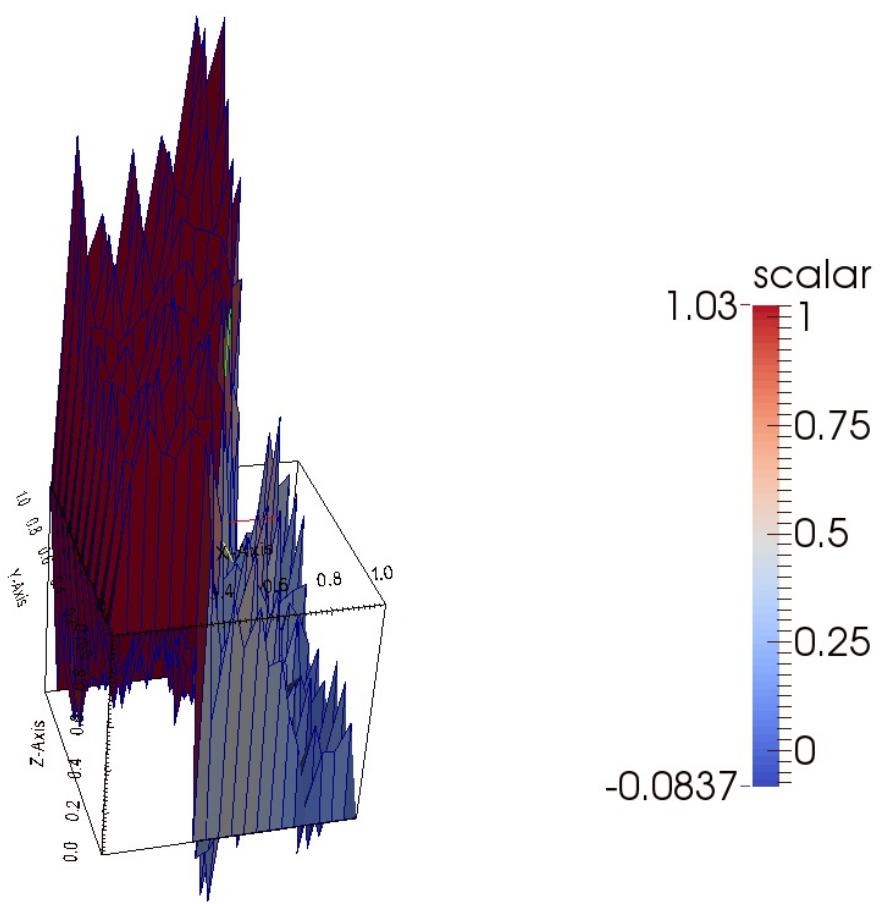


Fig. 364. External VEM for the advection-dominated test case: square mesh, no stabilization.

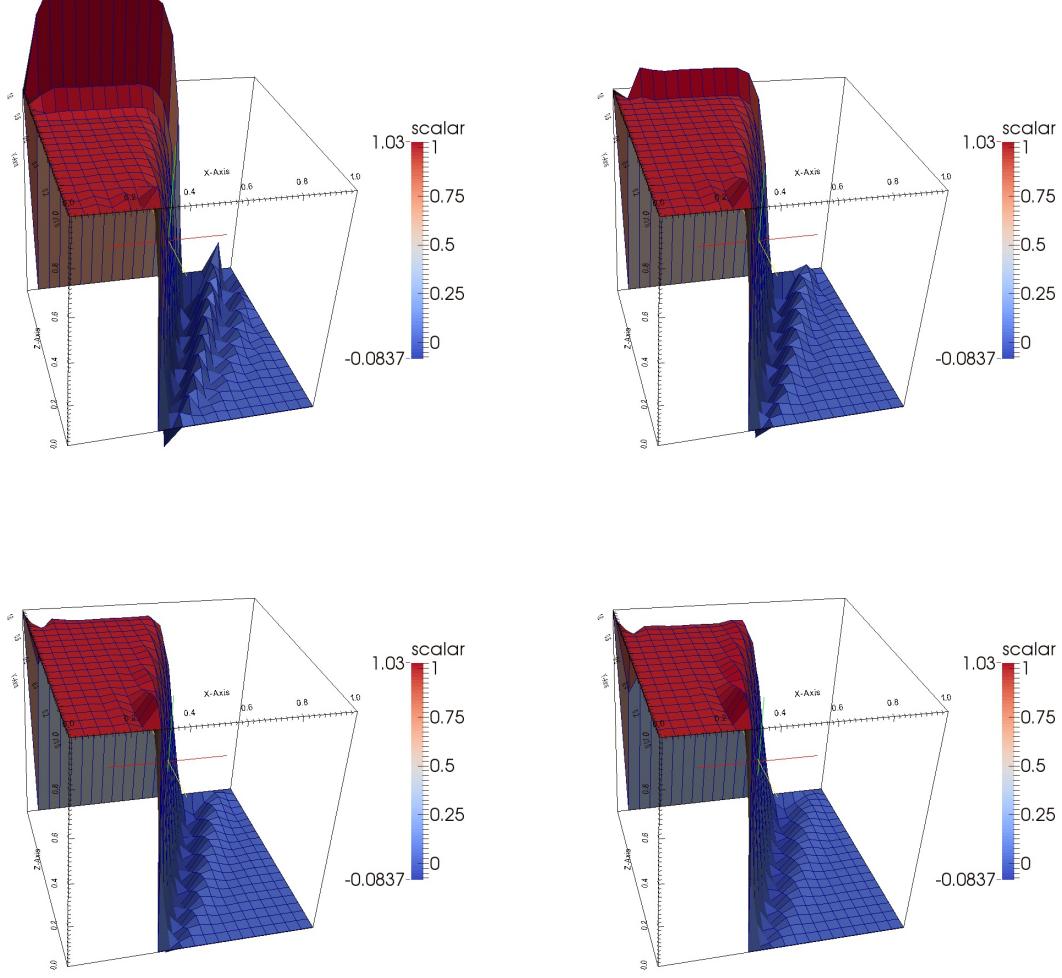


Fig. 365. External VEM for the advection-dominated test case: square mesh, streamline diffusion stabilization with  $\tau = 0.15, 0.3, 0.45, 0.6 h_E / |\beta|$  (from left to right, top to bottom), back view.

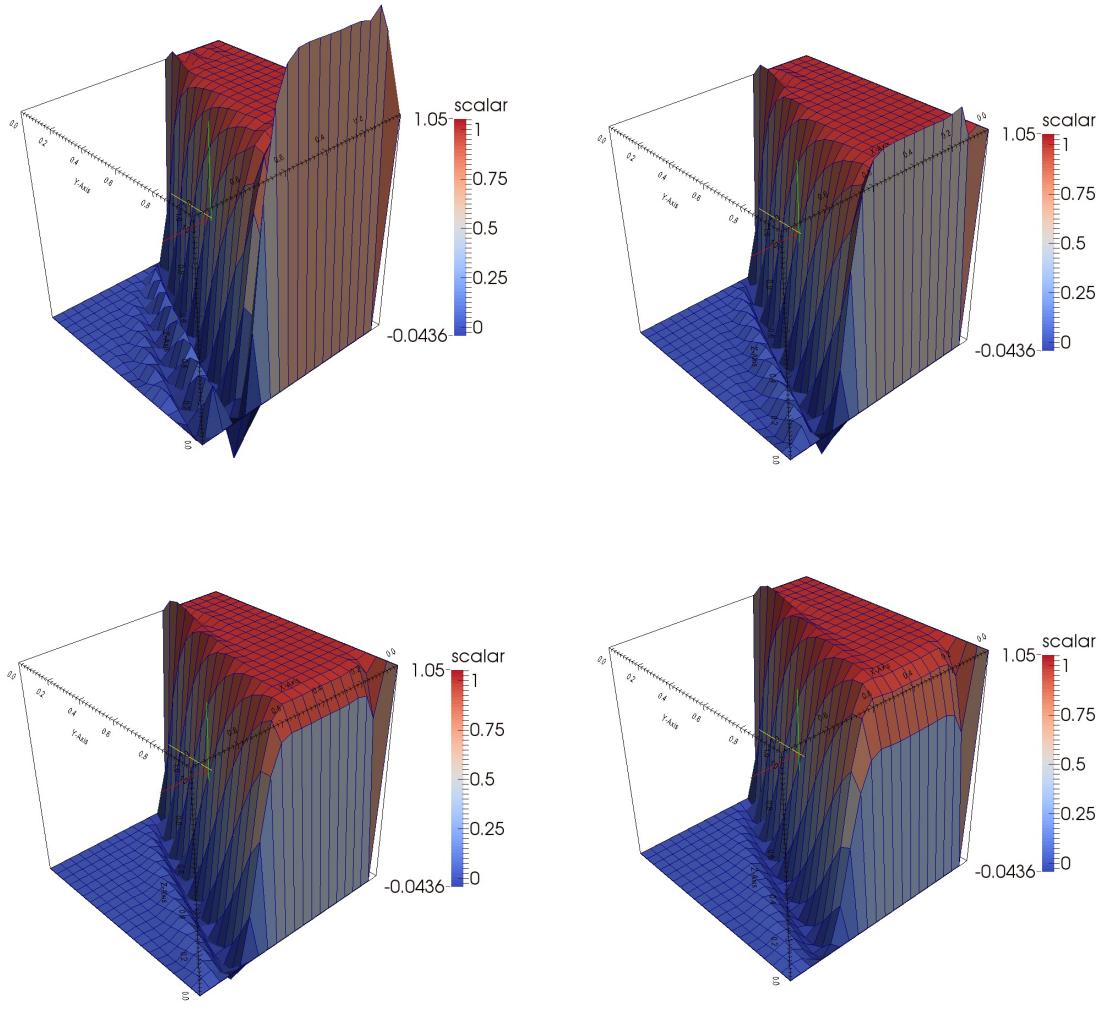


Fig. 366. External VEM for the advection-dominated test case: square mesh, streamline diffusion stabilization with  $\tau = 0.15, 0.3, 0.45, 0.6 h_E / |\beta|$  (from left to right, top to bottom), front view.

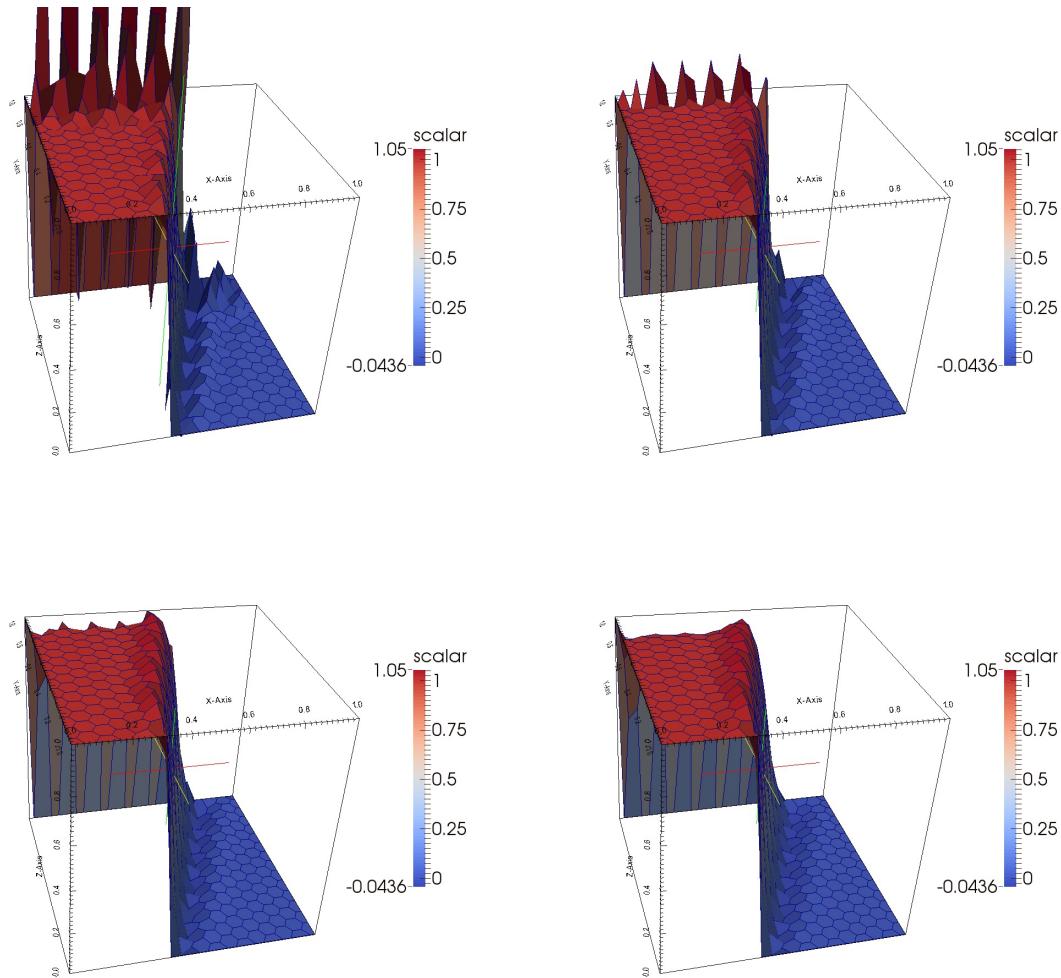


Fig. 367. External VEM for the advection-dominated test case: mesh of regular hexagons, streamline diffusion stabilization with  $\tau = 0.15, 0.3, 0.45, 0.6 h_E / |\beta|$  (from left to right, top to bottom), back view.

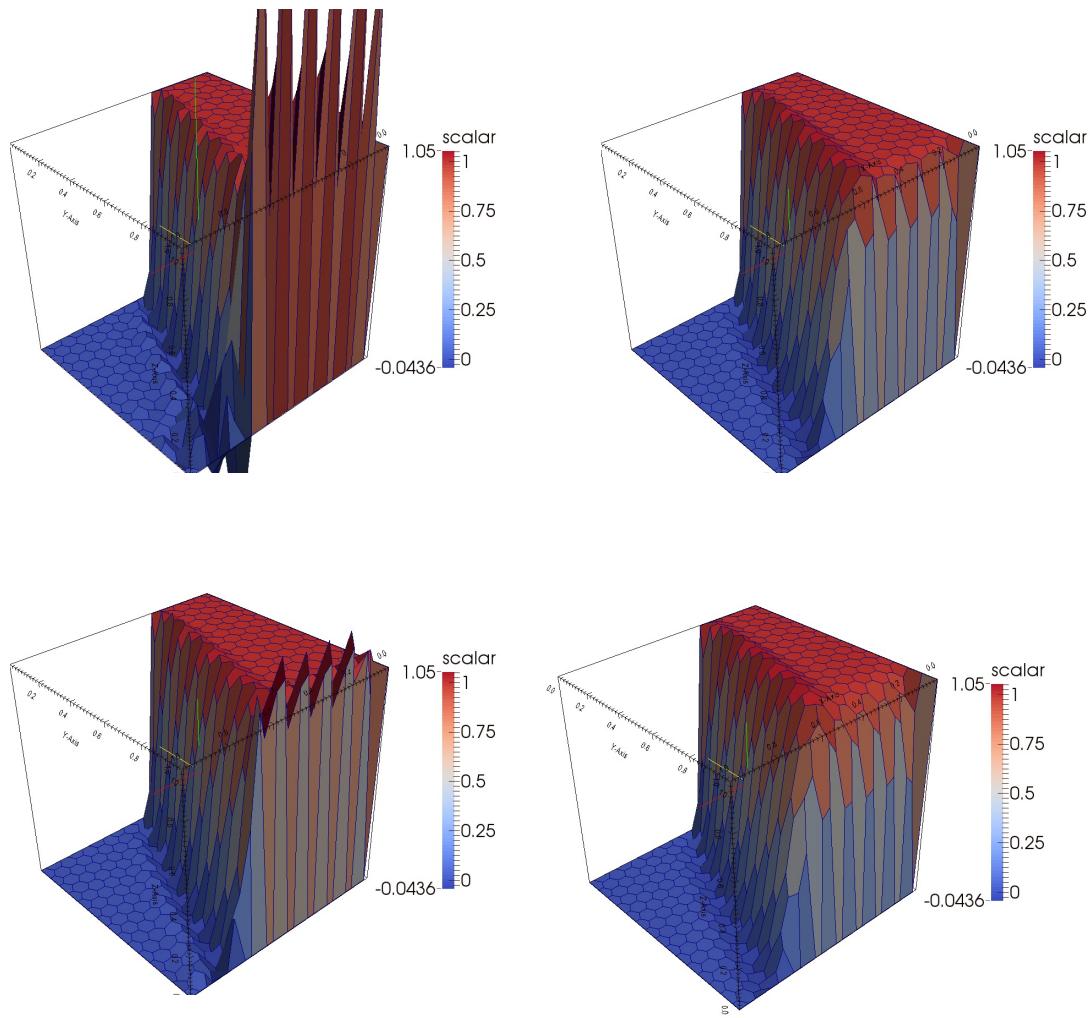


Fig. 368. External VEM for the advection-dominated test case: mesh of regular hexagons, streamline diffusion stabilization with  $\tau = 0.15, 0.3, 0.45, 0.6 h_E / |\beta|$  (from left to right, top to bottom), front view.

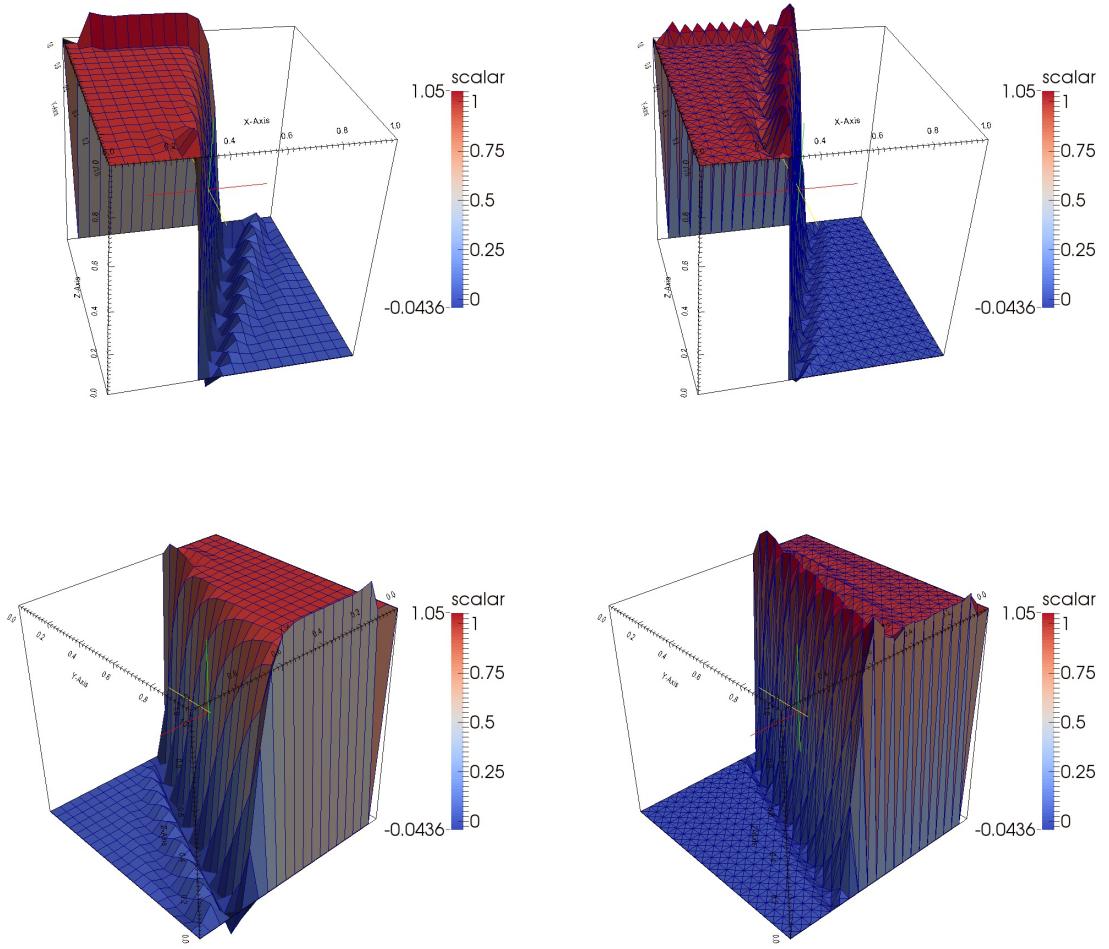


Fig. 369. External VEM for the advection-dominated test case: mesh of squares (left panels) and triangles (criss-cross) (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

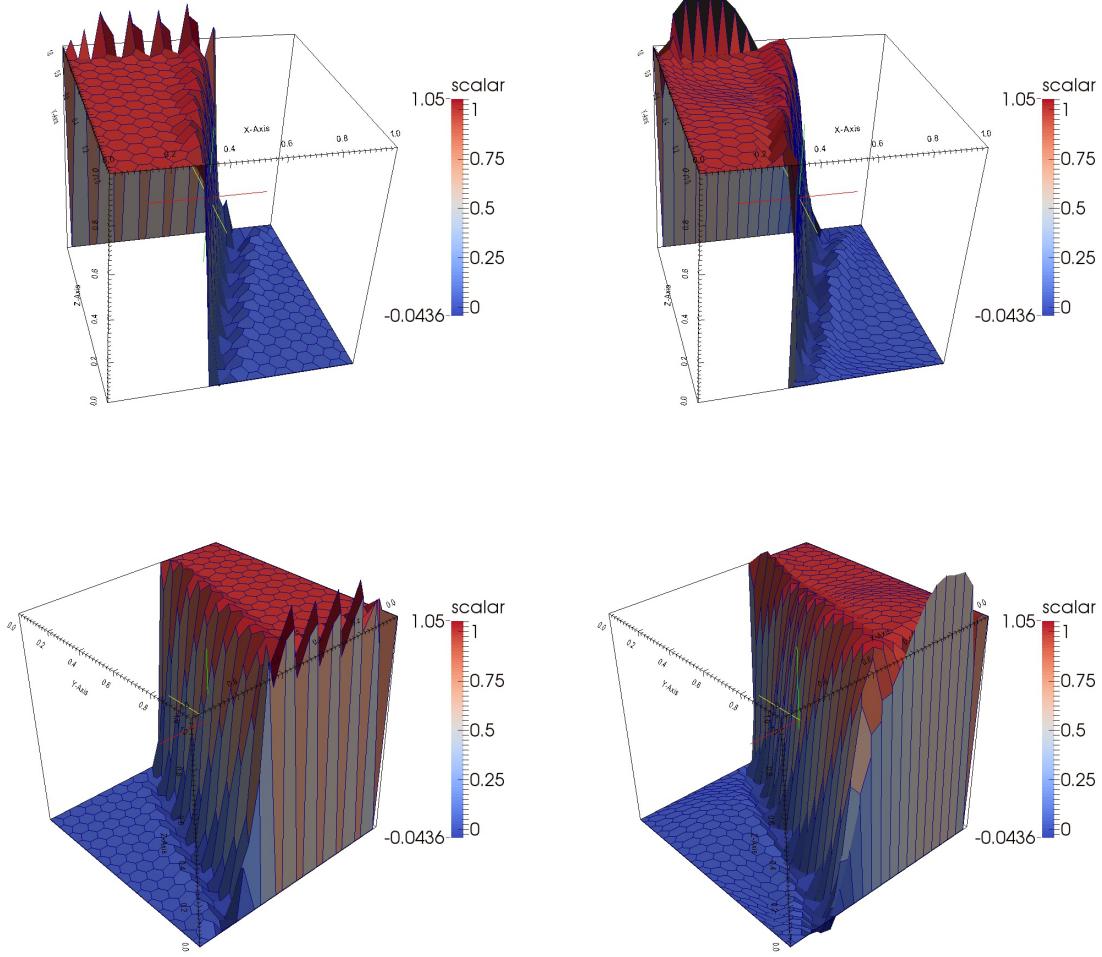


Fig. 370. External VEM for the advection-dominated test case: mesh of regular hexagons (left panels) and remapped hexagons (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

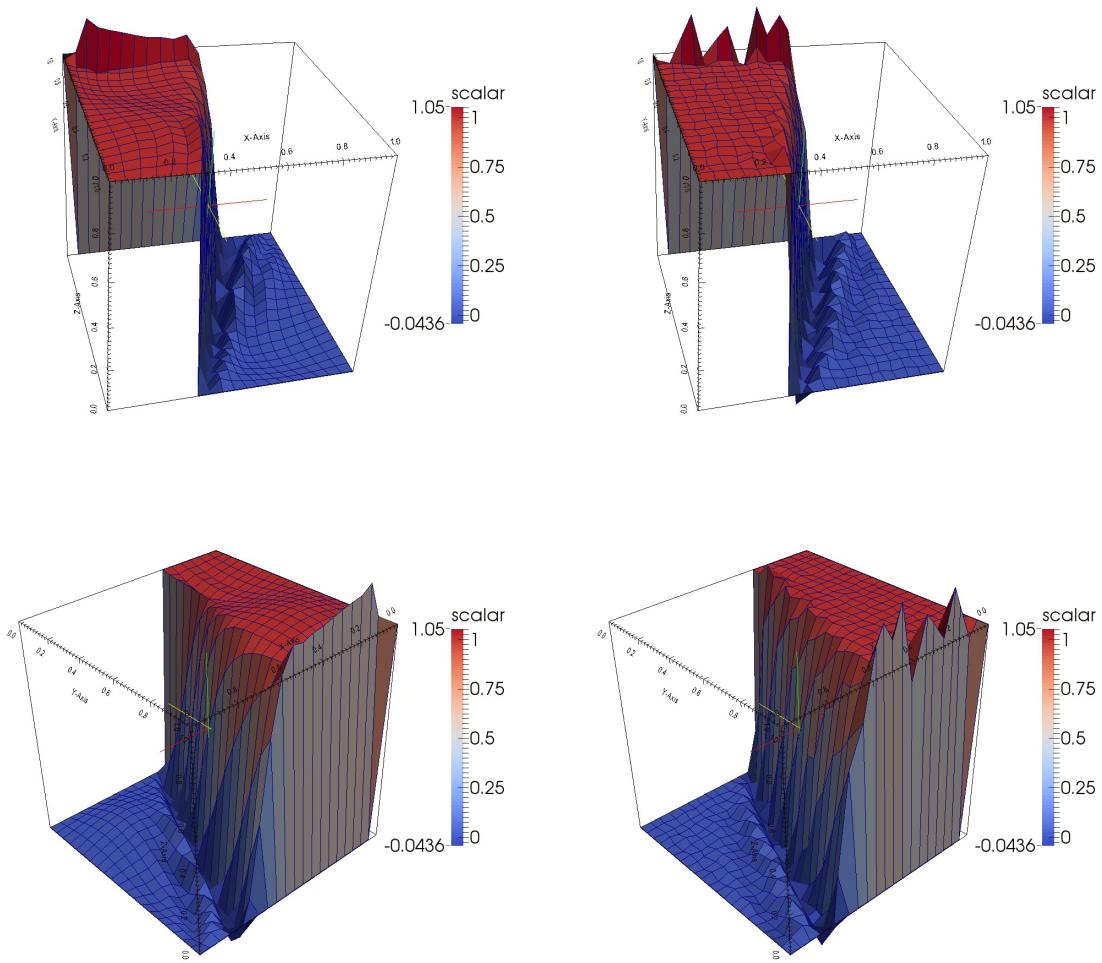


Fig. 371. External VEM for the advection-dominated test case: mesh of remapped quadrilaterals (left panels) and randomized quadrilaterals (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

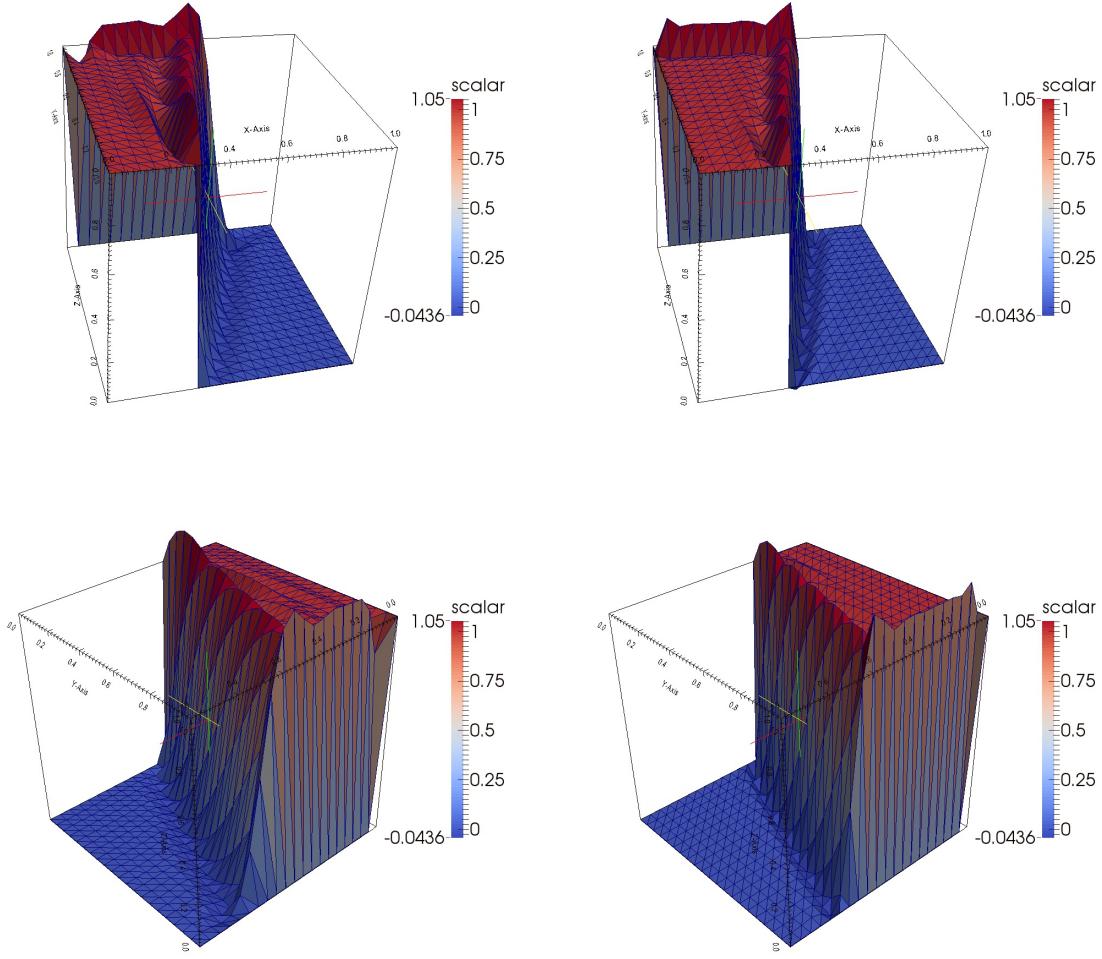


Fig. 372. External VEM for the advection-dominated test case: mesh of regular triangles (left and right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

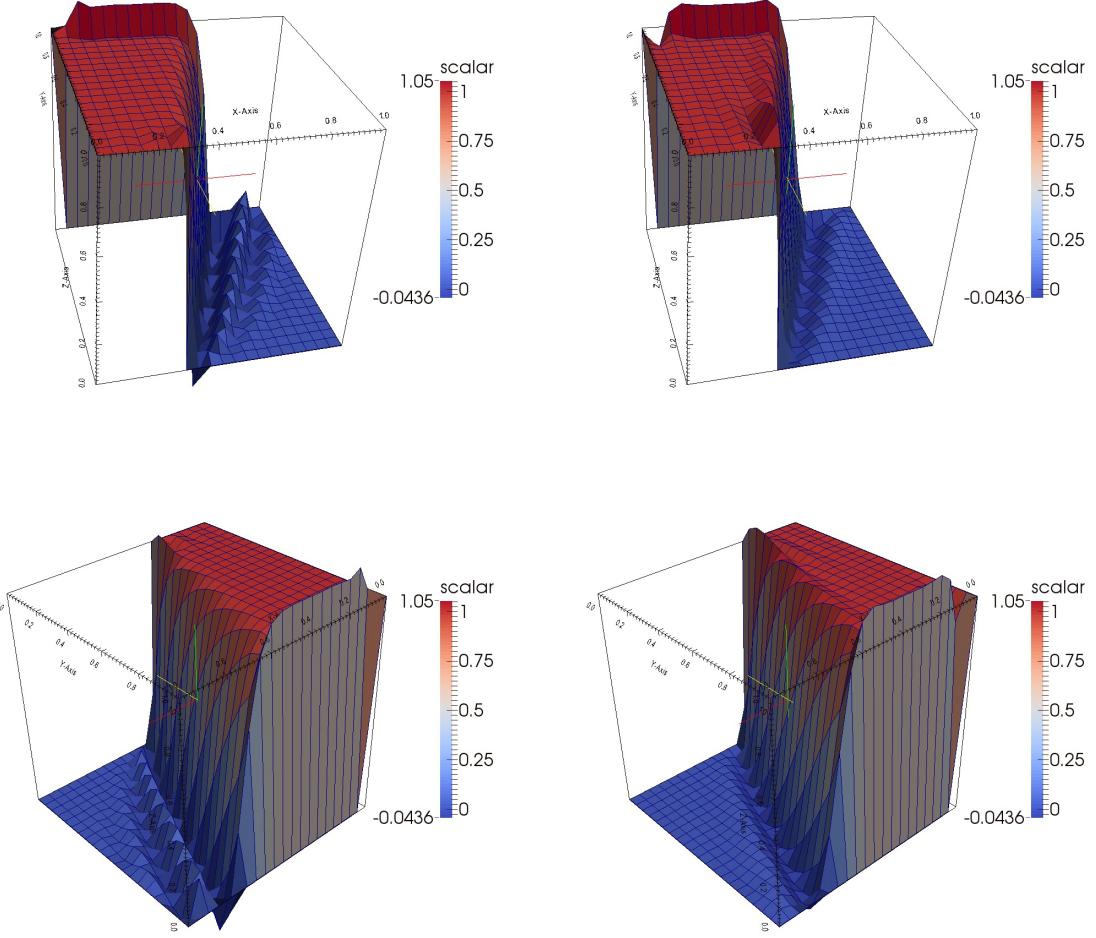


Fig. 373. External VEM for the advection-dominated test case: square mesh, streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$  and  $\tau' = 0.1 h_E / |\beta|$  (left panels) and  $\tau = 1.2 h_E / |\beta|$  (right panels); back view (top panels) and front view (bottom panels).

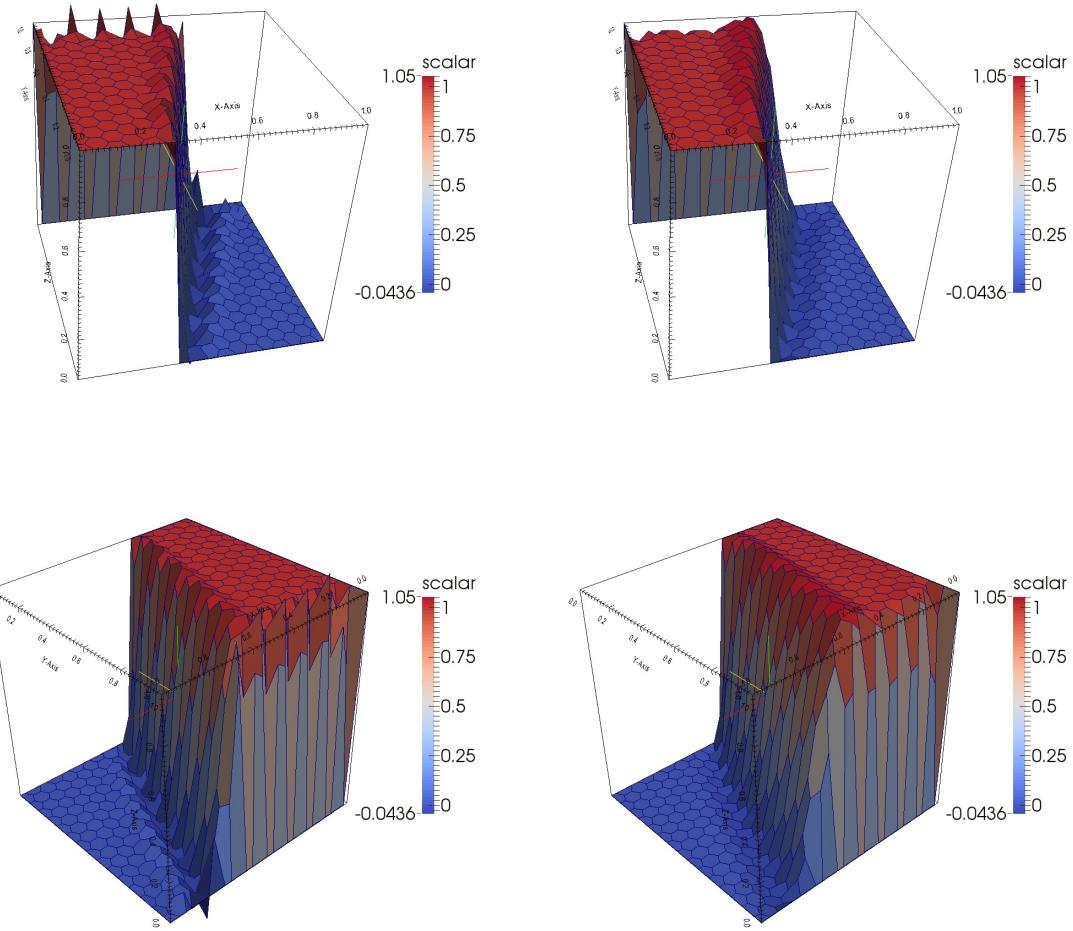


Fig. 374. External VEM for the advection-dominated test case: mesh of regular hexagons, streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$  and  $\tau' = 0.1 h_E / |\beta|$  (left panels) and  $\tau = 1.2 h_E / |\beta|$  (right panels); back view (top panels) and front view (bottom panels).

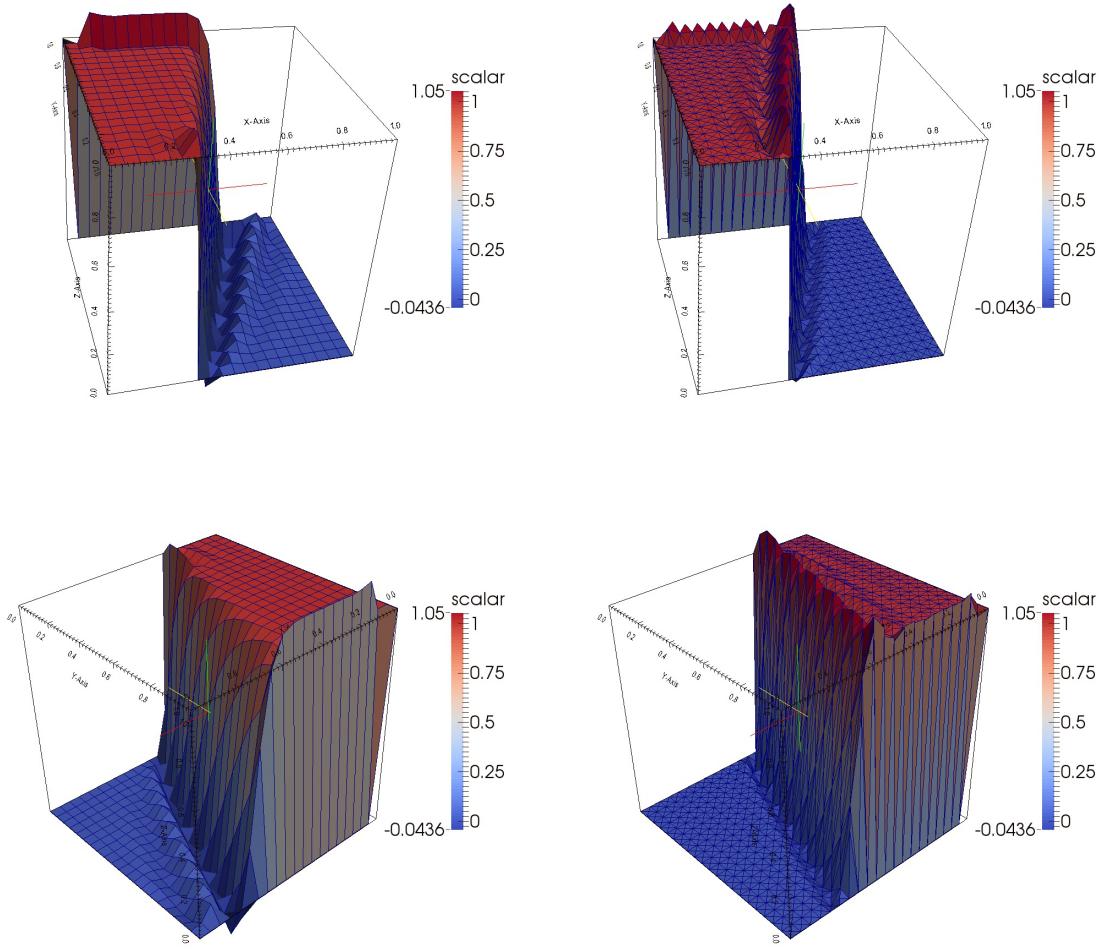


Fig. 375. Internal VEM for the advection-dominated test case: mesh of squares (left) and regular triangles (criss-cross) (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

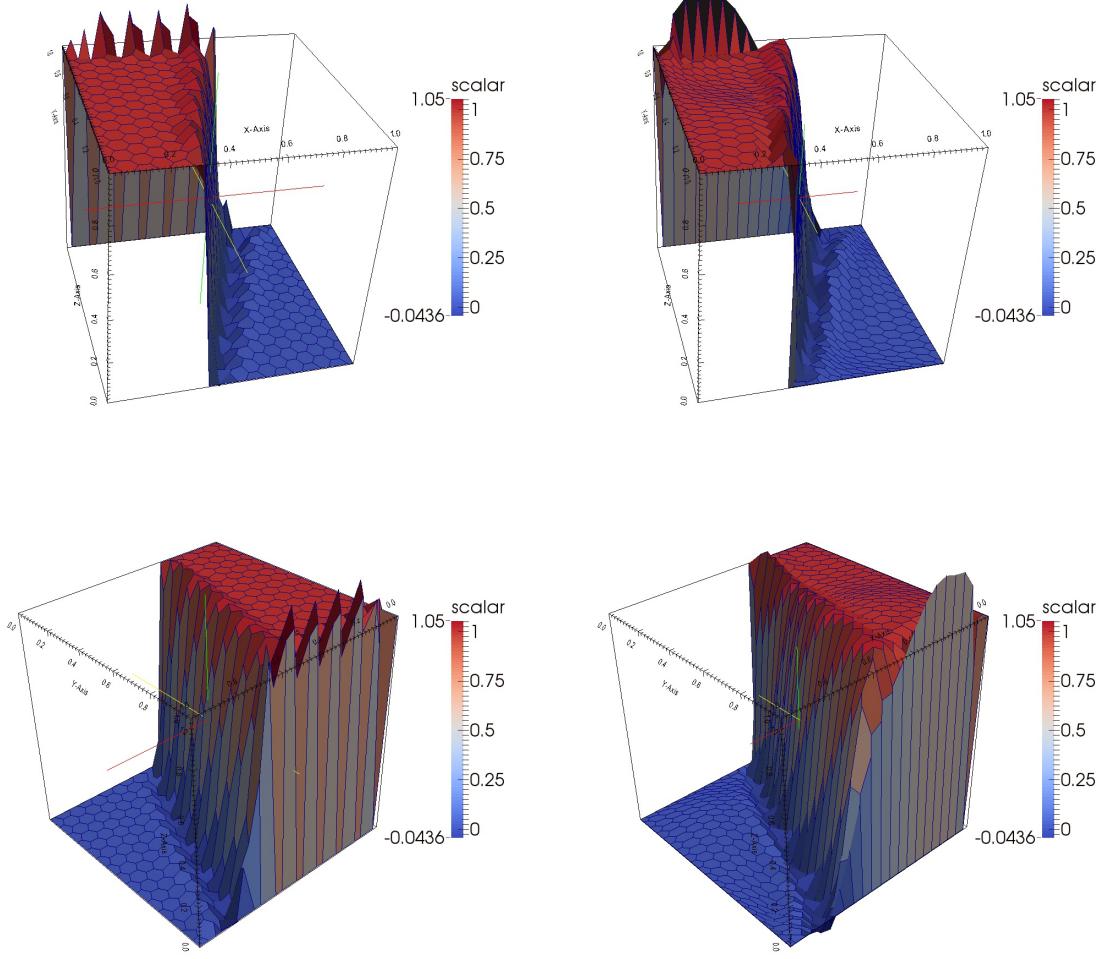


Fig. 376. Internal VEM for the advection-dominated test case: mesh of regular hexagons (left) and remapped hexagons (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

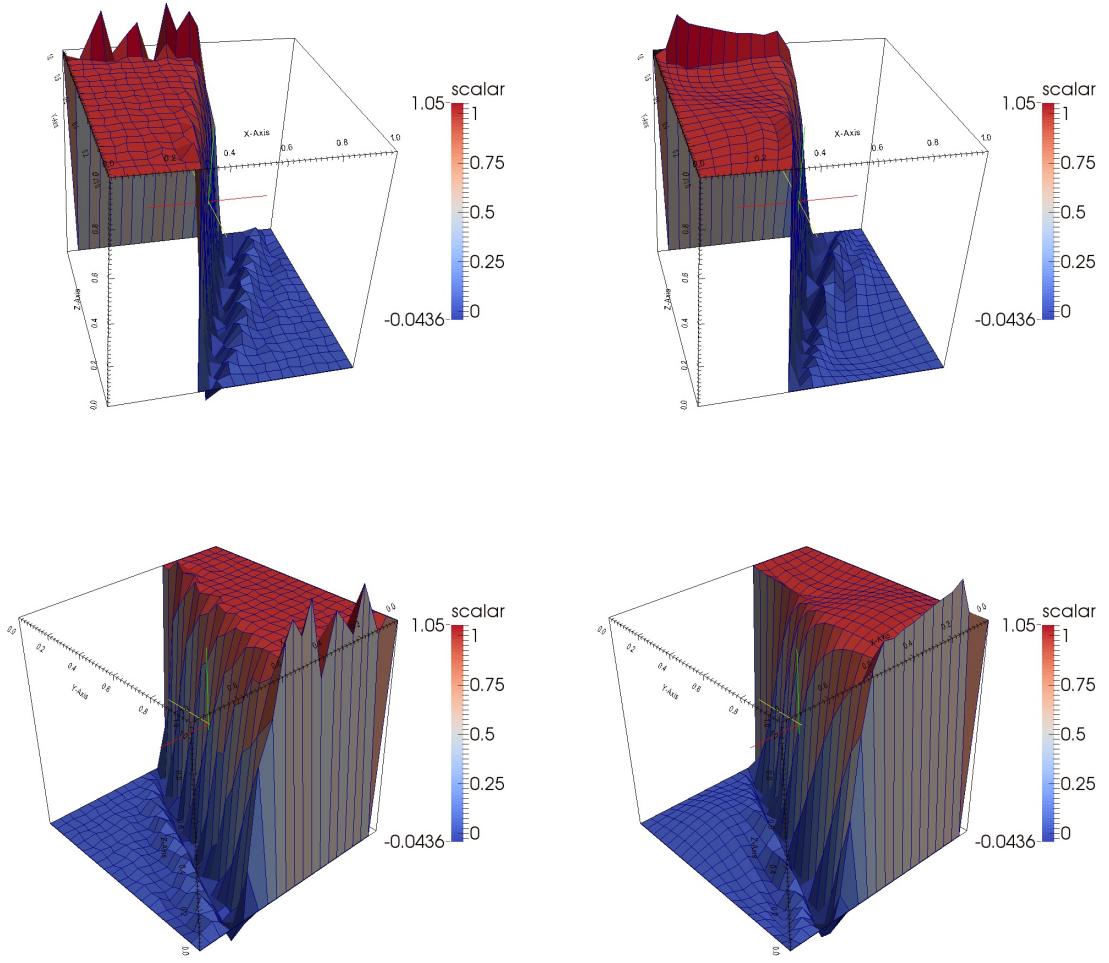


Fig. 377. Internal VEM for the advection-dominated test case: mesh of randomized quadrilaterals (left) and remapped quadrilaterals (right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).

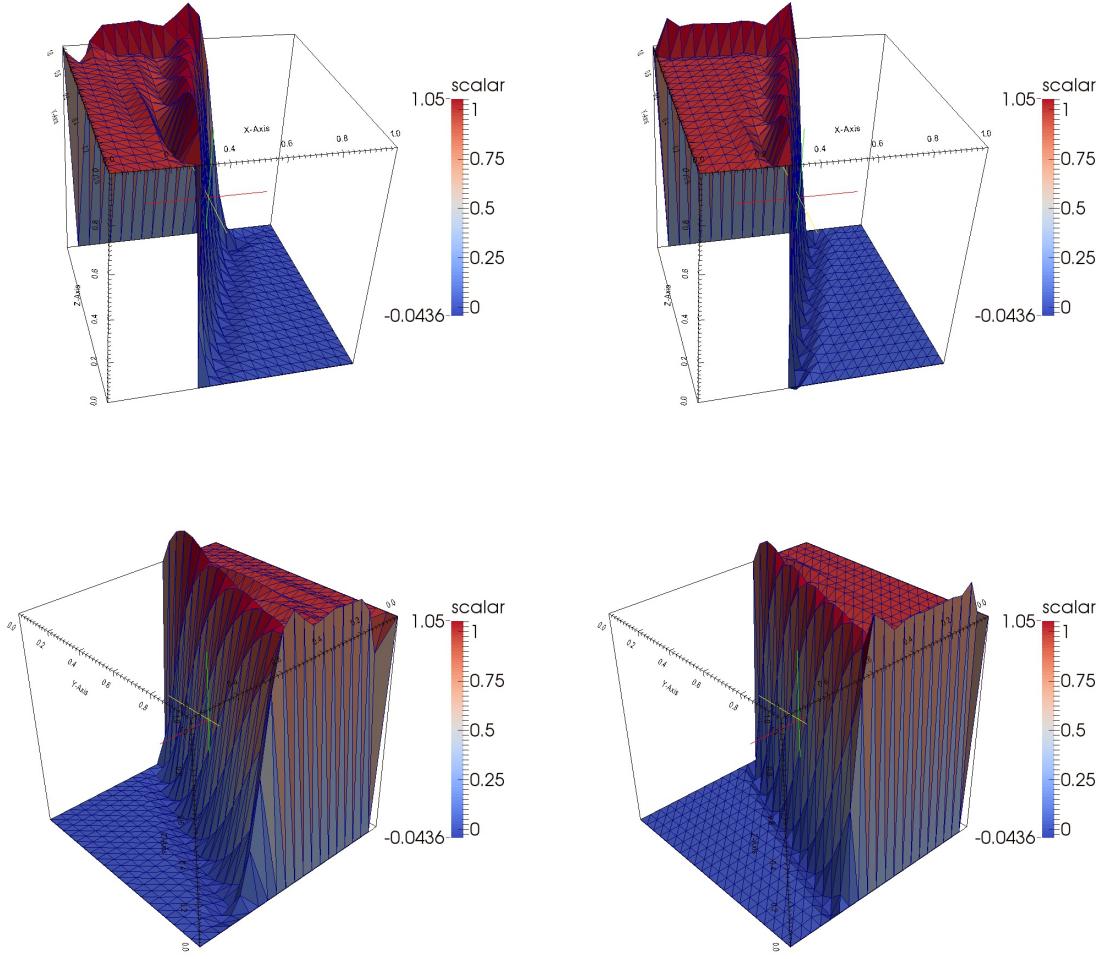


Fig. 378. Internal VEM for the advection-dominated test case: mesh of regular triangles (left and right panels), streamline diffusion stabilization with  $\tau = 0.3 h_E / |\beta|$ , back view (top panels) and front view (bottom panels).